

**Investigations on the breaking of left-right symmetry  
in light nuclei - the Poincaré instability**

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## The Menu

- **Breaking of left-right symmetry**  
a reminder of a famous publication
- **Modified Funny-Hills parametrisation**
- **Lublin-Strasbourg Drop**
- **Energy landscapes for rotating nuclei**
- **The Poincaré instability**
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# Breaking of left-right symmetry -

## a reminder of a famous publication

ANNALS OF PHYSICS 82, 557-596 (1974)

### Equilibrium Configurations of Rotating Charged or Gravitating Liquid Masses with Surface Tension. II\*

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Received April 10, 1973

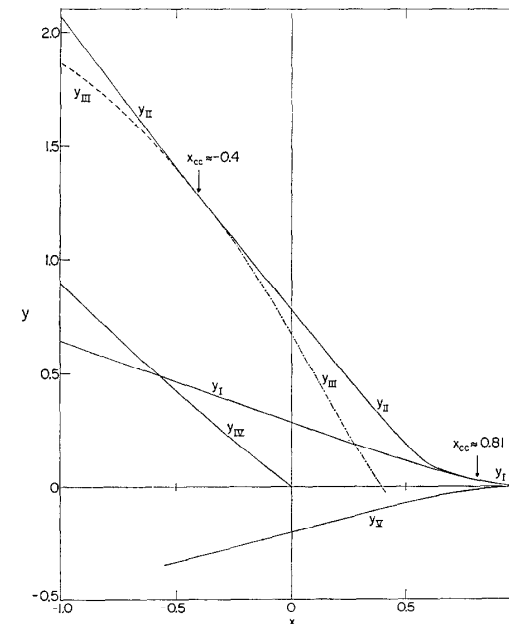
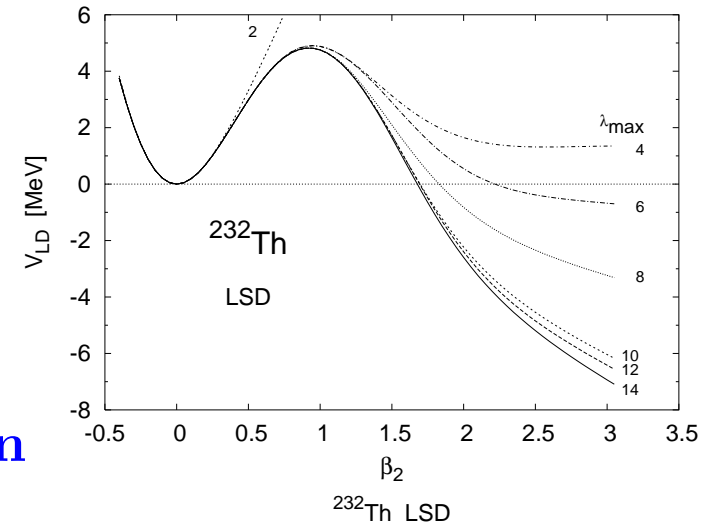


FIG. 2a. Various critical rotational parameters  $y$  in their dependence on the fissility parameter  $x$ . Triaxial shapes appear between  $y_I$  and  $y_{II}$ . Saddle shapes are stable against reflection asymmetric distortions to the right of the dot-dashed portion of  $y_{III}$ . Triaxial shapes are unstable against asymmetry between the dashed portion of  $y_{III}$  and  $y_{II}$ . The critical curves  $y_{IV}$  and  $y_V$  will be discussed in future installments of this series of papers.

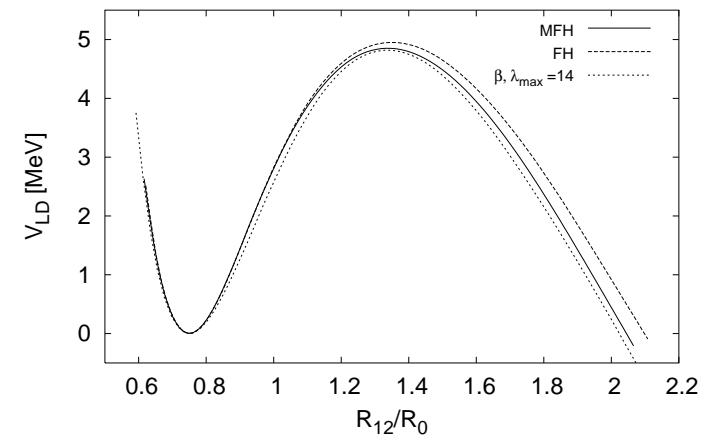
# Modified Funny-Hills Shape parametrisation

- Huge variety of nuclear shapes (g.s.  $\implies$  scission point)
- Only a very few relevant deformation parameters
  - Expansion in spherical harmonics



- Funny-Hills (FH) parametrisation

$$e_s^2(u) = \begin{cases} R_0^2 c^2 (1 - u^2) (A + \alpha u + B u^2) & , B \geq 0 \\ R_0^2 c^2 (1 - u^2) (A + \alpha u) \exp(B c^3 u^2) & , B < 0 \end{cases}$$

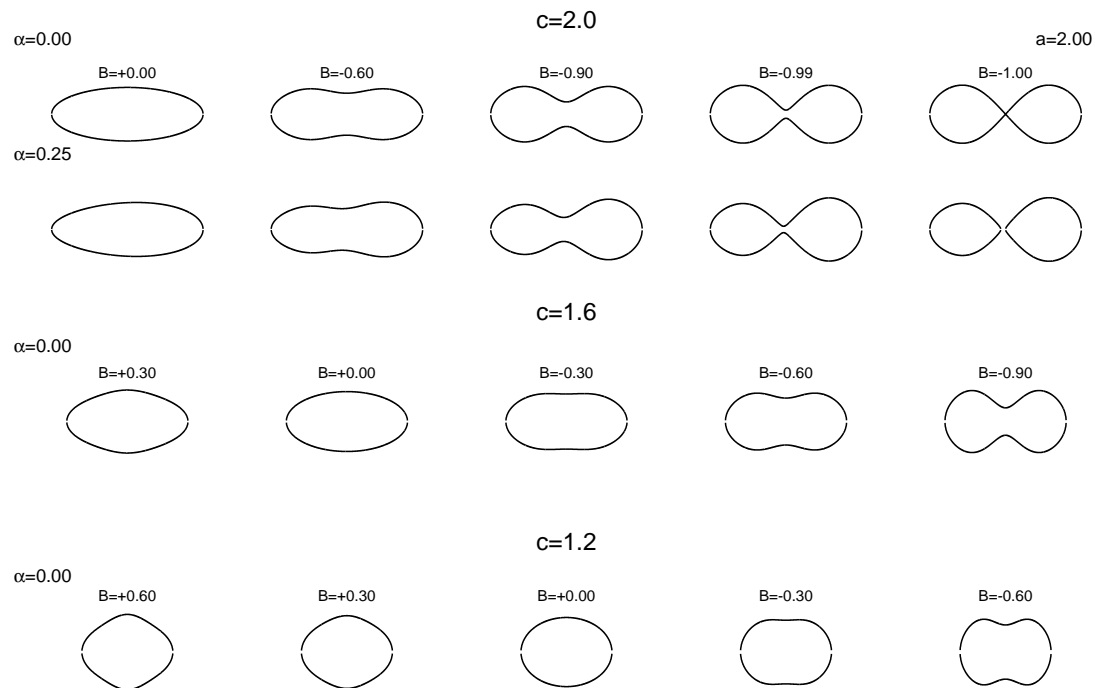


- Modified Funny-Hills shape parametrisation

$$Q_s^2(z) = \frac{R_o^2}{c f(a, B)} (1 - u^2) (1 + \alpha u) (1 - B e^{-a^2 u^2}) ,$$

where

$$f(a, B) = 1 - \frac{3B}{4a^2} \left[ e^{-a^2} + \sqrt{\pi} \left( a - \frac{1}{2a} \right) \text{Erf}(a) \right]$$



breaking axial symmetry

suppose ellipsoidal shape  $\perp$  to  $z$  axis

and introduce the non-axiality parameter

$$\eta = \frac{a_y - a_x}{a_y + a_x}$$

assume that  $\eta$  is independent of  $z$

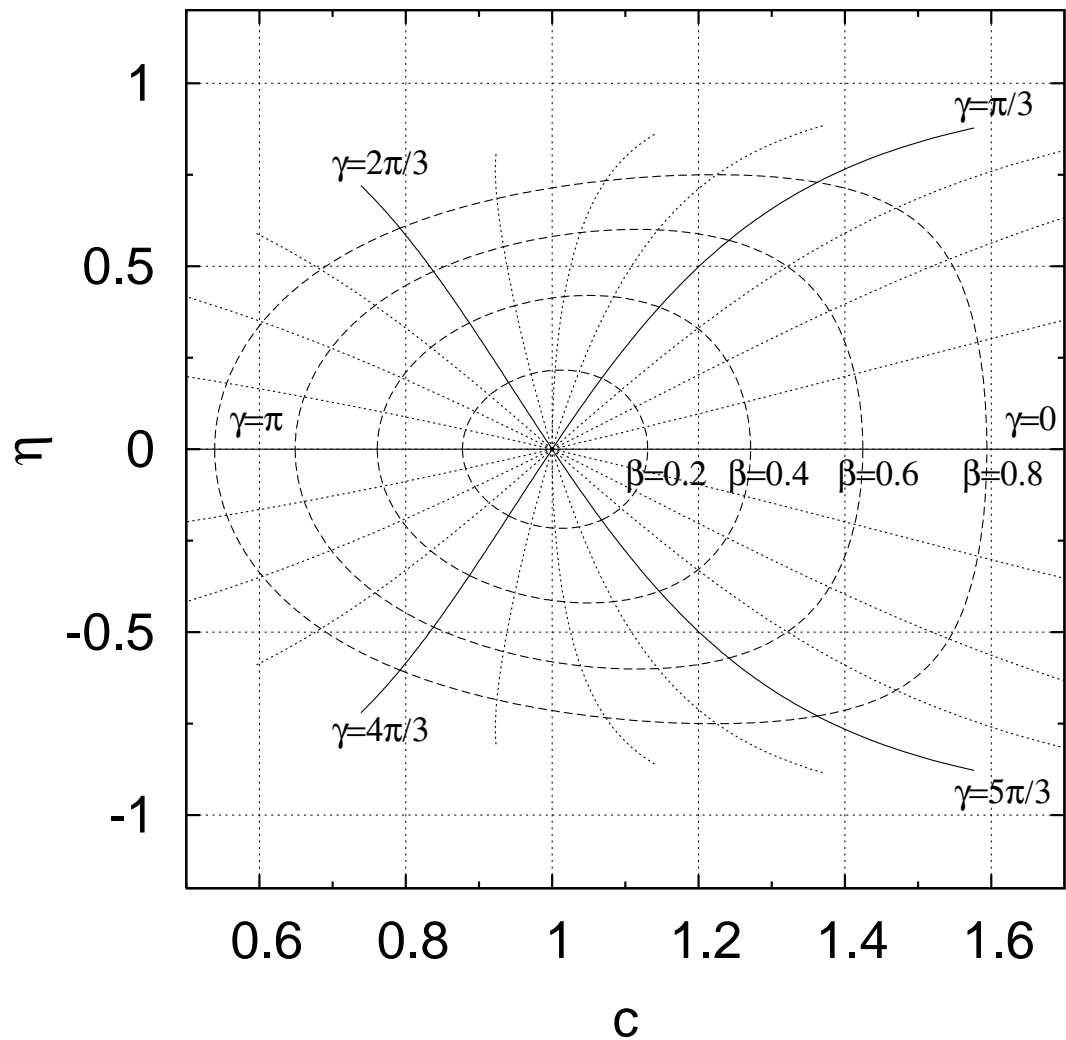
$$a_x(z) = \varrho_s(z) \left( \frac{1 - \eta}{1 + \eta} \right)^{1/2}$$

$$a_y(z) = \varrho_s(z) \left( \frac{1 + \eta}{1 - \eta} \right)^{1/2}$$

volume conservation then leads to

$$\tilde{\varrho}_s^2(z, \varphi) = \varrho_s^2(z) \frac{1 - \eta^2}{1 + \eta^2 + 2\eta \cos(2\varphi)}$$

**J. Bartel, F. Ivanyuk, K. Pomorski, IJMPE E19 (2010) 601**



Transformation from  $(c, \eta)$  to  $(\beta, \gamma)$  in the pure spheroidal case

# Lublin-Strasbourg Drop Model

According to the Strutinski theorem

$$E_{\text{tot}} = \int \mathcal{H}(\rho) d^3r = E_{\text{mac}} + \delta E_{\text{mic}}$$

with the Lublin-Strasbourg drop

$$\begin{aligned} E_{\text{mac}}(Z, N, def) = & a_{\text{vol}} (1 - \kappa_{\text{vol}} I^2) A + a_{\text{surf}} (1 - \kappa_{\text{surf}} I^2) A^{2/3} B_{\text{surf}}(def) \\ & + a_{\text{cur}} (1 - \kappa_{\text{cur}} I^2) A^{1/3} B_{\text{cur}}(def) + \frac{3e^2}{5r_0} \frac{Z^2}{A^{1/3}} B_{\text{Coul}}(def) \\ & - C_4 \frac{Z^2}{A} - E_{\text{cong}} \end{aligned}$$

with shell and pairing corrections

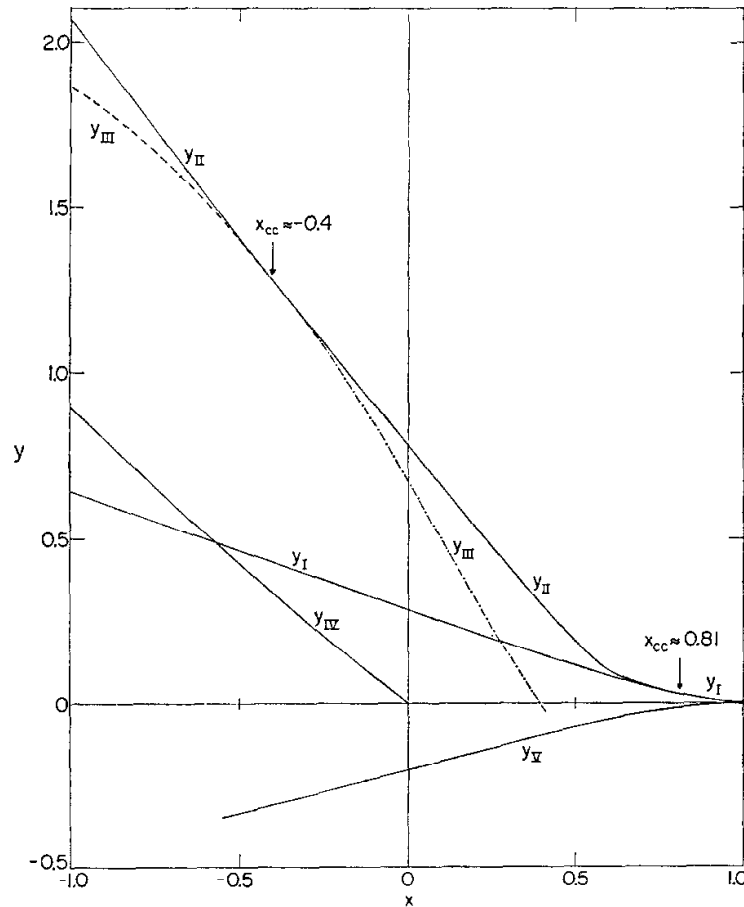
$$\delta E_{\text{mic}} = \delta E_{\text{shell}} + \delta E_{\text{pair}}$$
$$\delta E_{\text{shell}} = \sum_{\text{occ}} \varepsilon_{\nu} - \tilde{E}$$

$$\delta E_{\text{pair}} = E_{\text{BCS}} - \sum_{\text{occ}} \varepsilon_{\nu} - \langle E_{\text{pair}} \rangle$$



# Energy landscapes in light nuclei

studied nuclei:  $^{44}\text{Ti}$ ,  $^{64}\text{Zn}$ ,  $^{76}\text{Se}$ ,  $^{80}\text{Kr}$ ,  $^{84}\text{Sr}$ ,  $^{88}\text{Mo}$

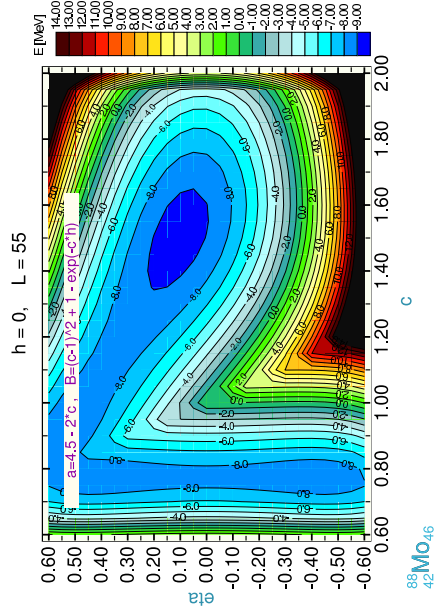
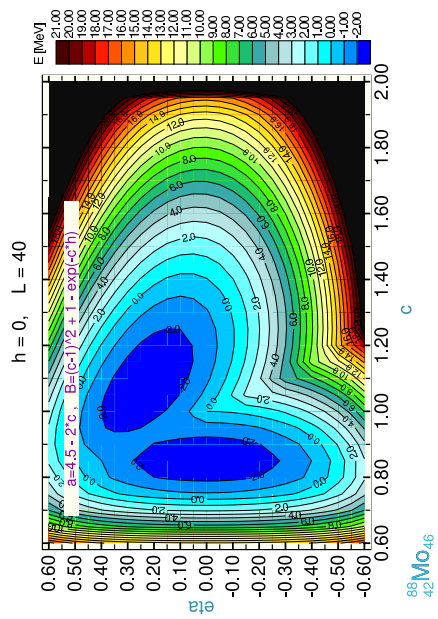
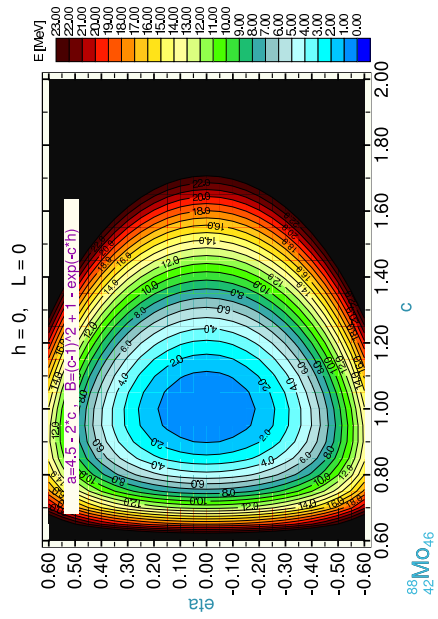
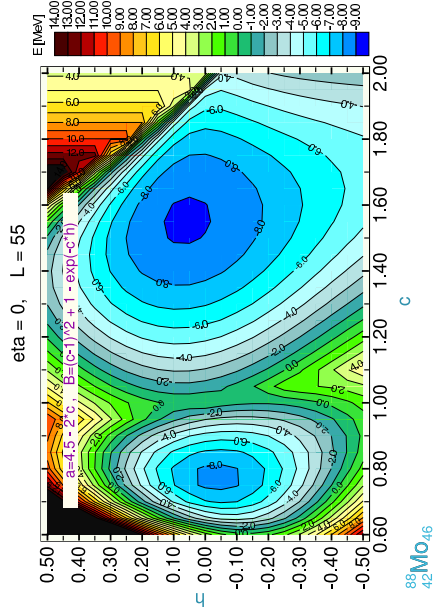
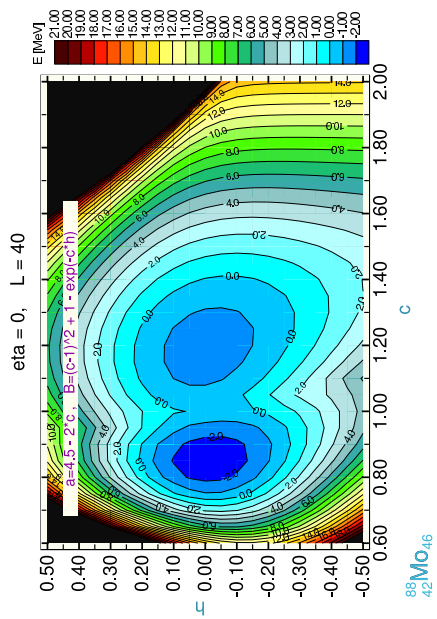
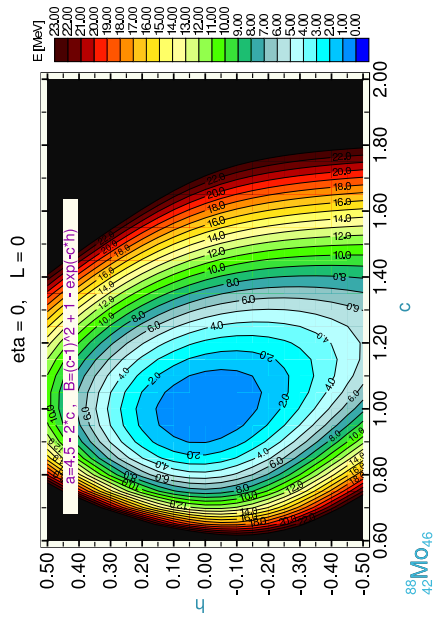


Instability against  
reflection asymmetry  
when  $x \leq 0.4$  for  $L=0$ ,  
decreasing for  $L > 0$

A rough estimate:

$$x \approx \frac{1}{50} \frac{Z^2}{A}, \quad y \approx 2 \frac{L^2}{A^{7/3}}$$

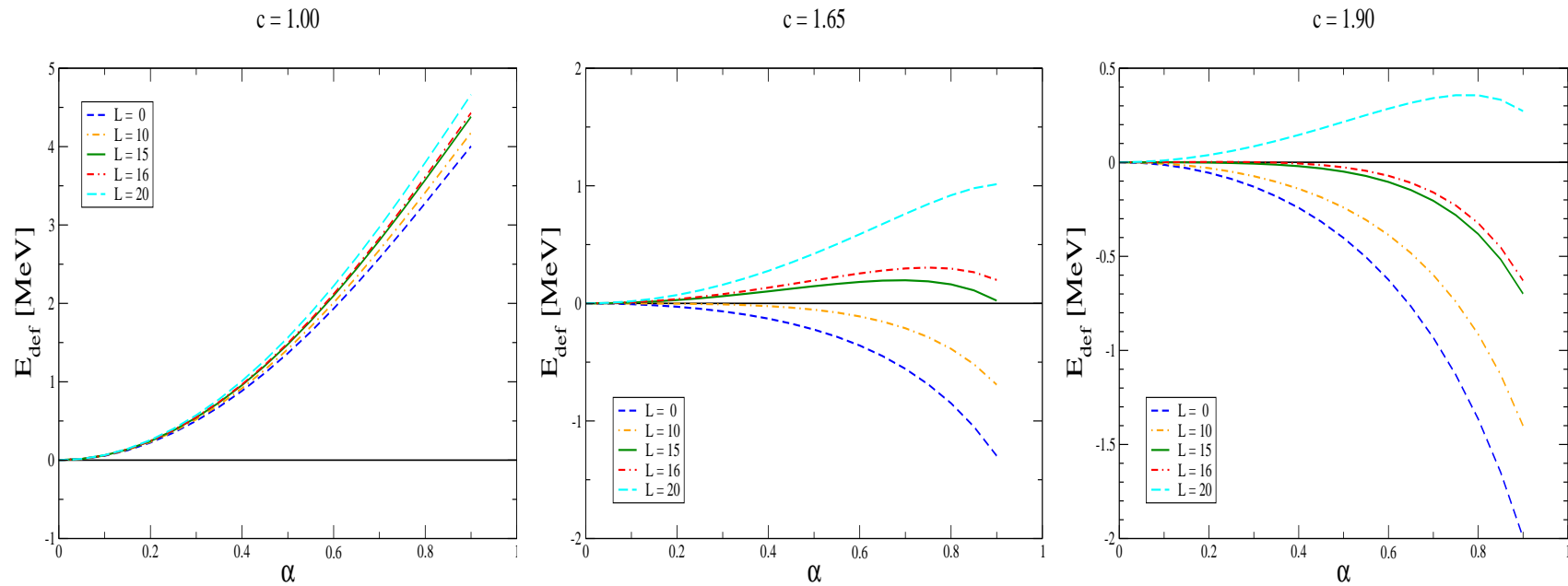
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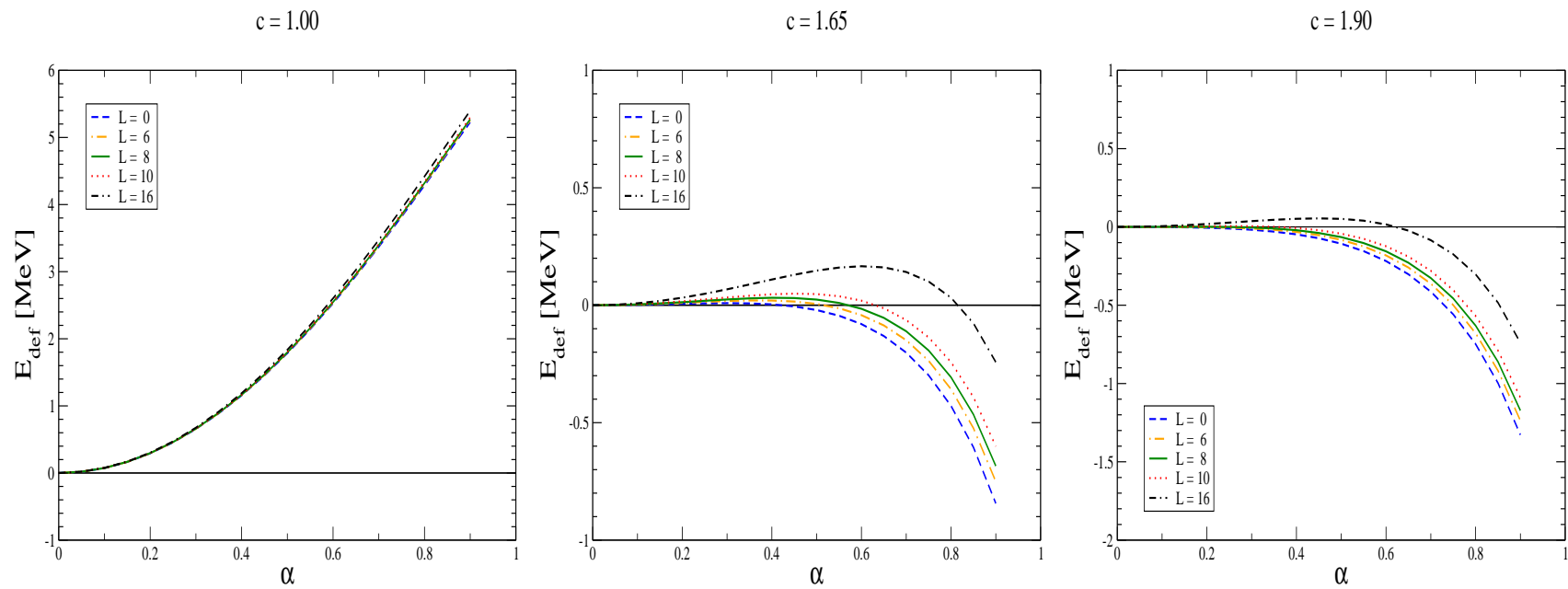
# The Poincaré instability

also motivated by the experimental studies of A. Maj & coworkers

(see XVIth Kazimierz workshop 2009, IJMP E19 (2010) 532)



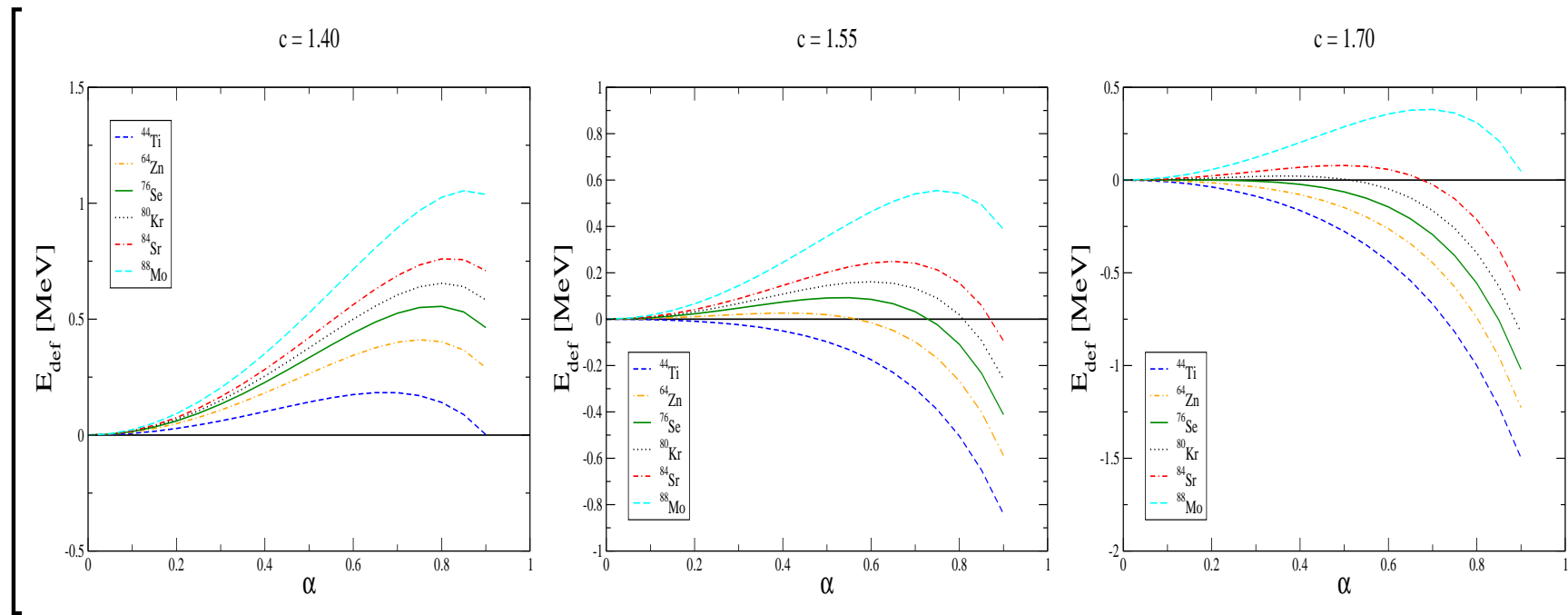
in  $^{44}\text{Ti}$



in  $^{76}\text{Se}$

But  $c = 1.65$  is already a pretty large deformation

# Study of Poincaré's instability for $L = 0$ at different deformations



## Conclusions

- Modified Funny-Hills (MFH) parametrisation describes nuclear shapes very well (already well known)
- Our LDM study with the LSD and the MFH parametrization allows for a systematic study of the Poincaré instability in light nuclei
- This instability against left-right asymmetry only present in light nuclei with mass  $A \leq 80 - 90$
- For higher angular momenta (or higher temperatures) in competition with the fission instability
- Higher angular momenta tend to stabilize the nucleus already pointed out by CPS 35 years ago
- Finite excitation of nuclei have a practically negligible effect on the Poincaré instability