Multi-reference Energy Density Functional method

Constraints from consistency requirements

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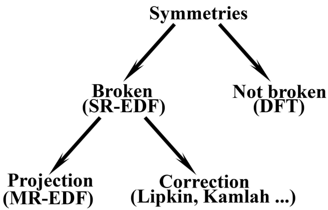
Outline

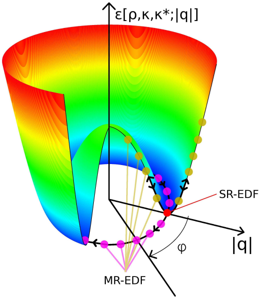
1. Introduction
   - Differences between wave-function and EDF methods
2. Constraining the MR-EDF method
   - Basic consistency requirements
   - Unexpected pathologies
   - Regularization method
   - New constraints?
3. Perspectives

Talk in one slide

Context

1. Two-step nuclear EDF method (i) single-reference (ii) multi-reference
2. Built by analogy with wave-function based methods (no existence theorem)
3. SR-EDF has both similarities and differences with (standard) DFT
4. Strongly relies on spontaneous symmetry breaking (SR) and restoration (MR)





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Take-away message

1. MR-EDF tackles long-range fluctuations and accesses collective excitations
2. MR-EDF calculations must be constrained through consistency requirements

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Ingredients of the nuclear EDF method

Two-level variational wave-function method

1^{st} level: HFB	2^{nd} level: projected HFB + GCM
Trial WF: $\vert\Phi_q\rangle = \prod_\mu \beta^q_\mu \vert 0\rangle$	Trial WF: $\vert\Psi\rangle = \sum_q f_q \vert\Phi_q\rangle = \sum_{\vert q\vert} g_{\vert q\vert} P_X \vert\Phi_{\vert q\vert}\rangle$
Sym. break. $q = \vert q\vert e^{i\varphi} \neq 0$	Sym. restor. $(\sum_\varphi)$ and zero-point fluct. $(\sum_{\vert q\vert})$

$$[\mathbf{X},\mathbf{H}] = 0 \text{ for } \{\mathbf{X}\} = \{\mathbf{N},\mathbf{Z},\mathbf{P},\mathbf{J^2},\mathbf{J_z},\mathbf{T^2},\mathbf{T_z},\mathcal{T}^2\}$$

Static collective correlations	*Dynamical* collective correlations
$E^{1^{\text{st}}}_{\vert q\vert} = \langle\Phi_q\vert H\vert\Phi_q\rangle$ $\Downarrow$ Standard Wick Theorem $\Downarrow$ $\langle\Phi_q\vert H\vert\Phi_q\rangle = E[\rho^{qq},\kappa^{qq},\kappa^{qq\,*}]$	$E^{2^{\text{nd}}}_X = \langle\Psi\vert H\vert\Psi\rangle = \sum_{qq'} f^*_q f_{q'} \langle\Phi_q\vert H\vert\Phi_{q'}\rangle$ $\Downarrow$ Generalized Wick Theorem $\Downarrow$ $\langle\Phi_q\vert H\vert\Phi_{q'}\rangle = E[\rho^{qq'},\kappa^{qq'},\kappa^{q'q\,*}]\langle\Phi_q\vert\Phi_{q'}\rangle$
$\mathbf{E^{1^{st}}_{\vert q\vert}}$ **is a functional of** *diagonal* **density matrices** $\rho^{\mathbf{qq}},\kappa^{\mathbf{qq}}$ and $\kappa^{\mathbf{qq}\,*}$	$\mathbf{E^{2^{nd}}_X}$ **invokes the SAME functional of (all)** *transition* **density matrices** $\rho^{\mathbf{qq'}},\kappa^{\mathbf{qq'}}$ and $\kappa^{\mathbf{q'q}\,*}$

Ingredients of the nuclear EDF method

Two-level energy density functional method

1^{st} **level: single-reference**	2^{nd} **level: multi-reference**
Trial state $\vert\Phi_q\rangle = \prod_\mu \beta_\mu^q \vert 0\rangle$	Trial **set of states** $\{\vert\Phi_q\rangle\} \neq \vert\Psi\rangle$
Sym. break. $q = \vert q\vert e^{i\varphi} \neq 0$	Sym. restor. $(\sum_\varphi)$ and zero-point fluct. $(\sum_{\vert q\vert})$

$$[\mathbf{X},\mathbf{H}] = \mathbf{0} \text{ for } \{\mathbf{X}\} = \{\mathbf{N},\mathbf{Z},\mathbf{P},\mathbf{J^2},\mathbf{J_z},\mathbf{T^2},\mathbf{T_z},\mathcal{T}^2\}$$

Static collective correlations	Dynamical collective correlations
$\mathcal{E}^{\text{SR}}_{\vert q\vert} \equiv \mathcal{E}[\Phi_q;\Phi_q] \neq \langle\Phi_q\vert H\vert\Phi_q\rangle$	$\mathcal{E}^{\text{MR}}_X \equiv \sum_{qq'} f_q^* f_{q'}\, \mathcal{E}[\Phi_q;\Phi_{q'}]\, \langle\Phi_q\vert\Phi_{q'}\rangle \neq \langle\Psi\vert H\vert\Psi\rangle$

Bulk of correlations resummed into $\mathcal{E}[\Phi_q;\Phi_{q'}] \equiv \mathcal{E}[\rho^{qq'},\kappa^{qq'},\kappa^{q'q\,*}]$

$\delta\left[\mathcal{E}^{\text{SR}} - \lambda \text{Tr}\{\rho\} - \lambda^{\vert\mathbf{q}\vert}\text{Tr}\{\rho\mathbf{Q}\}\right] = \mathbf{0}$ ⇓ **HFB-like equations**	$\delta\mathcal{E}^{\text{MR}}_{\mathbf{X}}/\delta\mathbf{f}_{\mathbf{q}}^* = \mathbf{0}$ ⇓ **Hill-Wheeler-Griffin-like equations**

Ingredients of the nuclear EDF method

Two-level energy density functional method

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Trial state $\vert\Phi_q\rangle = \prod_\mu \beta^q_\mu \vert 0\rangle$	Trial **set of states** $\{\vert\Phi_q\rangle\} \neq \vert\Psi\rangle$
Sym. break. $q = \vert q\vert e^{i\varphi} \neq 0$	Sym. restor. $(\sum_\varphi)$ and zero-point fluct. $(\sum_{\vert q\vert})$
$\mathcal{E}^{SR}_{\vert q\vert} \equiv \mathcal{E}[\Phi_q;\Phi_q] \neq \langle\Phi_q\vert H\vert\Phi_q\rangle$	$\mathcal{E}^{MR}_X \equiv \sum_{qq'} f^*_q f_{q'}\, \mathcal{E}[\Phi_q;\Phi_{q'}]\,\langle\Phi_q\vert\Phi_{q'}\rangle \neq \langle\Psi\vert H\vert\Psi\rangle$

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Relevant questions

1. Is the WF→EDF mapping efficient? Is it safe? How is it constrained?
2. Is the GWT-inspired mapping $\mathcal{E}[\Phi_q;\Phi_{q'}] \equiv \mathcal{E}[\rho^{qq'},\kappa^{qq'},\kappa^{q'q\,*}]$ appropriate?

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Set of constraints from consistency requirements

SR-EDF
[J. Dobaczewski, J. Dudek, APP B27 (1996) 45]

- $\mathcal{E}^{\text{SR}}_{|q|}$ must be real and a scalar under all $\mathcal{R}(g) \in$ symmetry group $\mathcal{G}$
- Example: rules to build local Skyrme EDF to 2^{nd} order in σ_ν and ∇

$$\mathcal{E}[\rho,\kappa,\kappa^*] \equiv \int d\vec{r}\,\mathcal{E}[\rho_T(\vec{r}),\tau_T(\vec{r}),J_{T,\mu\nu}(\vec{r}),\vec{s}_T(\vec{r}),\vec{j}_T(\vec{r}),\vec{T}_T(\vec{r}),\vec{F}_T(\vec{r}),\tilde{\rho}_T(\vec{r});\nabla]$$

MR-EDF
[L. Robledo, IJMP E16 (2007) 337; JPG 37 (2010) in press]

1. $\mathcal{E}^{\text{MR}}_{X}$ must be real and a scalar under all $\mathcal{R}(g) \in$ symmetry group $\mathcal{G}$
2. Consistency of MR and SR schemes
   1. $\mathcal{E}^{\text{MR}}_{X} = \mathcal{E}^{\text{SR}}_{|q|}$ when $\{|\Phi_q\rangle\} \longrightarrow |\Phi_q\rangle$
   2. Chemical potential λ must be recovered from Kamlah expansion of $\mathcal{E}^{\text{MR}}_{N}$
   3. QRPA must be recovered through harmonic limit of $\mathcal{E}^{\text{MR}}_{X}$

Extension to MR kernel must implicate *transition* densities only

Diagonal SR kernel		**Off-diagonal MR kernel**
$\mathcal{E}[\rho^{\mathbf{qq}},\kappa^{\mathbf{qq}},\kappa^{\mathbf{qq}\,*}]$	GWT-inspired connection (only) viable option?	$\mathcal{E}[\rho^{\mathbf{qq}'},\kappa^{\mathbf{qq}'},\kappa^{\mathbf{q}'\mathbf{q}\,*}]$

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Unexpected pathologies

Particle number restoration (PNR) as an example

1. Select appropriate MR set $\{|\Phi_\varphi\rangle \equiv e^{i\hat{N}\varphi}|\Phi_0\rangle\,;\varphi \in [0,2\pi]\}$
2. Define non-diagonal MR kernel $\mathcal{E}[\rho^{0\varphi},\kappa^{0\varphi},\kappa^{\varphi 0\,*}]\,\langle\Phi_0|\Phi_\varphi\rangle$ from SR functional
3. Expand MR kernel in a Fourier series over $U(1)$ Irreps

$$\mathcal{E}[\rho^{0\varphi},\kappa^{0\varphi},\kappa^{\varphi 0\,*}]\,\langle\Phi_0|\Phi_\varphi\rangle = \sum_{N\in\mathbb{Z}} c_N^2\,\mathcal{E}_N^{\mathrm{MR}}\,e^{iN\varphi}$$

$$\langle\Phi_0|\Phi_\varphi\rangle = \sum_{N\in\mathbb{Z}} c_N^2\,e^{iN\varphi}$$

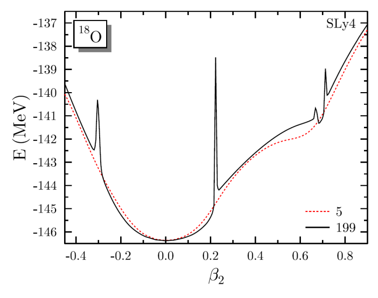
   to obtain real, scalar, particle-number-restored MR energies

$$\mathcal{E}_N^{\mathrm{MR}} \equiv \int_0^{2\pi} d\varphi\,\frac{e^{-i\varphi N}}{2\pi\,c_N^2}\,\mathcal{E}[\rho^{0\varphi},\kappa^{0\varphi},\kappa^{\varphi 0\,*}]\,\langle\Phi_0|\Phi_\varphi\rangle$$

4. Compute... so far so good...

Unexpected pathologies

$\mathcal{E}^{MR}_{Z=8,N=10}$ vs ρ_{20} for $\mathcal{E}[\rho\rho\rho^{1/6}]$



[M. Bender *et al.*, PRC79 (2009) 044319]

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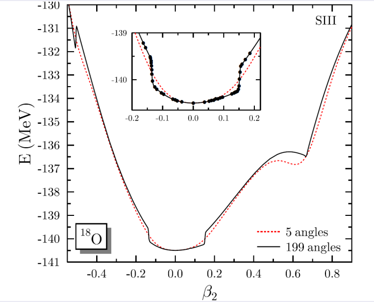

[M. Bender *et al.*, PRC79 (2009) 044319]

1. Divergencies and finite steps [J. Dobaczewski *et al.*, PRC76 (2007) 054315]
2. Non-analyticity of $\mathcal{E}[\rho^{0\varphi}, \kappa^{0\varphi}, \kappa^{\varphi 0\,*}]$ over $\mathbb{C}^*$-plane with $e^{i\varphi} \equiv z$
3. $\mathcal{E}^{MR}_N \neq 0$ for $N \leq 0$!! [M. Bender, T.D., D. Lacroix, PRC79 (2009) 044319]
4. Similar problems for other MR modes, e.g. angular momentum restoration

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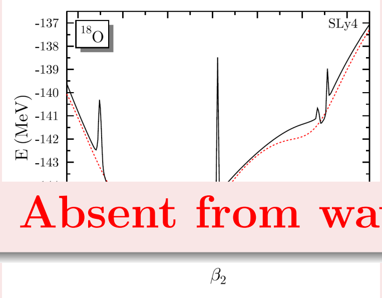

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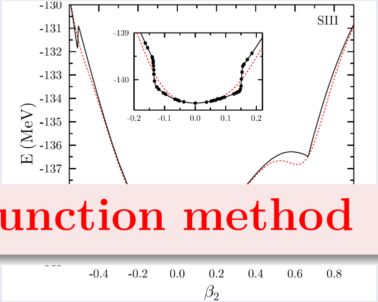

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Absent from wave-function method

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Regularization method

Origin of the problem and its solution

1. Alternative method to define $\mathcal{E}[\Phi_q;\Phi_{q'}]$ that relies on
   1. Considering Bogoliubov transformation connecting $|\Phi_q\rangle$ to $|\Phi_{q'}\rangle$
   2. Using Bloch-Messiah-Zumino decomposition to reach BCS-like connection

$$|\Phi_{q'}\rangle = \tilde{\mathcal{C}}_{qq'} \prod_{p>0} \left(\bar{A}^*_{pp} + \bar{B}^*_{p\bar{p}} \, \tilde{\alpha}^+_p \, \tilde{\alpha}^+_{\bar{p}} \right) |\Phi_q\rangle$$

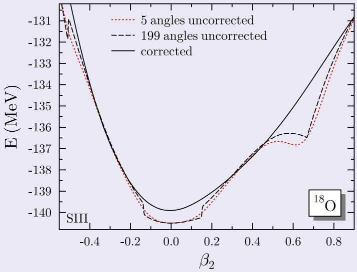
   3. Using SWT to compute $\langle\Phi_q|H|\Phi_{q'}\rangle / \langle\Phi_q|\Phi_{q'}\rangle$
   4. Extending to EDF disconnected from genuine operator H
2. GWT-inspired $\mathcal{E}[\Phi_q;\Phi_{q'}] \equiv \mathcal{E}[\rho^{qq'},\kappa^{qq'},\kappa^{q'q*}]$ unsafe in EDF context
   1. Provides dangerous terms with weights that are zero with SWT
   2. Such terms cancel for WF method but not for more general EDF
   3. Originates from self interaction and self pairing in the EDF kernel

[D. Lacroix, T. D., M. Bender, PRC 79 (2009) 044318]

Regularization method

Regularized PNR calculations

- $\mathcal{E}_{REG}[\Phi_0;\Phi_\varphi] \equiv \mathcal{E}[\rho^{0\varphi},\kappa^{0\varphi},\kappa^{\varphi 0\,*}] - \mathcal{E}_C[\langle\Phi_0|;|\Phi_\varphi\rangle]$
  1. Analytical over $\mathbb{C}^*$
  2. $\mathcal{E}_N^{MR}$ free from divergencies and steps while $\mathcal{E}_N^{MR} = 0$ for $N \leq 0$



- The correction
  1. is crucial at critical points but also *away* from them
  2. depends non-trivially on the quadrupole deformation ρ_{20}
  3. is on the MeV scale = mass accuracy/spectroscopic scales

[M. Bender, T. D., D. Lacroix, PRC 79 (2009) 044319]

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Regularized PNR calculations

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  1. Analytical over $\mathbb{C}^*$
  2. $\mathcal{E}_N^{\mathrm{MR}}$ free from divergencies and steps while $\mathcal{E}_N^{\mathrm{MR}} = 0$ for $N < 0$

Mathematical constraints from wave-function belonging to Irrep

1. Physics tell us that $\mathcal{E}_N^{\mathrm{MR}} = 0$ for $N \leq 0$ in PNR
2. What about expansion coefficients for other symmetry groups $\mathcal{G} = \{R(g)\}$?

$$\mathcal{E}[\rho^{gg'},\kappa^{gg'},\kappa^{g'g\,*}]\,\langle\Phi_g|\Phi_{g'}\rangle = \sum_{\lambda ab} c^*_{\lambda a}\,c_{\lambda b}\,\mathcal{E}^{\mathrm{MR}}_\lambda\,S^\lambda_{ab}(g-g')$$

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Angular momentum restoration

Expansion of MR kernel over Irreps of $SO(3)$

$$\mathcal{E}[\rho^{0\Omega}, \kappa^{0\Omega}, \kappa^{\Omega 0\,*}] \langle \Phi_0 | \Phi_\Omega \rangle = \sum_{LMK} c^*_{LM} c_{LK} \, \mathcal{E}^{\rm MR}_L \, D^L_{MK}(\Omega)$$

Wave-function method

[T. D., J. Sadoudi, JPG 37 (2010) 064009]

$$E^{\rm MR}_L = \frac{1}{2} \int d\vec{R}\, d\vec{r}\; V(r)\, \rho^{[2]}_{LMLM}(\vec{R}, \vec{r}) = \int d\vec{R} \sum_{L'=0}^{2L} \mathcal{V}^{L'0}_L(R)\; C^{LM}_{LML'0}\, Y^0_{L'}(\hat{R})$$

- Mathematical property of the angular-momentum-restored energy density

EDF method

- Properties of $\mathcal{E}[\rho^{0\Omega}, \kappa^{0\Omega}, \kappa^{\Omega 0\,*}]$ needed to verify the above property?

$$\mathcal{E}^{\rm MR}_L = \int d\vec{R} \sum_{L'=0}^{?} \mathcal{E}^{L'?}_L(R)\; Y^?_{L'}(\hat{R})$$

- Unfinished calculation so far

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Perspectives

To be done in the near future

1. Build correctable EDF of the form $\mathcal{E}[\rho^n, (\kappa^* \kappa)^m]$

   [J. Sadoudi, T. D., M. Bender, K. Bennaceur, in progress]

2. Perform systematic regularized MR calculations for AMR and $\Delta\rho_{20}$ mixing

   [J. Sadoudi, M. Bender, T.D., D. Lacroix, in progress]

3. Derive constraints on EDF kernel to satisfy AMR mathematical property

   [J. Sadoudi, T. D., in progress]

4. Need a constructive framework for MR-EDF method

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Ingredients of the nuclear EDF method

Non-negligible differences between EDF and WF methods

1. Energy $E^{1^{\rm st}}_{|q|} = \langle \Phi_q | T + V_{\rm skyrme} | \Phi_q \rangle = E[\rho^{qq}, \kappa^{qq}, \kappa^{qq\,*}] \, \langle \Phi_q | \Phi_q \rangle$
2. Replace $E[\rho^{qq}, \kappa^{qq}, \kappa^{qq\,*}]$ by $\mathcal{E}[\rho^{qq}, \kappa^{qq}, \kappa^{qq\,*}]$ to resum correlations

Skyrme average energy

$$
\begin{aligned}
E[\rho,\kappa,\kappa^*] \; = \; & \sum_{qq'} \int d^3r \Big[C^{\rho\rho}_{qq'} \, \rho_q \rho_{q'} + C^{\rho\Delta\rho}_{qq'} \, \rho_q \Delta \rho_{q'} + C^{\rho\tau}_{qq'} \left(\rho_q \tau_{q'} - \vec{j}_q \cdot \vec{j}_{q'} \right) \\
& + C^{ss}_{qq'} \, \vec{s}_q \cdot \vec{s}_{q'} + C^{s\Delta s}_{qq'} \, \vec{s}_q \cdot \Delta \vec{s}_{q'} + C^{\rho\nabla J}_{qq'} \left(\rho_q \vec{\nabla} \cdot \vec{J}_{q'} + \vec{j}_q \cdot \vec{\nabla} \times \vec{s}_{q'} \right) \\
& + C^{\nabla s \nabla s}_{qq'} \, (\nabla \cdot \vec{s}_q)(\nabla \cdot \vec{s}_{q'}) + C^{JJ}_{qq'} \left(\sum_{\mu\nu} J_{q,\mu\nu} J_{q',\mu\nu} - \vec{s}_q \cdot \vec{T}_{q'} \right) \\
& + C^{J\bar{J}} \left(\sum_{\mu} J_{q,\mu\mu} \sum_{\mu} J_{q,\mu\mu} + \sum_{\mu\nu} J_{q,\mu\nu} J_{q,\nu\mu} - 2 \, \vec{s}_q \cdot \vec{F}_q \right) \Big] \\
& + \sum_q \int d^3r \left[C^{\tilde\rho\tilde\rho}_{qq} \, |\tilde\rho_q|^2 + \ldots \right] + \sum_q \int d^3r \, \frac{\hbar^2}{2m} \, \tau_q
\end{aligned}
$$

Ingredients of the nuclear EDF method

Non-negligible differences between EDF and WF methods

1. Energy $E^{1^{\mathrm{st}}}_{|q|} = \langle \Phi_q | T + V_{\mathrm{skyrme}} | \Phi_q \rangle = E[\rho^{qq}, \kappa^{qq}, \kappa^{qq*}] \langle \Phi_q | \Phi_q \rangle$
2. Replace $E[\rho^{qq}, \kappa^{qq}, \kappa^{qq*}]$ by $\mathcal{E}[\rho^{qq}, \kappa^{qq}, \kappa^{qq*}]$ to resum correlations

Skyrme energy density functional

$$
\begin{aligned}
\mathcal{E}[\rho,\kappa,\kappa^*] \; = \; & \sum_{qq'} \int d^3r \Big[C^{\rho\rho}_{qq'}\, \rho_q \rho_{q'} + C^{\rho\Delta\rho}_{qq'}\, \rho_q \Delta \rho_{q'} + C^{\rho\tau}_{qq'} \left(\rho_q \tau_{q'} - \vec{j}_q \cdot \vec{j}_{q'} \right) \\
& + C^{ss}_{qq'}\, \vec{s}_q \cdot \vec{s}_{q'} + C^{s\Delta s}_{qq'}\, \vec{s}_q \cdot \Delta \vec{s}_{q'} + C^{\rho\nabla J}_{qq'} \left(\rho_q \vec{\nabla} \cdot \vec{J}_{q'} + \vec{j}_q \cdot \vec{\nabla} \times \vec{s}_{q'} \right) \\
& + C^{\nabla s \nabla s}_{qq'}\, (\nabla \cdot \vec{s}_q)(\nabla \cdot \vec{s}_{q'}) + C^{JJ}_{qq'} \Big(\sum_{\mu\nu} J_{q,\mu\nu} J_{q',\mu\nu} - \vec{s}_q \cdot \vec{T}_{q'} \Big) \\
& + C^{J\bar{J}} \Big(\sum_{\mu} J_{q,\mu\mu} \sum_{\mu} J_{q,\mu\mu} + \sum_{\mu\nu} J_{q,\mu\nu} J_{q,\nu\mu} - 2\, \vec{s}_q \cdot \vec{F}_q \Big) \Big] \\
& + \sum_q \int d^3r \Big[C^{\tilde{\rho}\tilde{\rho}}_{qq}\, |\tilde{\rho}_q|^2 + \dots \Big] + \sum_q \int d^3r \, \frac{\hbar^2}{2m} \tau_q
\end{aligned}
$$

Link with QRPA

QRPA from Time-Dependent SR-EDF calculations

1. Adiabatic-type scheme (omitting anomalous densities κ)

$$A_{minj} = (\epsilon_m - \epsilon_i)\,\delta_{mn}\,\delta_{ij} + \frac{\partial^2 \mathcal{E}[\rho]}{\partial \rho_{im} \partial \rho_{nj}} \quad ; \quad B_{minj} = \frac{\partial^2 \mathcal{E}[\rho]}{\partial \rho_{im} \partial \rho_{jn}}$$

2. Extensions (e.g. second RPA…) needed to access *spreading width*

QRPA from Projection+GCM in WF method

- QRPA (WF)

$$A_{minj} = (\epsilon_m - \epsilon_i)\delta_{mn}\delta_{ij} + \bar{v}_{mjin} \quad ; \quad B_{minj} = \bar{v}_{mnij}$$

- QRPA (WF) is recovered from the harmonic limit of GCM (WF)
  [B. Jancovici, D. H. Schiff, NP58 (1964) 678]
  1. Use Thouless parameterization $|\Phi_q\rangle = \exp[\sum_{im} z^{q\,*}_{mi}\, a^\dagger_m\, a_i]|\Phi_0\rangle$
  2. Expand $\langle\Phi_q|H|\Phi_{q'}\rangle / \langle\Phi_q|\Phi_{q'}\rangle$ to second order in $\mathbf{z^q}/\mathbf{z^{q'}}^*$
  3. Assume gaussian overlap $\langle\Phi_q|\Phi_{q'}\rangle \propto \exp[\mathrm{Tr}(\mathbf{z^q}\mathbf{z^{q'}}^\dagger)]$
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Ground-state correlation energy associated with $X = J^2$

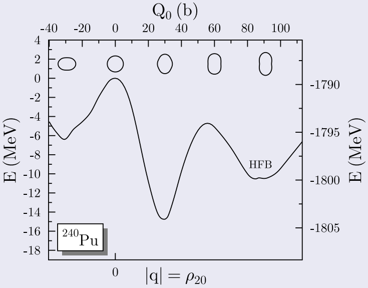
Collective coordinates = "Order parameter"

1. $|q|$ = multipole moments of $\rho(\vec{r})$
2. φ = Euler angles (α, β, γ)

From static deformation

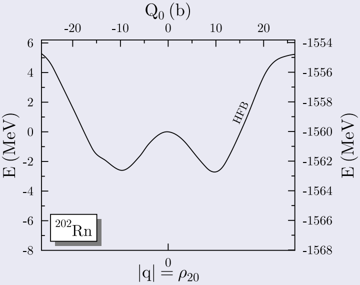
$$\Delta\mathcal{E}^{\mathrm{SR}}_{|q|_{\mathrm{min}}} = \mathrm{Min}_{|q|}\left\{\mathcal{E}^{\mathrm{SR}}_{|q|}\right\} - \mathcal{E}^{\mathrm{SR}}_{0}$$

Well-deformed nucleus for $|q| = \rho_{20}$



[M. Bender, private communication]

Transitional nucleus for $|q| = \rho_{20}$



[M. Bender, private communication]

Ground-state correlation energy associated with $X = J^2$

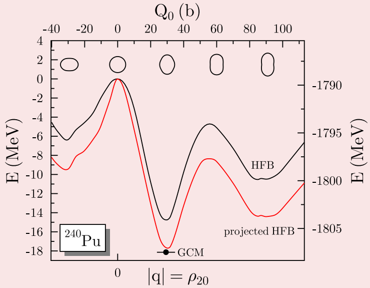
Collective coordinates = "Order parameter"

1. $|q|$ = multipole moments of $\rho(\vec{r})$
2. φ = Euler angles (α, β, γ)

From dynamical fluctuations

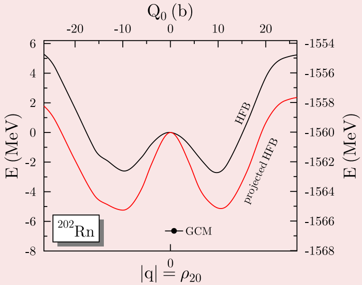
$$\Delta\mathcal{E}^{\mathrm{MR}}_{J=0} = \mathcal{E}^{\mathrm{MR}}_{J=0} - \mathrm{Min}_{|q|}\left\{\mathcal{E}^{\mathrm{SR}}_{|q|}\right\}$$

Stiff nucleus for $|q| = \rho_{20}$



[M. Bender, private communication]

Soft nucleus for $|q| = \rho_{20}$

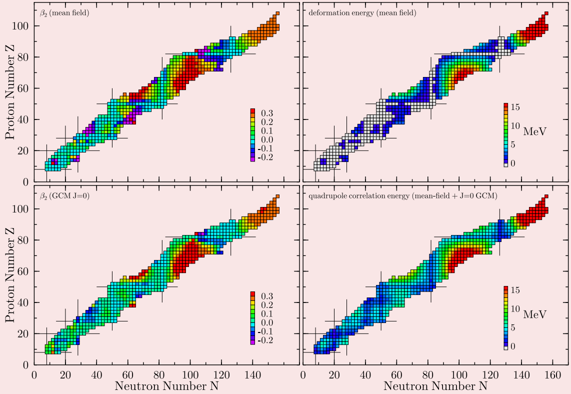


[M. Bender, private communication]

Ground-state correlation energy associated with $X = J^2$

Systematic of quadrupole correlations: $\rho_{20} \neq 0 + J = 0 + \Delta\rho_{20}$

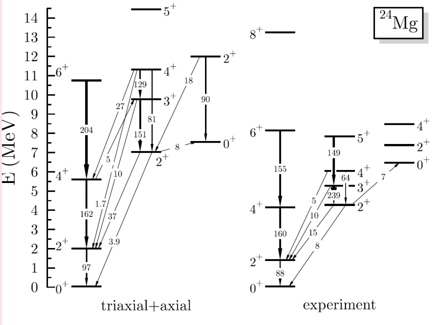
[M. Bender, G.-F. Bertsch, P.-H. Heenen, PRC73 (2006) 034322]



1. Deformation/fluctuations dominant in heavy/light nuclei
2. Right balance between open- and closed-shell nuclei from single EDF
3. Improve ground-state observables systematically, e.g. $\sigma^{\text{mass}}_{2149} = 800\text{keV}$

Collective excitations

Restoration of N, Z, J^2, M + $\Delta\rho_{20}$ + $\Delta\rho_{22}$



[M. Bender, P.-H. Heenen, PRC78 (2008) 024309]

Energy spectrum

- Vibrational + rotational states
- Nicely aligned with experiment
- Too spread out spectrum

Electromagnetic transitions

- Restoration of (J^2, J_z) essential
- Selection rules recovered
- Good in- and out-band $B(E2)$