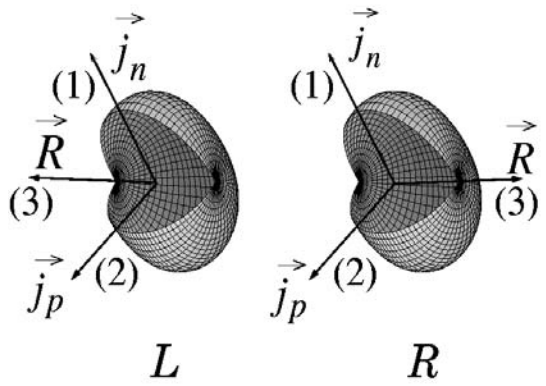
wfj-sept10

T.koike, K.Starosta and I.H., PRL, **93**, 172502 (2004)

Selection rule for electromagnetic transitions in the chiral geomtry

Ikuko Hamamoto

Mathematical physics, LTH, University of Lund, Sweden



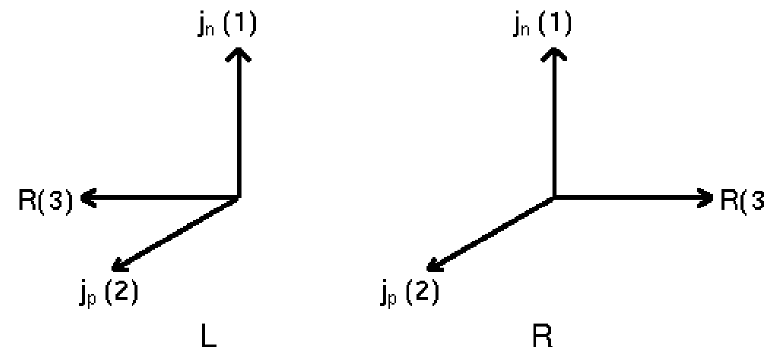
Chirality in triaxial nuclei
is characterized by the presence of
noncoplanar 3 angular-momentum vectors.

S.Frauendorf and J.Meng, Nucl.Phys., **A617**, 131 (1997)

In chiral geometry

$$\vec{I} = \vec{R} + \vec{j}_n + \vec{j}_p$$

Observed states
$$|I+\rangle = \frac{1}{\sqrt{2}}\left(|IL\rangle + |IR\rangle\right)$$
$$|I-\rangle = \frac{i}{\sqrt{2}}\left(|IL\rangle - |IR\rangle\right)$$



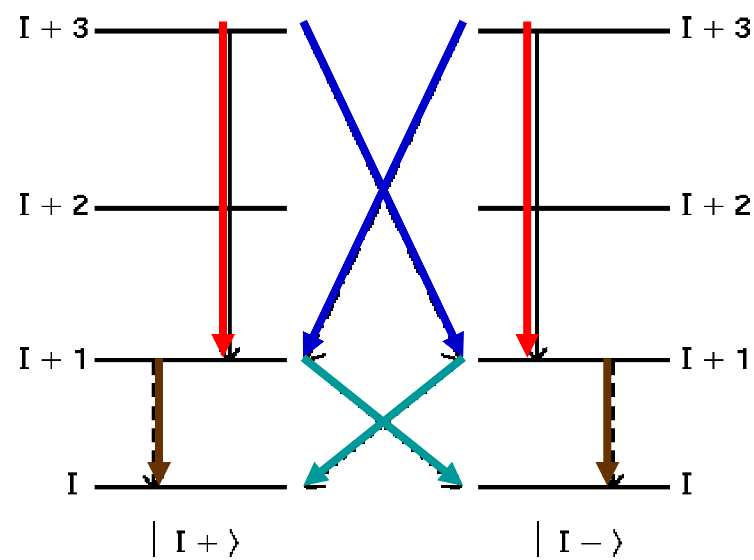
If no tunneling between L and R, then
$|I+\rangle$ and $|I-\rangle$ are degenerate.

For *I* » 1 (|***R***| » 1)

$$\langle IL|E2|IR\rangle \approx 0$$
$$\langle IL|M1|IR\rangle \approx 0$$

Then, for *EM* = *E2* or *M*1

$$B(EM; I'+ \to I+) \approx B(EM; I'- \to I-)$$
$$B(EM; I'+ \to I-) \approx B(EM; I'- \to I+)$$



(Thinking of odd-odd nuclei such as $^{130}_{55}Cs_{75}$,)

A simple particle-rotor model

Core with γ=+90° (equivalently, γ=–30° in Lund convention), assuming the γ-dependence of J_{hydro}

$$H_{rot} = \sum_k \frac{\hbar^2}{2J_k} R_k^2 \Rightarrow \frac{\hbar^2}{8J_0}\left(R_3^2 + 4(R_1^2 + R_2^2)\right)$$

Note the triaxial shape;
$\langle r_2^2 \rangle < \langle r_3^2 \rangle < \langle r_1^2 \rangle$

One proton-particle in j (=$h_{11/2}$) shell for γ=+90°

$$V_{sp}^{\pi} \propto \left[3j_3^2 - j(j+1)\right]\cos\gamma + \sqrt{3}(j_1^2 - j_2^2)\sin\gamma \;\Rightarrow \left(j_{p1}^2 - j_{p2}^2\right)$$

One neutron-hole in j (=$h_{11/2}$) shell for γ=+90°

$$V_{sp}^{\nu} \propto \left[3j_3^2 - j(j+1)\right]\cos\gamma + \sqrt{3}(j_1^2 - j_2^2)\sin\gamma \;\Rightarrow (j_{n2}^2 - j_{n1}^2)$$

Note that in respective Hamiltonians the energetically preferred directions are ;

$\vec{j}_p$ // ± 2-axis, (s-axis) $\quad \vec{j}_n$ // ± 1-axis, (l-axis) $\quad \vec{R}$ // ± 3-axis (i-axis)

However, the energy minimum for a given total angular momentum $\vec{I} = \vec{R} + \vec{j}_p + \vec{j}_n$

Triaxial one-body potential

$$V(r,\theta,\phi) = k(r)\,\beta\left\{ Y_{20}\cos\gamma + \frac{1}{\sqrt{2}}(Y_{22}+Y_{2-2})\sin\gamma \right\} \qquad (1)$$

For a single-j shell

$$(1) \longrightarrow \frac{\kappa}{j(j+1)}\left\{ \left[3 j_3^2 - j(j+1)\right]\cos\gamma + \sqrt{3}(j_1^2 - j_2^2)\sin\gamma \right\}$$

where we use, for example,

$$\frac{\langle j,\Omega \,|\, Y_{20} \,|\, j,\Omega\rangle}{\langle j,\Omega+2 \,|\, Y_{22} \,|\, j,\Omega\rangle} \propto \frac{\langle j,\Omega \,|\, 3 j_3^2 - j(j+1) \,|\, j,\Omega\rangle}{\langle j,\Omega+2 \,|\, j_+^2 \,|\, j,\Omega\rangle}$$

Invariance properties of $H=H_{rot}+V_{sp}^{\pi}+V_{sp}^{\nu}$

$$H_{rot} \propto R_3^2 + 4\left(R_1^2 + R_2^2\right)$$
$$V_{sp}^{\pi} + V_{sp}^{\nu} \propto \left(j^2_{p1} - j^2_{p2}\right) + \left(j^2_{n2} - j^2_{n1}\right)$$

1) D_2 symmetry $\rightarrow$ $R_3 = 0, \pm 2, \pm 4, \ldots\ldots$

2) Invariant under the operation *A* :

2.1) rotation $\exp\left(i(\frac{\pi}{2}) R_3\right)$ or $\exp\left(i(\frac{3\pi}{2}) R_3\right)$ combined with

2.2) charge symmetry [*C* : *n*↔*p*]

C = +1 symmetric for *n*↔*p*

- 1 anti-symmetric for *n*↔*p* ,

$\therefore$ $\boxed{\text{Eigenstates of } \boldsymbol{H} \text{ have quantum-number } A=\pm 1 .}$

Eigenstates of ***H*** are

$A=+1$ $\begin{cases} R_3 = 0, \pm 4, \pm 8, \ldots \ \& \ C = +1 \\ R_3 = \pm 2, \pm 6, \ldots \ \& \ C = -1 \end{cases}$

$A=-1$ $\begin{cases} R_3 = 0, \pm 4, \pm 8, \ldots \ \& \ C = -1 \\ R_3 = \pm 2, \pm 6, \ldots \ \& \ C = +1 \end{cases}$

Selection rule for E2 transitions

Core contributions only are considered, then, in order to have $B(E2) \neq 0$;

$$\begin{cases} \Delta C = 0 & \because) \text{ Neutron and proton are spectators.} \\ \Delta R_3 \neq 0 & \because) \ \gamma = +90^\circ \longrightarrow [\text{E2 matrix elements with } \Delta R_3 = 0] = 0 \end{cases}$$

$\therefore$

$$\boxed{B(E2) = 0 \quad \text{for } \Delta A = 0}$$

$A=+1$ $\begin{cases} R_3 = 0, \pm 4, \pm 8, \ldots \ \& \ C = +1 \\ R_3 = \pm 2, \pm 6, \ldots \ \& \ C = -1 \end{cases}$ $A=-1$ $\begin{cases} R_3 = 0, \pm 4, \pm 8, \ldots \ \& \ C = -1 \\ R_3 = \pm 2, \pm 6, \ldots \ \& \ C = +1 \end{cases}$

Selection rule for M1 transitions

$$(M1)_\mu = \sqrt{\frac{3}{4\pi}}\frac{e\hbar}{2mc}\left(\underline{(g_\ell - g_R)}\, \ell_\mu + \underline{(g_s^{eff} - g_R)}\, s_\mu \right)$$

where $\underline{g_\ell - g_R} =$ 0.5 (-0.5) and $\underline{g_s^{eff} - g_R} =$ 2.848 (-2.792)

for *p* (*n*).

Then, B(M1) ≈ 0 for $\Delta C = 0$. ∵) M1 is almost "antisymmetric" under p ↔ n

Since B(M1) = 0 for $|\Delta R_3| \geq 2$,

[B(M1) with ΔA=0] « [B(M1) with ΔA ≠ 0]

A=+1 $\begin{cases} R_3 = 0, \pm 4, \pm 8, \ldots \;\&\; C = +1 \\ R_3 = \pm 2, \pm 6, \ldots \;\&\; C = -1 \end{cases}$

A=–1 $\begin{cases} R_3 = 0, \pm 4, \pm 8, \ldots \;\&\; C = -1 \\ R_3 = \pm 2, \pm 6, \ldots \;\&\; C = +1 \end{cases}$

If ideal chiral partner bands appear in the present model

$C\,|L> \;\propto\; |R>$ and $C\,|R> \;\propto\; |L>$

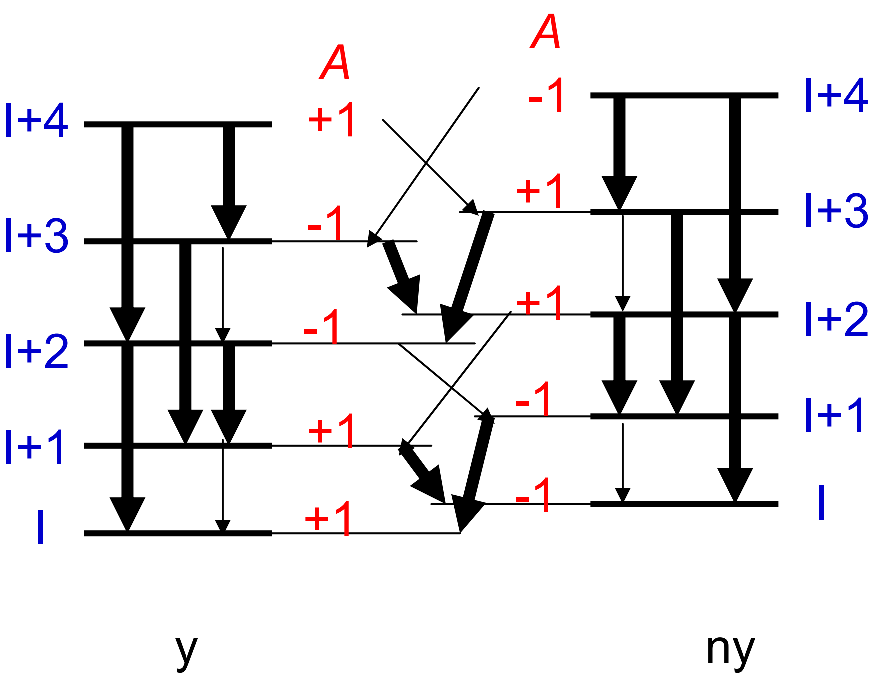
Since the rotation, $\exp\left[i\left(\frac{\pi}{2}\right)R_3\right]$ or $\exp\left[i\left(\frac{3\pi}{2}\right)R_3\right]$, does not affect chirality,

$A\,|L> \;\propto\; |R>$ and $A\,|R> \;\propto\; |L>$

Thus, two degenerate states, $|I+>$ and $|I->$, have different eigenvalues of A

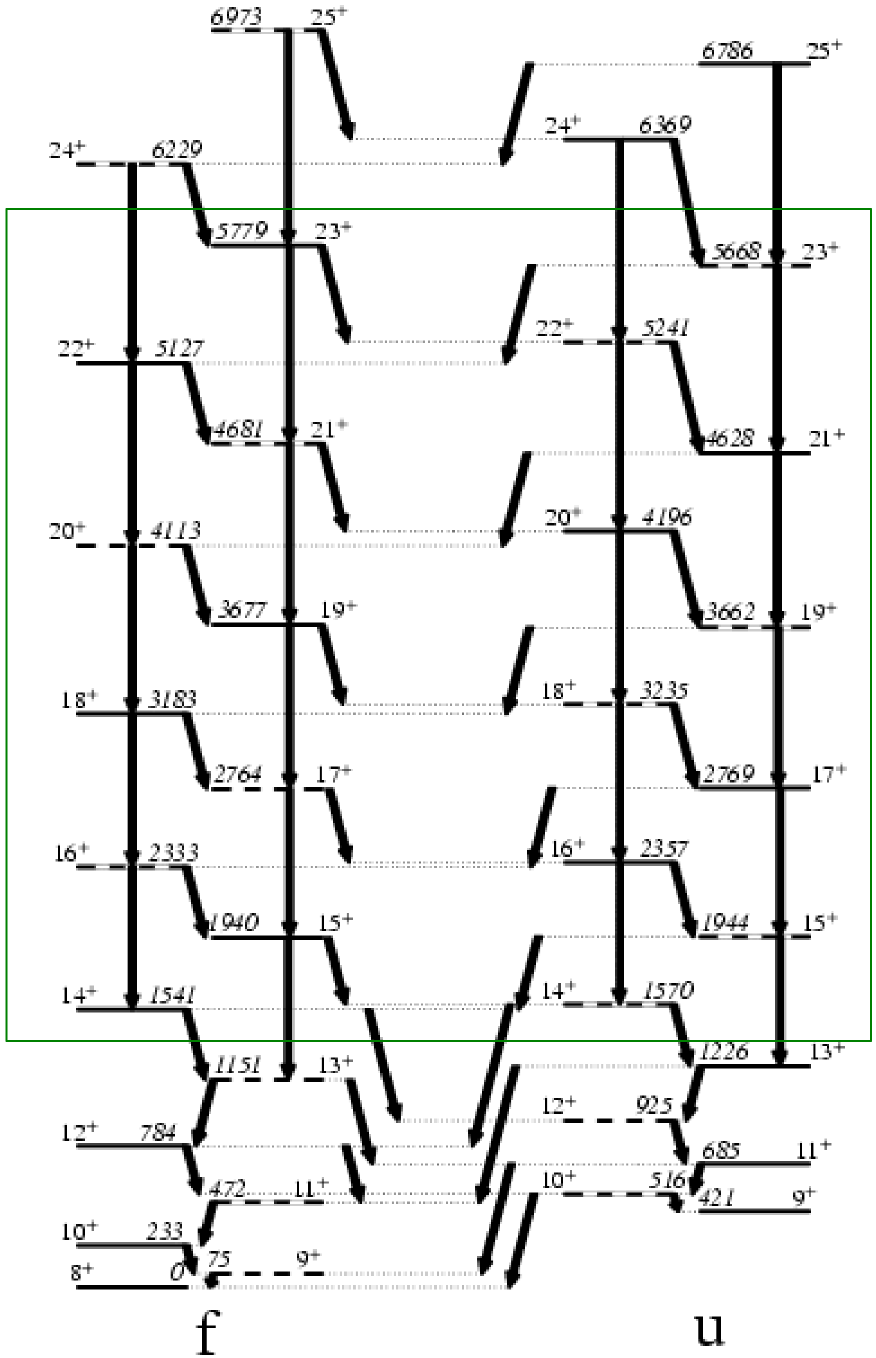
$A\,|I+> = \pm\,|I+> \quad \leftrightarrow \quad A|I-> = \mp\,|I->$

When bands are arranged so that ΔI = 2 E2 transitions are always allowed within respective bands, the sign of A in a given band must change at every increase of I by 2.



: allowed E2 (with ΔI = 1 and 2) and stronger M1 (with ΔI = 1) transitions.

: much weaker ΔI = 1 M1 transitions.



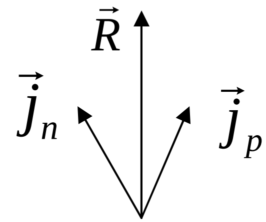
$$\vec{I} = \vec{R} + \vec{j}_n + \vec{j}_p$$

A numerical diagonalization of ***H*** with

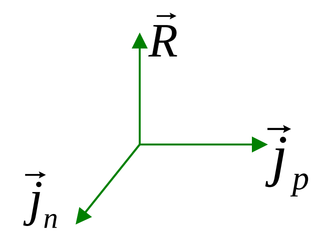
$$\begin{cases} j_n = j_p = h_{11/2} \\ J_0 = 8.55 \quad \hbar^2 \ / \ \text{MeV} \end{cases}$$

(A = 130, Z = 55, β = 0.3)

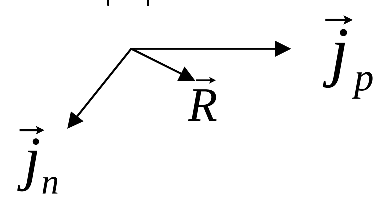
I : large ($|\vec{R}|$: large)

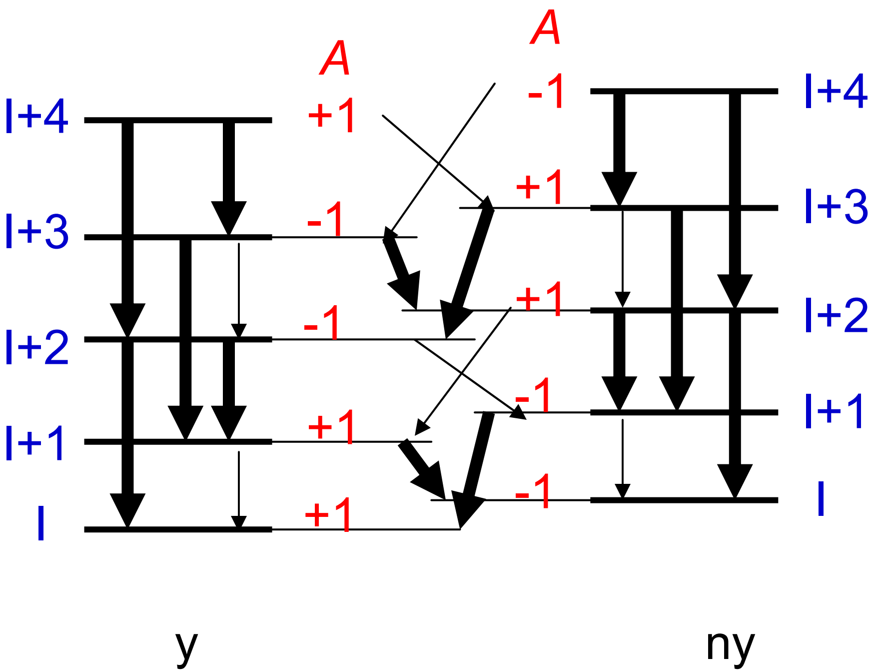


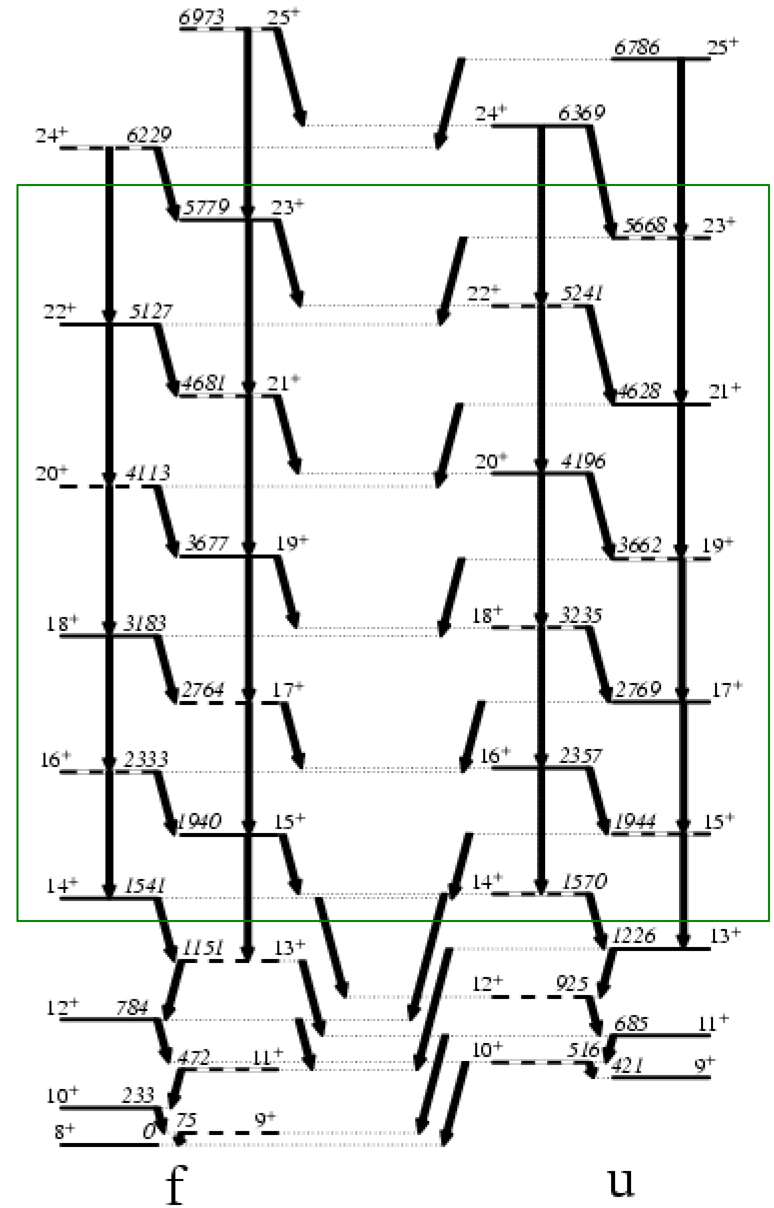
Chiral geometry ?



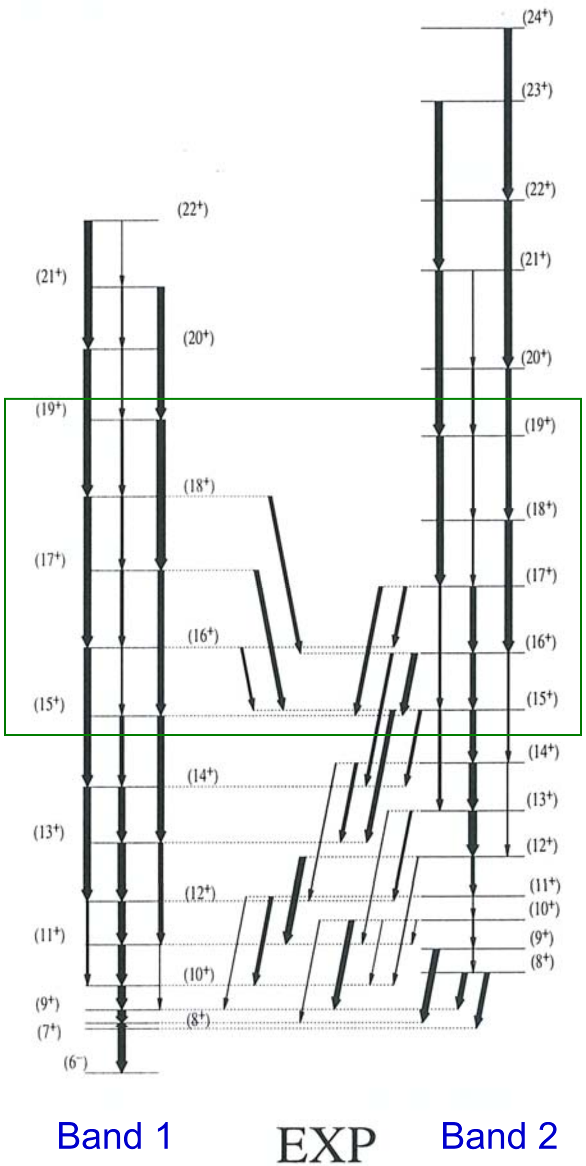
I : small ($|\vec{R}|$: small)




I+4
I+3
I+2
I+1
I
A
A
+1
-1
-1
+1
+1
+1
+1
-1
-1
I+4
I+3
I+2
I+1
I
y
ny


6973 25+
6786 25+
24+ 6229
24+ 6369
5779 23+
5668 23+
22+ 5127
22+ 5241
4681 21+
4628 21+
20+ 4113
20+ 4196
3677 19+
3662 19+
18+ 3183
18+ 3235
2764 17+
2769 17+
16+ 2333
16+ 2357
1940 15+
1944 15+
14+ 1541
14+ 1570
1151 13+
1226 13+
12+ 784
12+ 925
472 11+
685 11+
10+ 233
10+ 516
421 9+
75 9+
8+ 0
f
u

$^{134}_{59}\mathrm{Pr}_{75}$



D.Tonev et al., PRC **76**, 044313 (2007)

In the “partner” bands;

(1) Measured B(E2: I→I–2) values differ by a factor of 2.

(2) Measured inband B(M1) values are large, while the interband B(M1) values are small.

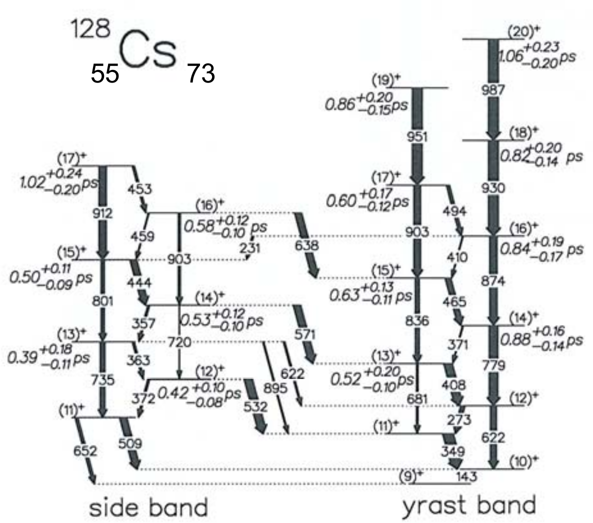
Petrache, Hagemann, Hamamoto and Starosta, PRL **96**, 112502 (2006)

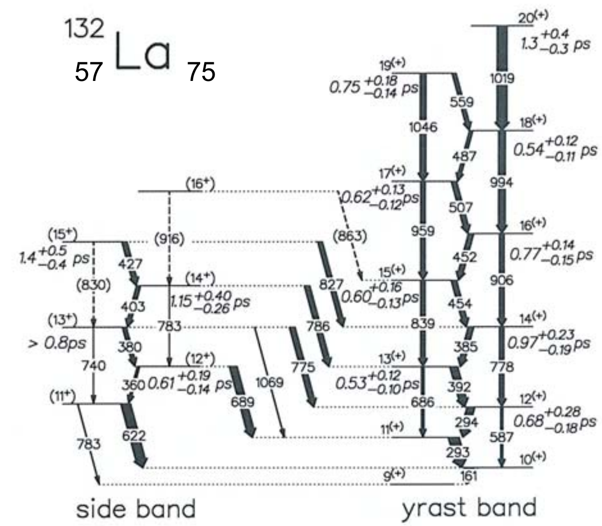
In a totally independent study with other data:

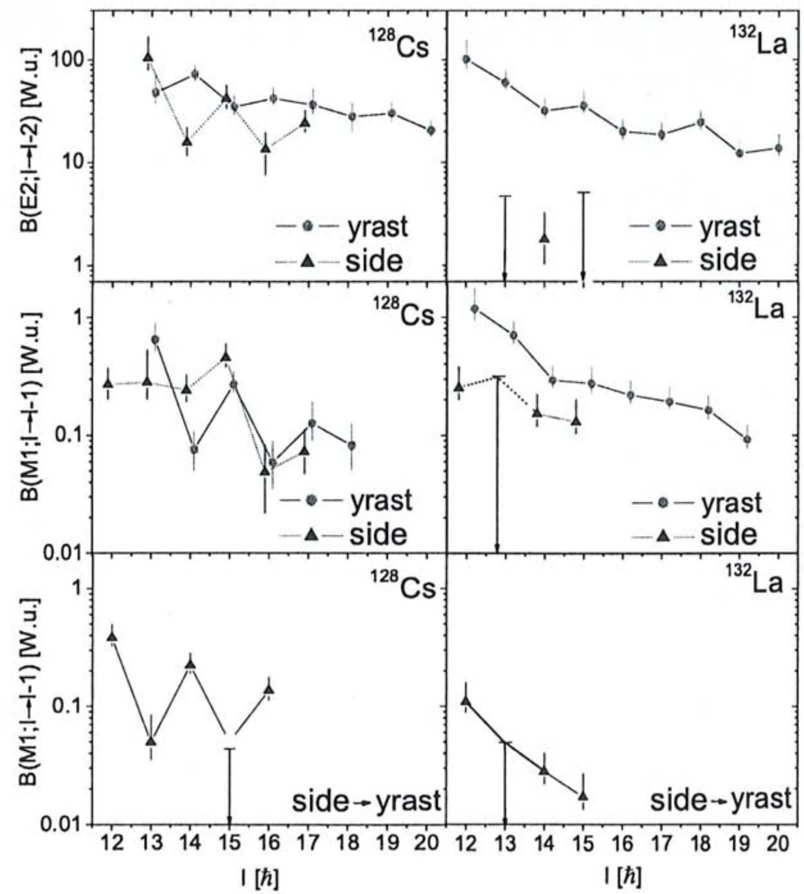
$Q_0(1) / Q_0(2) = 2.0 \pm 0.4$

is obtained from the analysis of the two-bands crossing at I = 15-16, using measured $B(E2, I\to I\text{-}2)_{out} / B(E2, I\to I\text{-}2)_{in}$ values.

Used data : GS2K009 Collaboration; K.Starosta et al., AIP Conf. Proc. No.610 (AIP, New York, 2002), p.815







E.Grodner et al., PRL, **97**, 172501 (2006)

C.Petrache, G.B.Hagemann, I.H. and K.Starosta, PRL, **96**, 112502 (2006)

Analysis of $^{134}Pr_{75}$ data around I = 14 - 18

1) Energies

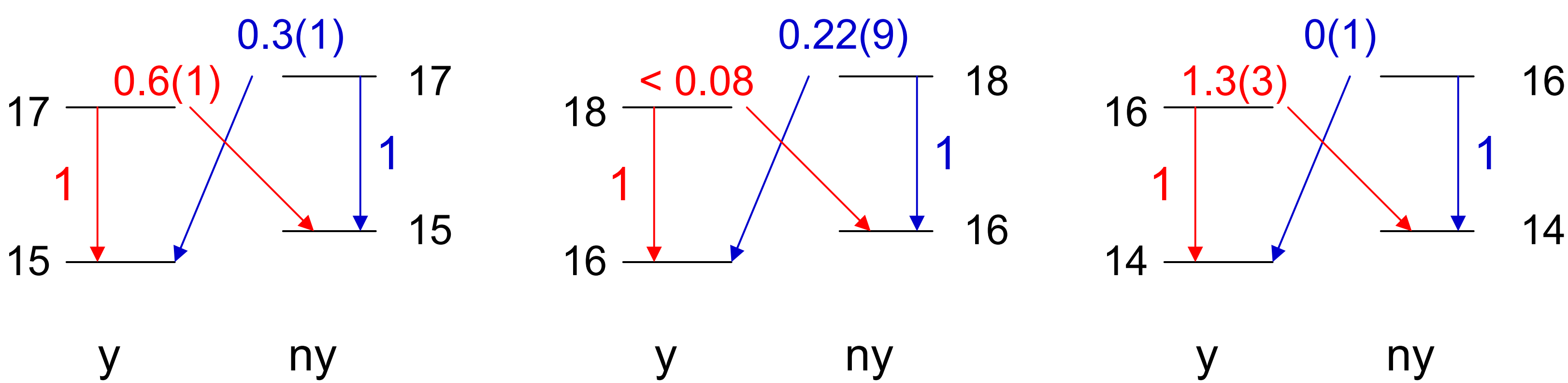
I	14	15	16	17	18
$E(I)_{ny} - E(I)_{y}$ in keV	163	36	44	118	173

2) In the case of two-bands (bands 1 and 2) mixing, the B(E2) ratios

$$\frac{B(E2, I \rightarrow I\text{-}2)_{out}}{B(E2, I \rightarrow I\text{-}2)_{in}}$$

should be equal for the yrast and non-yr stats, if $Q_{0,1} = Q_{0,2}$

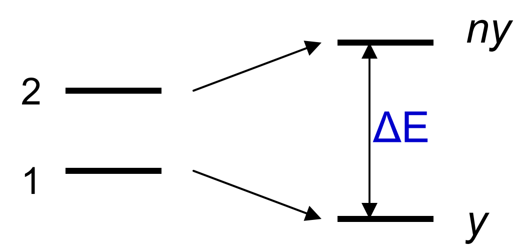
In contrast, measured values (K.Starosta et al.) are



3) Assuming two-bands (1 and 2) crossing,

$$|y\rangle = \alpha_I |1\rangle + \sqrt{1-\alpha_I^2}\,|2\rangle \qquad \text{(a)}$$

$$|ny\rangle = \sqrt{1-\alpha_I^2}\,|1\rangle - \alpha_I |2\rangle \qquad \text{(b)}$$



We obtain

$$\frac{B[E2, I^{ny} \to (I-2)^{y}]}{B[E2, I^{ny} \to (I-2)^{ny}]} = \left(\frac{Q_{0,1}\alpha_{I-2}\sqrt{1-\alpha_I^2} - Q_{0,2}\alpha_I\sqrt{1-\alpha_{I-2}^2}}{Q_{0,1}\sqrt{1-\alpha_{I-2}^2}\sqrt{1-\alpha_I^2} + Q_{0,2}\alpha_I\alpha_{I-2}} \right)^2 \qquad \text{(c)}$$

$$\frac{B[E2, I^{y} \to (I-2)^{ny}]}{B[E2, I^{y} \to (I-2)^{y}]} = \left(\frac{Q_{0,1}\alpha_I\sqrt{1-\alpha_{I-2}^2} - Q_{0,2}\alpha_{I-2}\sqrt{1-\alpha_I^2}}{Q_{0,1}\alpha_I\alpha_{I-2} + Q_{0,2}\sqrt{1-\alpha_I^2}\sqrt{1-\alpha_{I-2}^2}} \right)^2 \qquad \text{(d)}$$

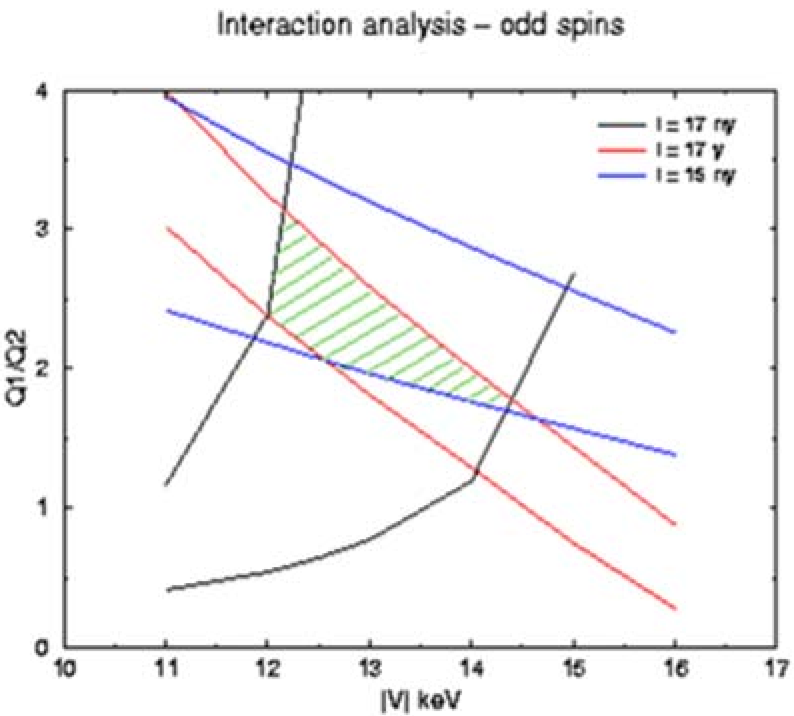
On the other hand, for a given I

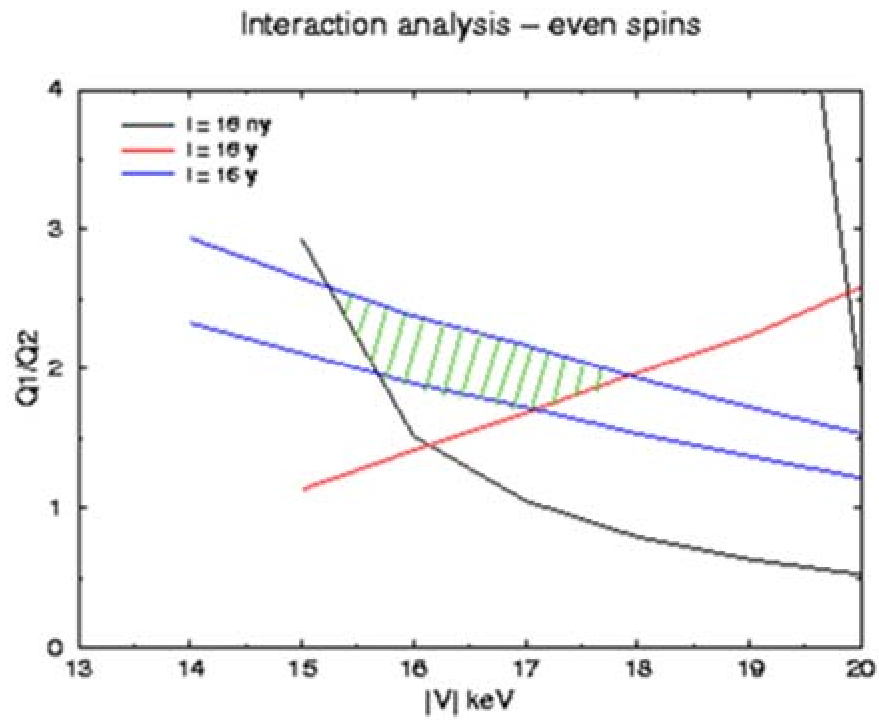
$$\frac{2V}{\Delta E_I} = \sin\left(2\sin^{-1}(\alpha_I)\right) \qquad \text{(e)}$$

where V : interaction strength between bands 1 and 2,
ΔE_I : observed level spacing for I

$$\left.\begin{array}{l}\text{Observed B(E2) ratios, (c) and (d)}\\ \text{Observed } \Delta E_I \text{ in (e)}\end{array}\right\} \longrightarrow \left\{\begin{array}{l} V,\ \alpha_I \\ (Q_{0,1}/Q_{0,2})\end{array}\right.$$

Relatively large ambiguity in measured B(E2) ratios !





V ~ 13.5 keV

V ~ 16.5 keV

$Q_{0,1} / Q_{0,2} = 2.0 \pm 0.4$

We have obtained a possible range of V and $Q_{0,1}/Q_{0,2}$

V ~ 13.5 keV for odd I bands
V ~ 16.5 keV for even I bands
$Q_{0,1}/Q_{0,2} = 2.0 \pm 0.4$

Comparison of exp and calculated values of $B(E2, I \rightarrow I-2)_{out}/B(E2, I \rightarrow I-2)_{in}$ in the crossing region of the pair bands in ^{134}Pr.
Using $Q_{0,1}/Q_{0,2} = 2$, we obtain

I	$E_{\gamma,out}$ (keV)	$E_{\gamma,in}$ (keV)	$B(E2)_{out}/B(E2)_{in}$ Exp.	$B(E2)_{out}/B(E2)_{in}$ Calc.	|V| (keV)	α_I
17^{ny}	991	1027	0.3(1)	0.30	13.5	0.114
17^{y}	909	873	0.6(1)	0.61	13.5	-
15^{y}	671	892	0(1)	0.030	13.5	0.905
15^{ny}	928	707	1.1(5)	0.79	13.5	-
18^{ny}	1113	1069	0.22(9)	0.16	16.5	0.096
18^{y}	896	940	< 0.08	0.060	16.5	-
16^{ny}	813	976	0(1)	0.11	16.5	0.411
16^{y}	933	770	1.3(3)	1.23	16.5	-

Exp data from K.Starosta et al.

$V \sim 15$ keV $\longrightarrow$ a large difference in the structure of the two bands ;

ex. a large shape difference ?
their chiral character ?
a difference in some other quantum numbers ?

Obs. $V \sim 15$ keV is obtained in many cases from the crossing between SD and ND bands in the decay-out region for A ~ 130 nuclei. (Gudrun)

$Q_{0,1} / Q_{0,2} \sim 2.0 \longrightarrow$ the shapes of nearly deg. bands in ^{134}Pr are very different.

Thus, they cannot be interpreted as chiral bands.

For a given I ; $\left| y \right\rangle = \alpha \left| 1 \right\rangle + \sqrt{1-\alpha^2} \left| 2 \right\rangle$

$$\left| ny \right\rangle = \sqrt{1-\alpha^2} \left| 1 \right\rangle - \alpha \left| 2 \right\rangle$$

Writing interaction matrix element *V*,

$$\left(H_{11} - E_y \right)\alpha + V\sqrt{1-\alpha^2} = 0$$

$$V\alpha + \left(H_{22} - E_y \right)\sqrt{1-\alpha^2} = 0$$

$$\left(H_{11} - E_{ny} \right)\sqrt{1-\alpha^2} - V\alpha = 0$$

$$V\sqrt{1-\alpha^2} - \left(H_{22} - E_{ny} \right)\alpha = 0$$

One obtains

$$E_{y/ny} = \frac{1}{2}\left[\left(H_{11} + H_{22} \right) \mp \sqrt{\left(H_{11} - H_{22} \right)^2 + 4V^2} \right]$$

and

$$\frac{H_{11} - H_{22}}{V} = -\frac{\sqrt{1-\alpha^2}}{\alpha} + \frac{\alpha}{\sqrt{1-\alpha^2}} = -\frac{\cos(x)}{\sin(x)} + \frac{\sin(x)}{\cos(x)} = \frac{-2}{\tan(2x)}$$

where $x \equiv \sin^{-1}(\alpha)$

Then,

$$\frac{2V}{E_{ny} - E_y} = \sin(2x) = \sin\left(2\sin^{-1}(\alpha) \right)$$