XIII Nuclear Physics Workshop
Maria and Pierre Curie
Pairing & Beyond - 50 Years of the BCS Model
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Superconductivity in metallic nanograins (study report)

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50 years ago:

- John Bardeen, Leon Cooper and Robert Schrieffer – in 1957 formulated theory of superconductivity (BCS).
(Cooper solution – 1956)
Nobel – 1972.

20 years ago:

- Karl Alex Müller and Georg Jorg Bednorz – in 1986 discovered high temperature superconductivity (HTS) in copper oxides. Nobel – 1987.

Outline

- *Why study of nanograins might be interesting ?*
  - *Superconducting nanograins in controlled conditions*
  - *Nanograins exist in HTS (e.g. STM study of underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$)*
  - *Sr_2RuO_4 – the role of small regions in exotic superconductors*
  - *"Shape resonances" : old effect – new possibilities*
- *Richardson's solution of BCS problem in canonical ensemble*
  - *The finite size effects in a two-level model*
  - *The role of temperature*

Why study of superconducting nanograins might be interesting ?

- Scientific interests:
  - The spectrum of small system differs from the bulk one
  - The parity (2n vs. 2n+1 electrons) effects important
  - Novel quantum behaviour is to be expected (negative U center as an analog of the charge Kondo model)
- Applications:
  - Small superconducting box as a qubit
  - Shape resonances to enhance superconducting transition temperatures or other parameters

Superconducting nanograins in controlled conditions

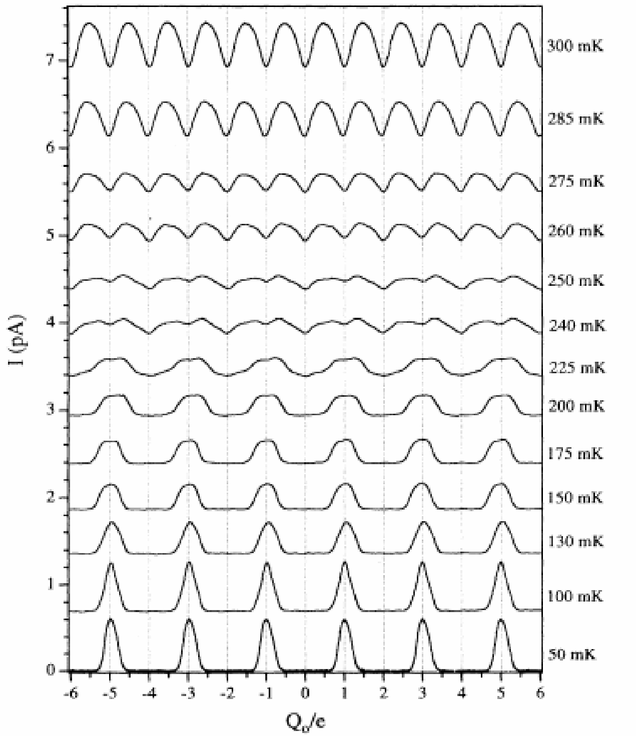


FIG. 2. Current through an *NSN* single-electron transistor with Al island vs gate charge Q_0 at temperatures from 50 to 300 mK (bias voltage V=125 μV). Curves have been displaced upward successively for clarity. Note that the transition from 2*e* periodicity to *e* periodicity occurs in the rather narrow temperature range 240–270 mK near T^*, where the even-odd free energy difference F_0 is going to zero.

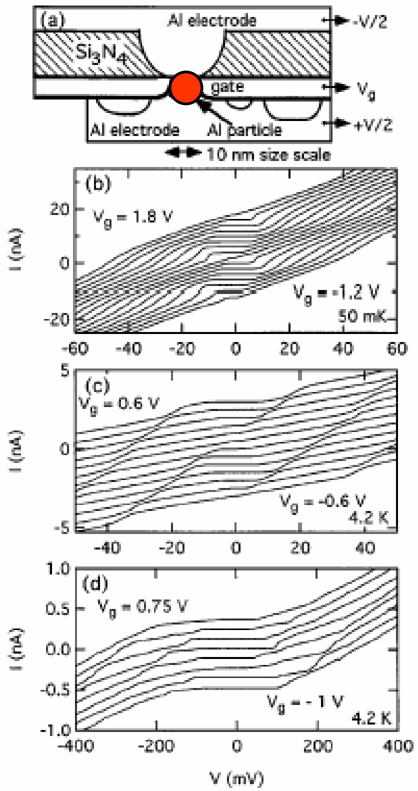


FIG. 1. (a) Schematic cross section of device geometry. (b)–(d) Current-voltage curves displaying Coulomb-staircase structure for three different samples, at equally spaced values of gate voltage. Data for different V_g are artificially offset on the current axis.

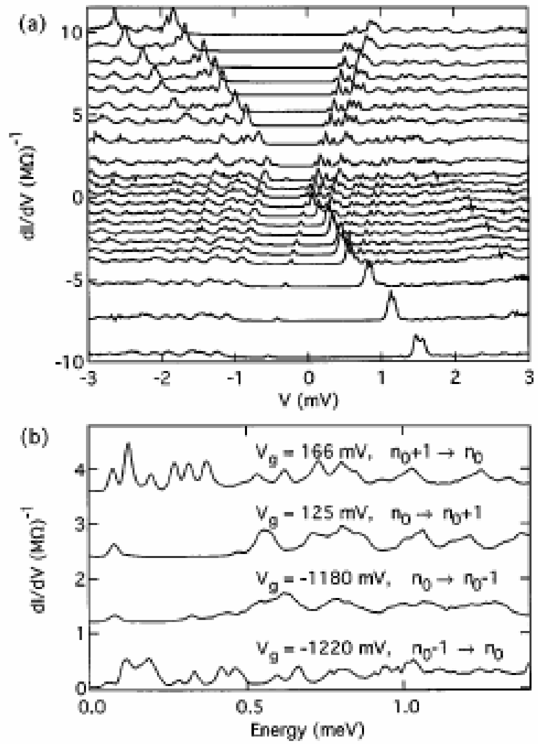


FIG. 2. (a) dI/dV vs source-drain voltage, plotted for V_g ranging from 75 mV (bottom) to 205 mV (top), for the device of Fig. 1(b). Curves are offset on the dI/dV axis. (b) Tunneling spectra for this sample, labeled by the number of electrons in the initial and final states. (n_0 is odd.) All data are for T = 50 mK, H = 0.05 T, to drive the Al leads normal.

M. Tinkham, J.M. Hergenrother, J.G. Lu
Phys. Rev. B **51** 12 649 (1995).

D. C. Ralph, C. T. Black, M. Tinkham
Phys. Rev. Lett. **21**, 4087 (1997).

Superconducting nanograins in controlled conditions – tunnelling conductance

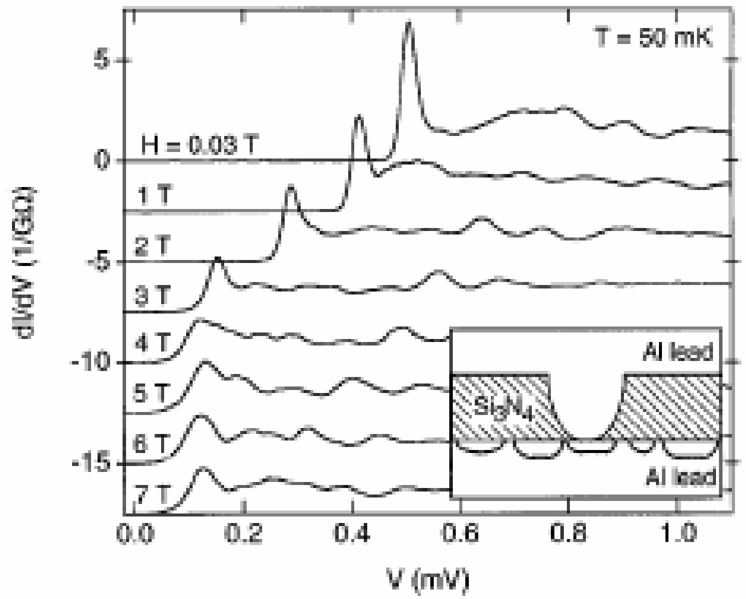
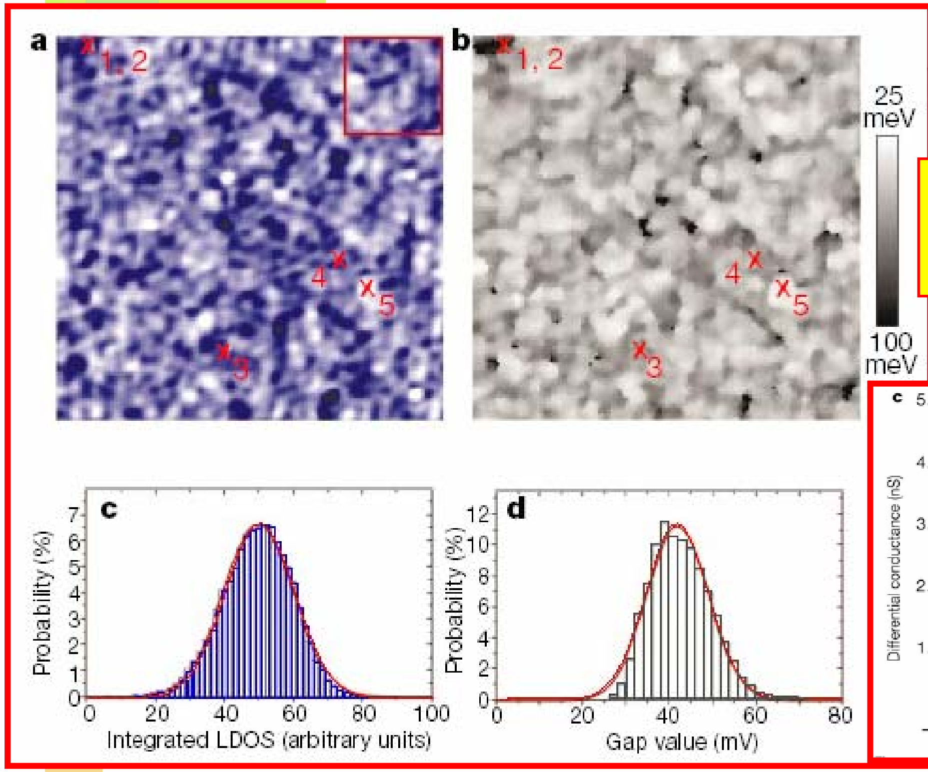


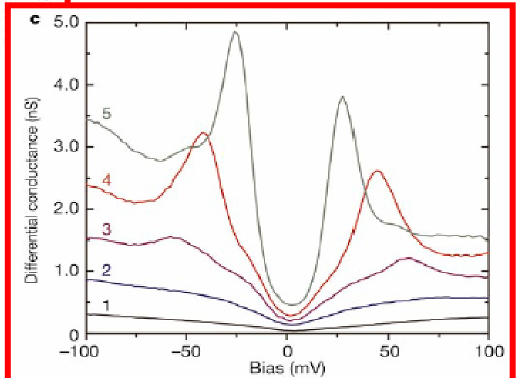
FIG. 1. Differential conductance vs voltage for sample 1, for a range of applied magnetic fields. Curves are offset. Inset: cross-sectional schematic of device.

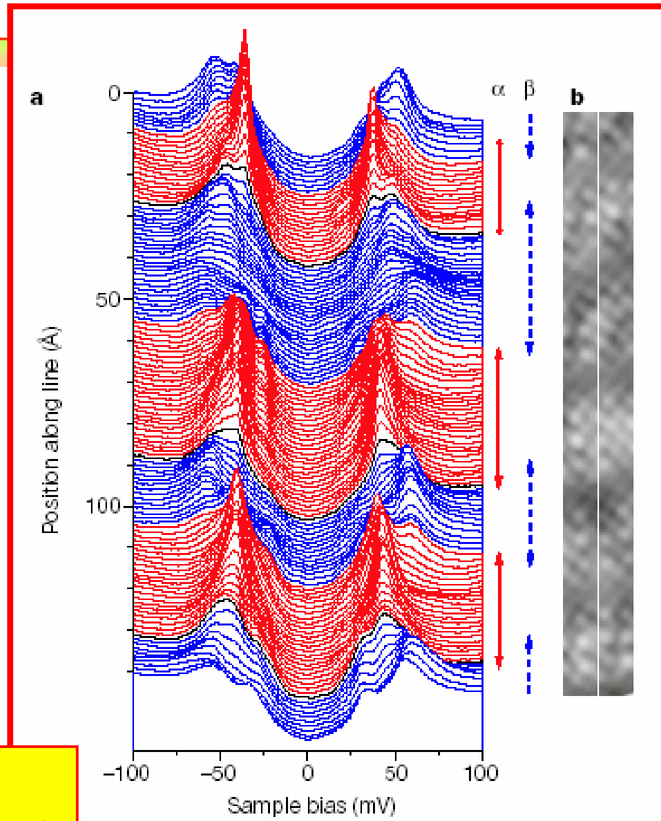
D. C. Ralph, C. T. Black, M. Tinkham
Phys. Rev. Lett. **21**, 4087 (1997).

STM study of $Bi_2Sr_2CaCu_2O_{8+\delta}$



S.H. Pan, et al.
Nature 413 282 (2001)





K.M. Lang et al.
Nature **415** 412 (2002)

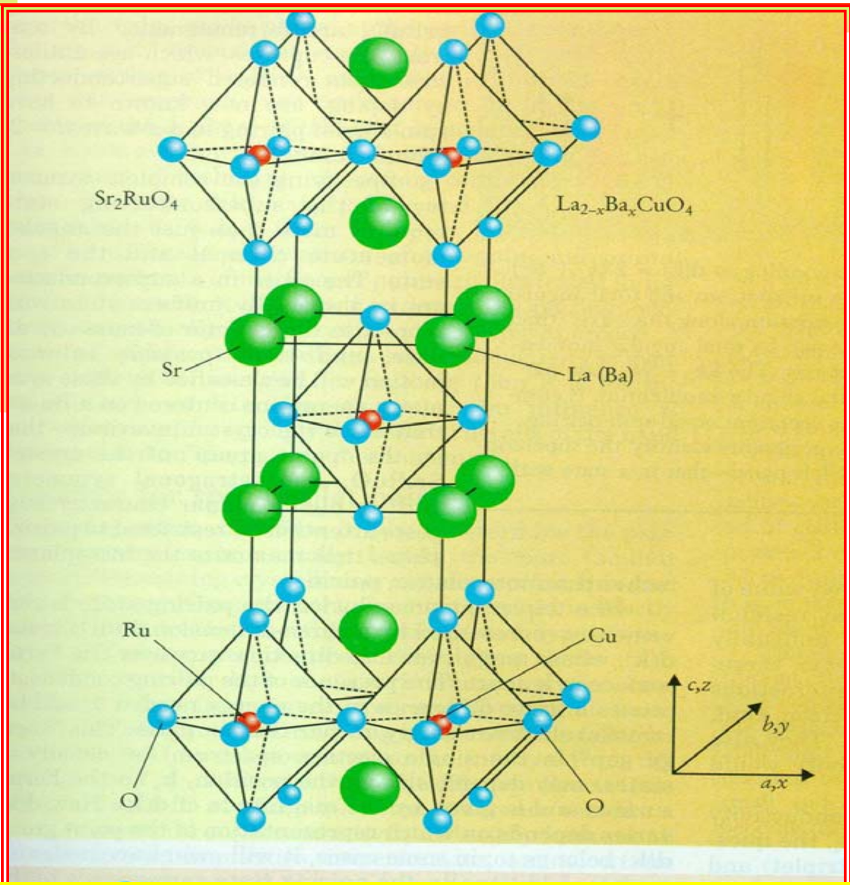
The inhomogeneity is manifested as spatial variations in both the local density of states spectrum and the superconducting energy gap. These variations are correlated spatially and vary on the surprisingly short length scale of ≈14 Å.

These observations suggest that underdoped $Bi_2Sr_2CaCu_2O_{8+d}$ is a mixture of two different short-range electronic orders with the long-range characteristics of a granular supercond.

Sr_2RuO_4 – the role of small regions in exotic superconductors

- Sr_2RuO_4 – the only perovskite superconductor without Cu
- The first member of the Ruddlesden-Popper homologous series $Sr_{n+1}Ru_nO_{3n+1}$
- The first spin-triplet, odd-parity superconductor - solid state analogue of ^3He

Sr_2RuO_4 – the only perovskite superconductor without Cu



Y. Maeno, H. Hashimoto, K. Yoshida, S. Nishizaki, T. Fujita, J.G. Bednorz, and F. Lichtenberg, Nature 372, 532 (1994).

Important parameters

$T_c = 1.5$ K

$\mu_0 H_{c2}^{ab}(0) = 1.5$ T
$\mu_0 H_{c2}^{c}(0) = 0.075$ T

$\xi_{ab}(0) = 660$ Å
$\xi_c(0) = 33$ Å
$\lambda_{ab}(0) = 1800$ Å
$\lambda_c(0) = 3.7$ μm

$\ell_{ab} > 6000$ Å

Results:

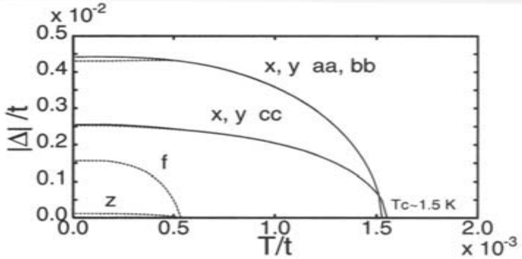


FIG. 1. Order parameters, $|\Delta^{z}_{aa}|$, $|\Delta^{z}_{cc}|$, $|\Delta^{z}_{aa}|$, $|\Delta^{f}_{aa}|$ as functions of temperature, (dashed lines), and excluding z and f (full lines).

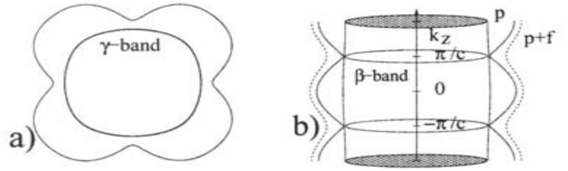
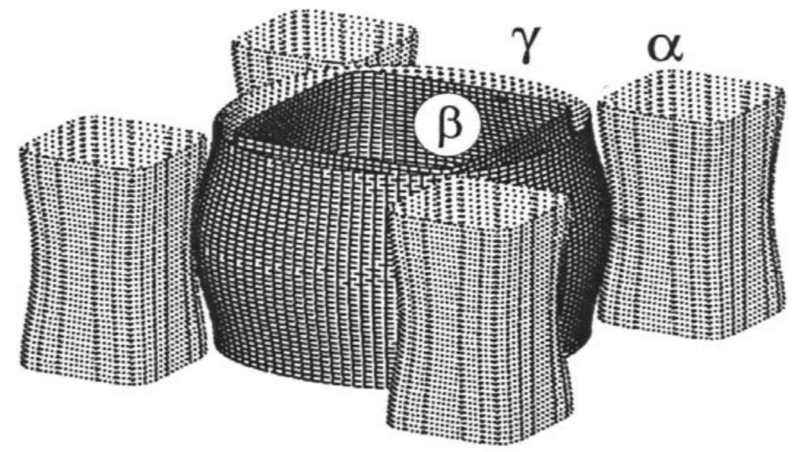


FIG. 2. Lowest energy eigenvalues, $E^{\nu}(\mathbf{k})$ on the Fermi surface; (a) γ sheet in the plane $k_z = 0$, $E_{\gamma,max}(\mathbf{k}_F) = 0.25$meV, $E_{\gamma,min}(\mathbf{k}_F) = 0.064$meV (b) β sheet in the plane $k_x = k_y$, $E_{\beta,max}(\mathbf{k}_F) = 0.32$meV. The f−wave order parameter lifts the p-wave line nodes (dotted lines)



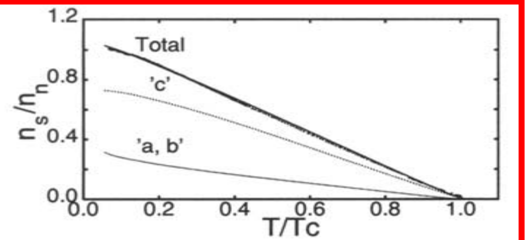
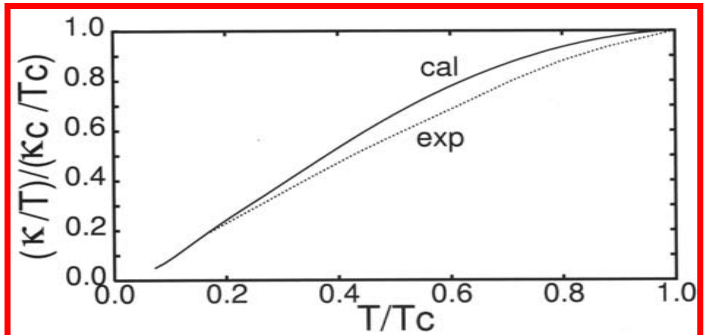


FIG. 4. Superfluid density as a function of T (solid line), and experimental points from Sample 1 of Bonalde *et al.*[15]. The relative contributions of c and a, b order parameters are indicated (dashed lines).



J. F. Annett. G. Litak, B. L. Gyorffy, K.I.W.
Phys. Rev. B 66, 134514 (2002).
Eur. Phys. J. B 36, 301-312 (2003).
phys. stat. sol. (b) 236, 325 (2003). ,
phys. stat. solidi B 241, 983-989 (2004).
Phys. Rev. B 73, 134501 (2006).

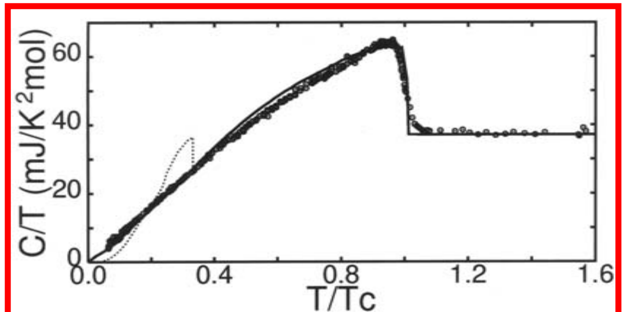


Figure 1. Calculated specific heat, compared to the experimental data of NishiZaki *et al.*. $U_{\perp}/t = 0.590$, $U_{\|}/t = 0.494$, $U_{int}/t = 0.0$

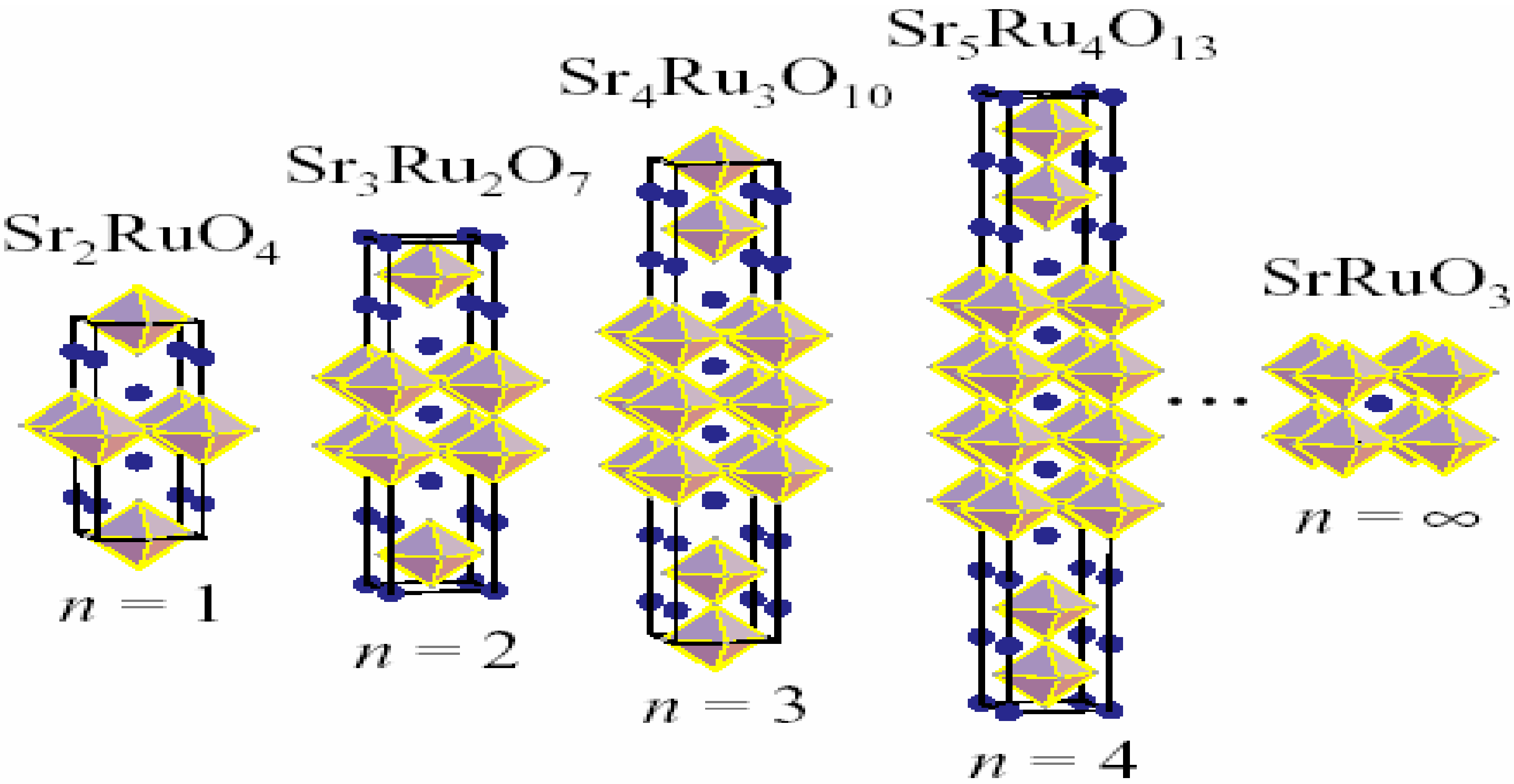
Parameters:

$U_{\|}$= 40 meV
$U_{\perp}$=48 meV
fitted to T_c=1.5 K
give **correctly**:
- T dependence
- jump of C/T

Too small penetration depth @0K 450A instead of 1900A

$$\Delta(\vec{k}) = \Delta^{c}_{\|}(\sin k_x + i \sin k_y) + \Delta^{b/a}_{\perp}(\sin \frac{k_x}{2} \cos \frac{k_y}{2} + i \sin \frac{k_y}{2} \cos \frac{k_x}{2}) \cos \frac{k_z c'}{2}$$

Ruddlesden-Popper homologous series $Sr_{n+1}Ru_nO_{3n+1}$



$Sr_2RuO_4 - Sr_3Ru_2O_7$ eutectics

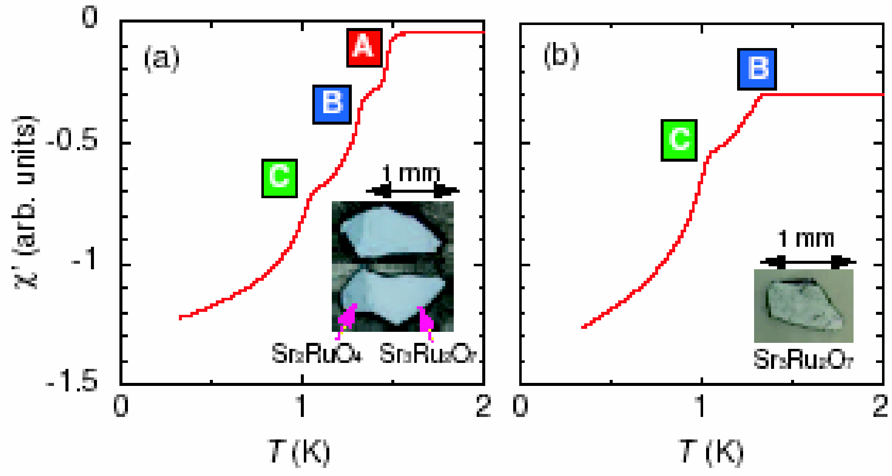


FIG. 1: (Color online) Temperature dependence of the real part of the AC susceptibility at $\mu_0 H_{AC} = 0.58\ \mu$T-rms and $\mu_0 H_{DC} = 0$ T. (a) For Sample 1 (Sr_2RuO_4-$Sr_3Ru_2O_7$ eutectic); (b) For Sample 2 (only the $Sr_3Ru_2O_7$ region cut from Sample 1). Insets are PLOM images of the samples.

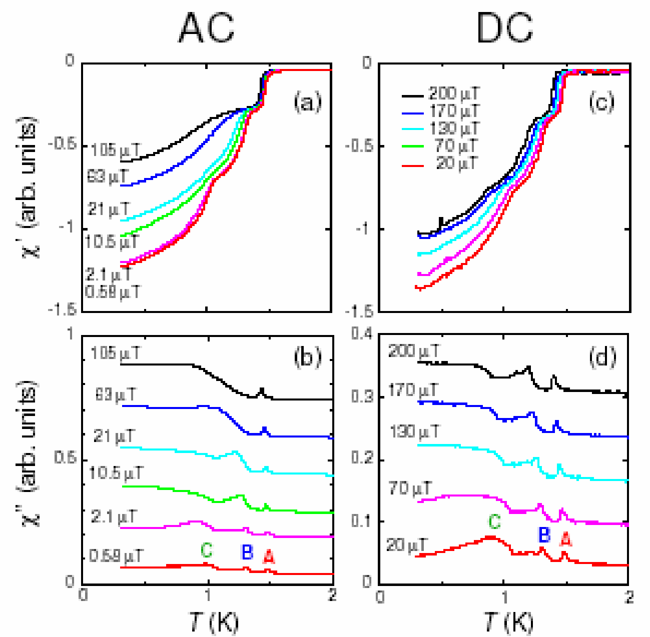


FIG. 2: (Color online) Temperature dependence of the real and imaginary parts of the AC susceptibility of a eutectic crystal of Sr_2RuO_4-$Sr_3Ru_2O_7$ (Sample 1). (a), (b) Data at various AC magnetic fields ($\mu_0 H_{DC} = 0$ T). The AC field amplitudes $\mu_0 H_{AC}$ are indicated in μT-rms. (c), (d) Data at various DC magnetic fields ($\mu_0 H_{AC} = 0.58\ \mu$T-rms). The DC field amplitudes $\mu_0 H_{DC}$ are indicated in μT.

S. Kittaka, S. Fusanobori, H. Yaguchi, S. Yonezawa, Y. Maeno, R. Fittipaldi, A. Vecchione, cond-mat 0607151.

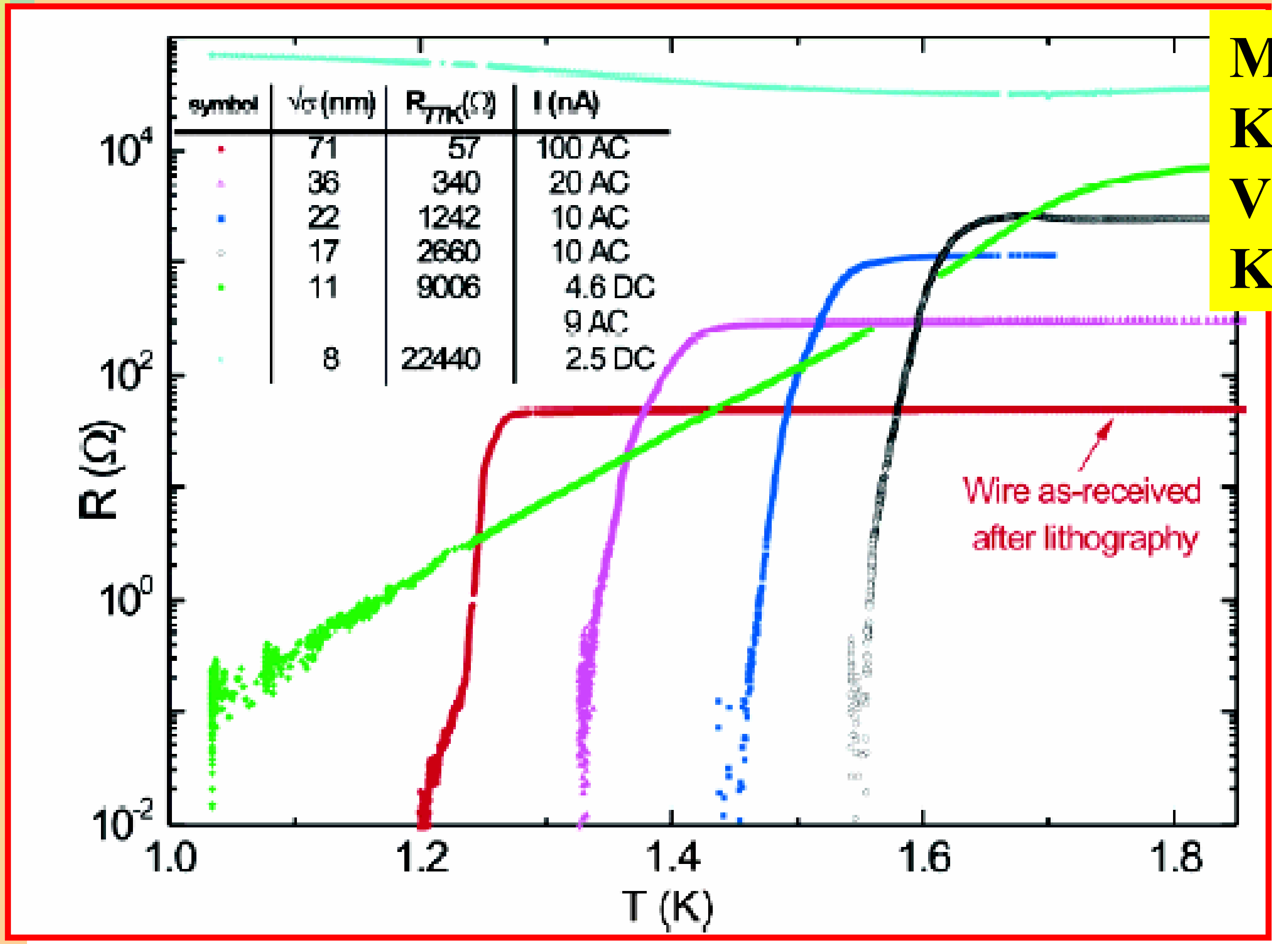
Single crystals of $Sr_3Ru_2O_7$ do not show signs of superconductivity down to 20 mK (*R. S. Perry, et al. Phys. Rev. Lett. 92, 166602 (2004).*) but

"- three superconducting transitions observed in the Sr_2RuO_4-$Sr_3Ru_2O_7$

-the lower two transitions originate from the $Sr_3Ru_2O_7$ region alone.

-The superconductivity observed in the $Sr_3Ru_2O_7$ region is not caused by a proximity effect at the bulk boundary of Sr_2RuO_4 and $Sr_3Ru_2O_7$"

"Shape resonances" : old effect – new possibilities (J. M. Blatt and C. J. Thompson, Phys. Rev. Lett. **10**, 332 (1963))



Maciek Zgirski, Karri-Pekka Riikonen, Vladimir Touboltsev, Konstantin Arutyunov

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The superconducting transition temperature first increases and then decreases with decreasing nanowire thickness.

“Shape resonances” : old effect – new possibilities

A. A. Shanenko, et al.

PHYSICAL REVIEW B **74**, 052502 (2006)

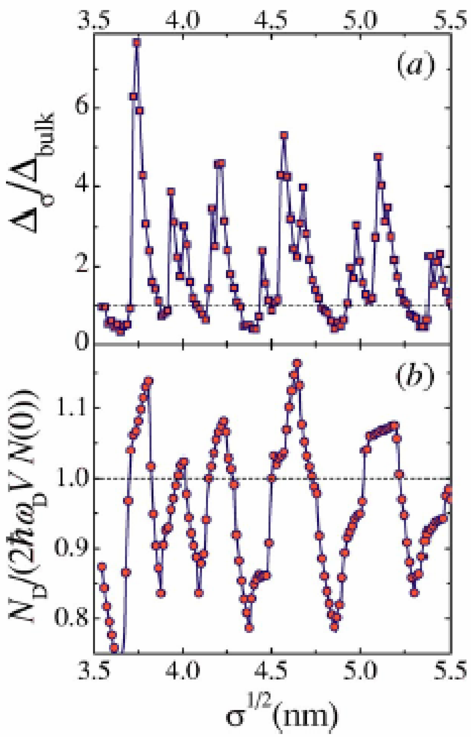


FIG. 1. (Color online) The width-dependent relative gap $\Delta_\sigma/\Delta_{bulk}$ (a) and the relative mean density of the single-electron states in the Debye window $N_D/[2\hbar\omega_D VN(0)]$ (b) versus $\sigma^{1/2}$ for cylindrical Al nanowires with cross section σ at zero temperature.

PHYSICAL REVIEW B **74**, 052502 (2006)

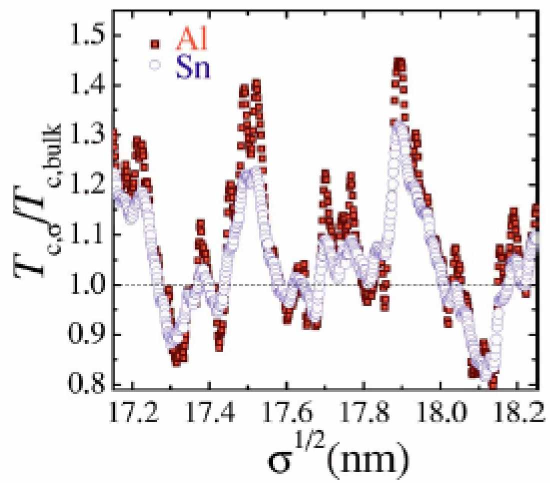


FIG. 3. (Color online) The shape resonances in the relative transition temperature for Al and Sn cylindrical nanowires.

“the size-dependent increase of the superconducting temperature of (Al and Sn) nanowires is well explained by the shape resonance effect.”

Richardson's solution of BCS problem in canonical ensemble

(R.W.Richardson, N. Sherman, Nucl. Phys. B**52**,221(1964))

The Hamiltonian: $$H = \sum_{i,\sigma=+,-}^{N} \varepsilon_i c_{i\sigma}^{+} c_{i\sigma} - g \sum_{i,j}^{M} c_{i+}^{+} c_{i-}^{+} c_{j-} c_{j+}$$

N – no. of doubly degenerate single particle states, M – no. of pairs, N_e – no. of electrons=2M

Richardson's solution: (see J.M.Roman, G. Sierra, J. Dukelsky, Nucl. Phys. B**634** [FS] 483-510 (2002).)

Eigenstates:

$$|M\rangle = \prod_{\nu=1}^{M} \sum_{j=1}^{N} \frac{1}{\varepsilon_j - E_\nu} b_j^{+}$$

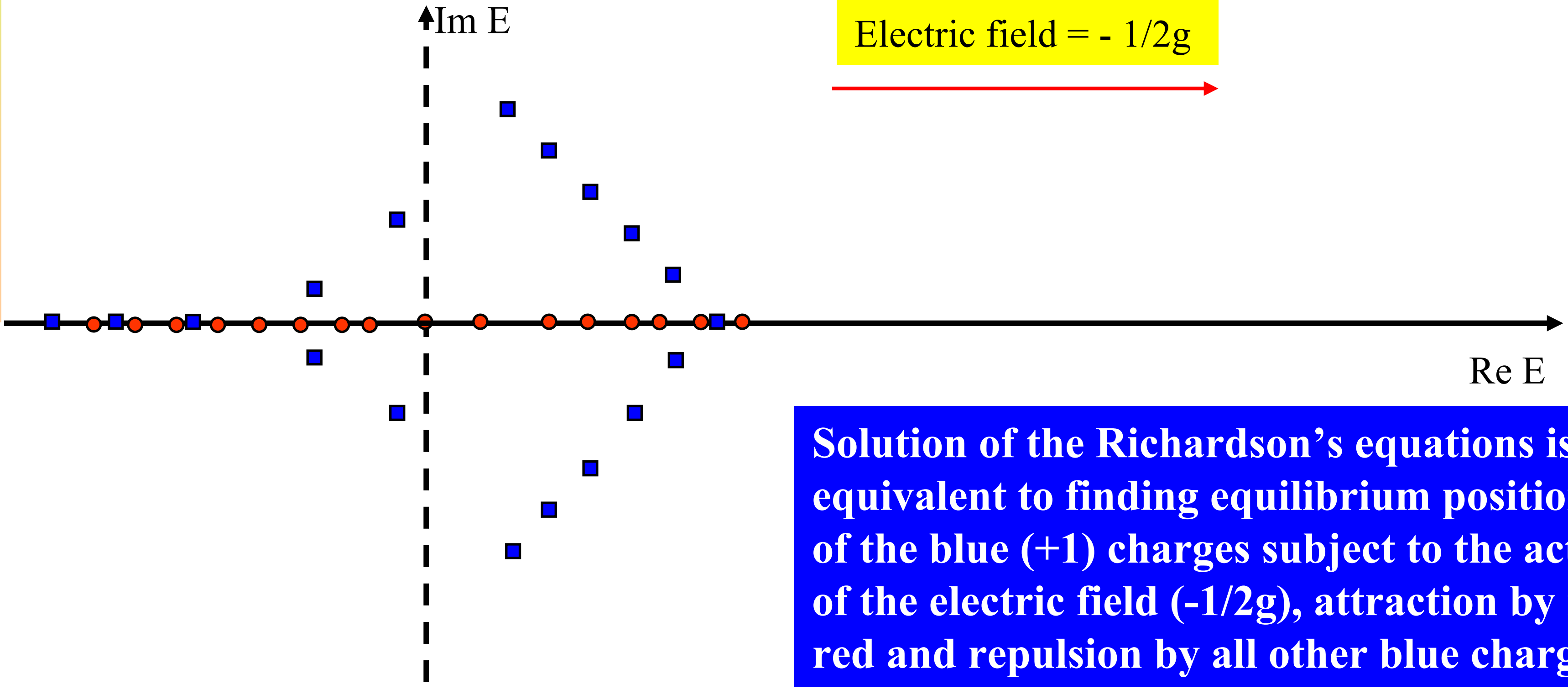
Eigenvalues:

$$E(M) = \sum_{\nu=1}^{M} E_\nu$$

Parameters:

$$\frac{1}{2g} = \sum_{j=1}^{N} \frac{1/2}{\varepsilon_j - E_\nu} - \sum_{\mu=1(\mu\neq\nu)}^{M} \frac{1}{E_\mu - E_\nu}, \quad \nu = 1,\ldots,M$$

Richardson's solution - electrostatic analogy



Solution of the Richardson's equations is equivalent to finding equilibrium positions of the blue (+1) charges subject to the action of the electric field (-1/2g), attraction by fixed red and repulsion by all other blue charges.

- ● -set of N **(– ½)** charges located at fixed positions ε_j
- ■ -set of M **(+1)** charges located at (unknown) equilibrium positions E_ν
- → - electric field = **- 1/2g** acting on ■

Two level model

$\varepsilon_1 = \varepsilon, \quad \Omega_1$

$\varepsilon_0 = 0, \quad \Omega_0$

ε_0 and ε_1 –positions of energy levels
Ω_0 and Ω_0 – their pair degeneracies
N – actual no. of pairs in the system.

Richardson has shown that the solutions: eigenenergies E and eigenfunctions φ(μ) of the total interacting system, where μ is the number of pairs in the upper level (=0,1,2,….N_1) are solutions of the following set of equations:

$$(\omega_\mu - E)\phi(\mu) - A_\mu \phi(\mu+1) - B_\mu \phi(\mu-1) = 0$$

where

$$\omega_\mu = 2\mu\varepsilon - g(N-\mu)(\Omega_0 - N + \mu + 1) - g\mu(\Omega_1 - \mu + 1)$$

$$A_\mu = g(N-\mu)(\Omega_1 - \mu)$$

$$B_\mu = g\mu(\Omega_0 - N + \mu)$$

This is rare example of the exactly soluble interacting many body system.

Two level model – the results

Finite size corrections:

scale g → g/N

find ground state $e=E_{GS}(N)/N$

and excitation en. $E^{(1)}_{ex}=E_1-E_{GS}$

$$e(N, h=g=1) = -\frac{h}{2} - \frac{A_1}{N} + \frac{B}{N^{\beta}} + \ldots$$

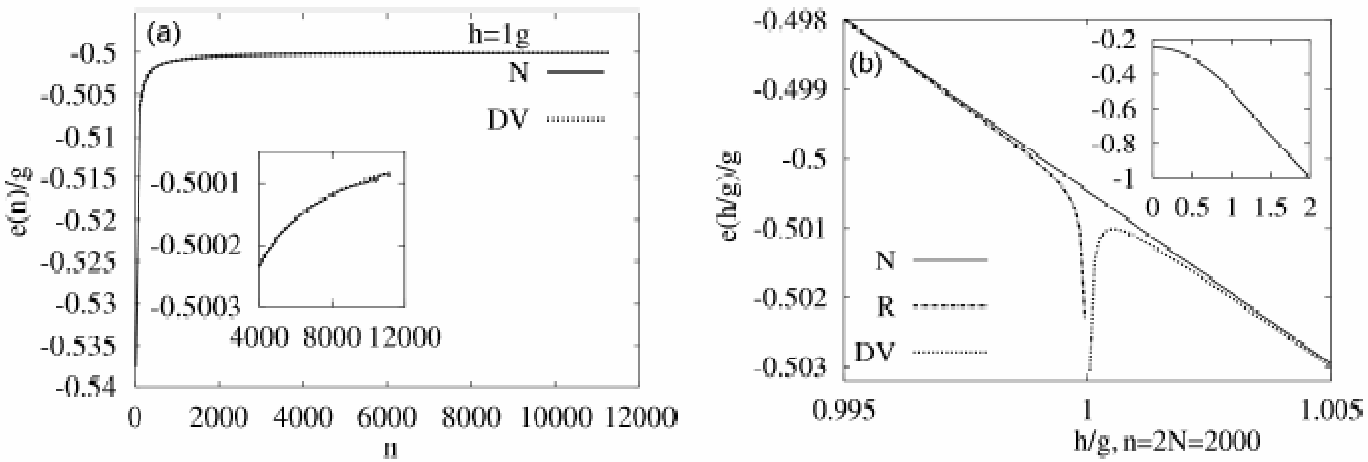


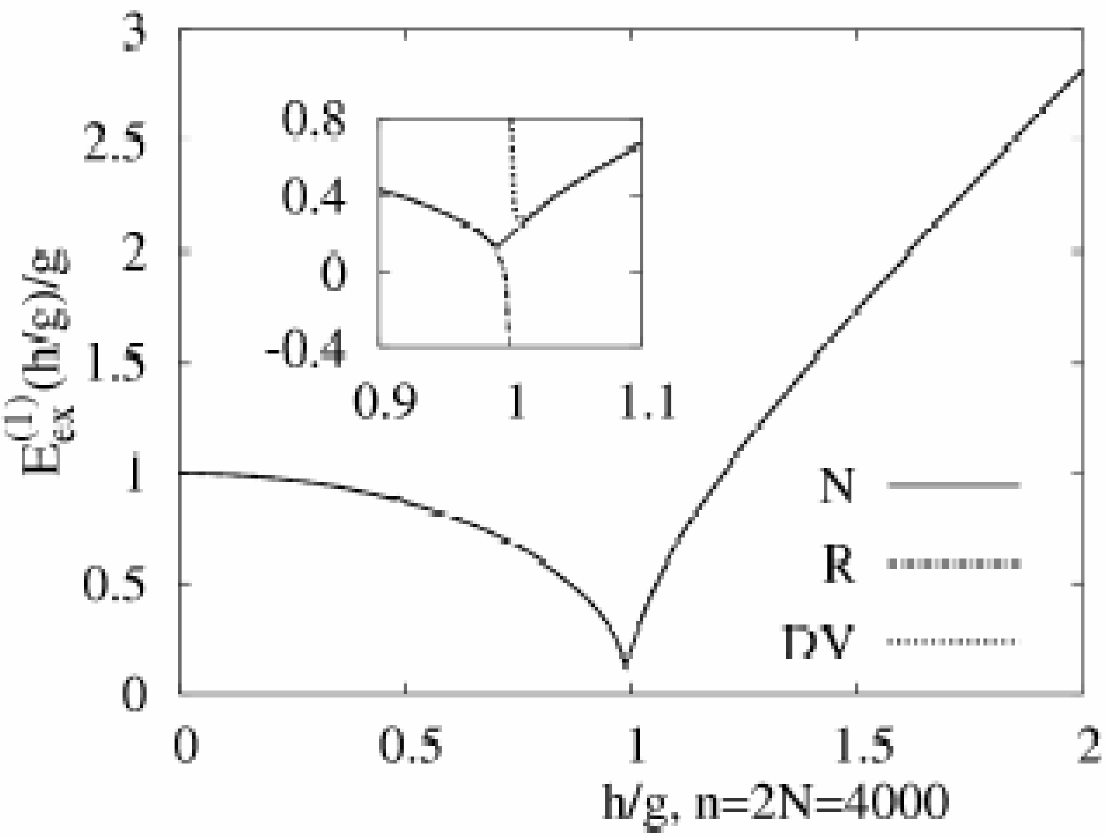
Fig. 1. The ground state energy per electron $e = E_{GS}/2N$ in units of g as a function of the number of electrons $n = 2N$ for $h = g = 1$ (a) and h/g for $n = 2000$ (b). Our results (solid lines) are compared with those of [5] (dashed line — R) and [6] (dotted lines — DV). The insets show respective data on an expanded scales.

β=4/3, at the critical point (both Dusuel,Vidal and our numerical work). Average distance between pairs ~$N^{-1/3}$. Corrections ~ 1/volume, $(1/\text{surface})^2$, etc.

(R): R.W.Richardson (1965).
(V): S. Dusuel, J. Vidal, Phys. Rev A71(2005).

A. Ciechan, KIW,
Acta Phys. Pol. **109**, 569 (2006)

Excitation energy $E^{(1)}_{ex}=E_1-E_{GS}$



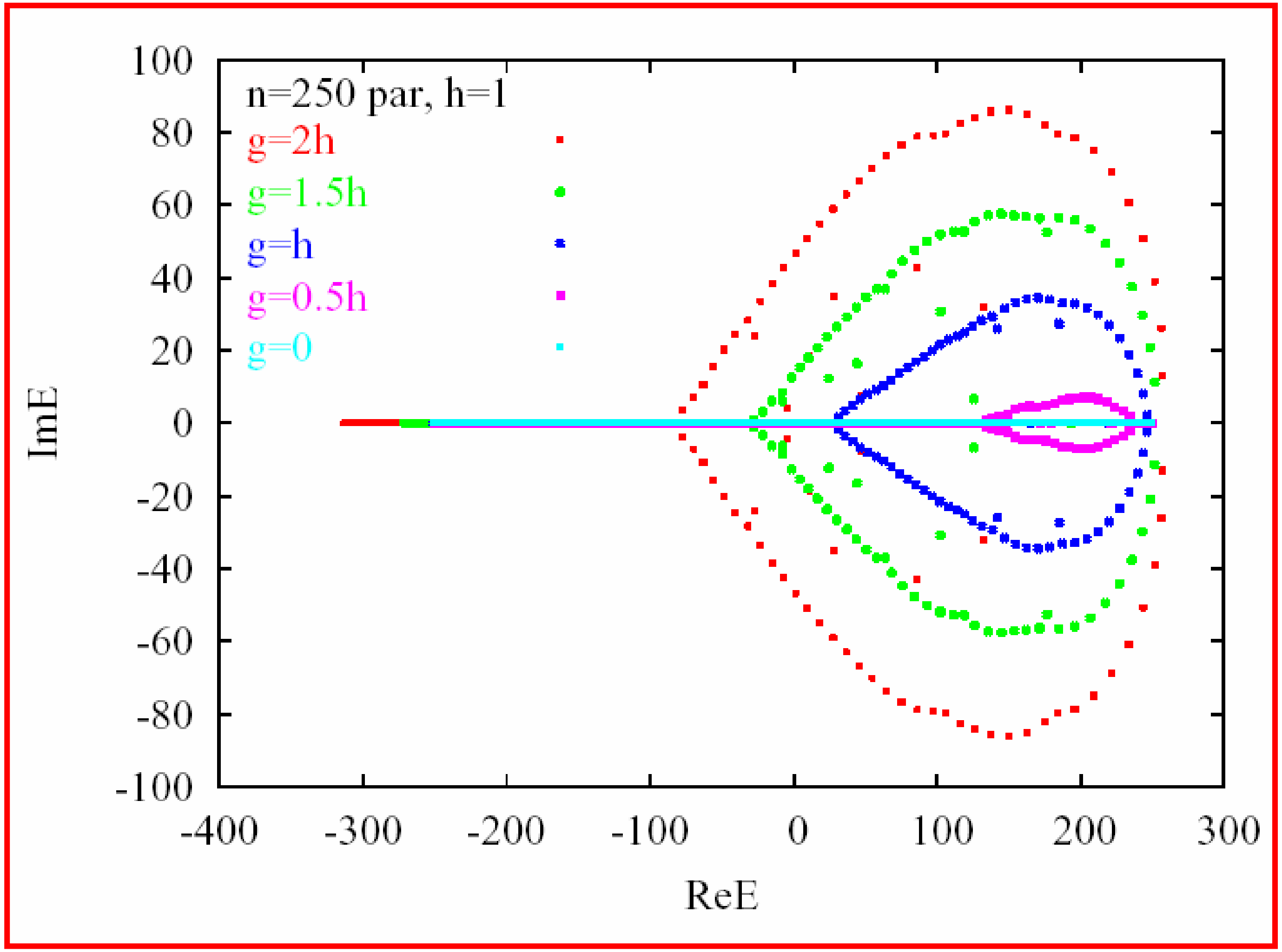
Two level model – thermodynamics. How to formulate it?

- One knows all eigenenergies and eigenfunctions of the interacting many body system!
- Calculate the partition function of the canonical ensemble

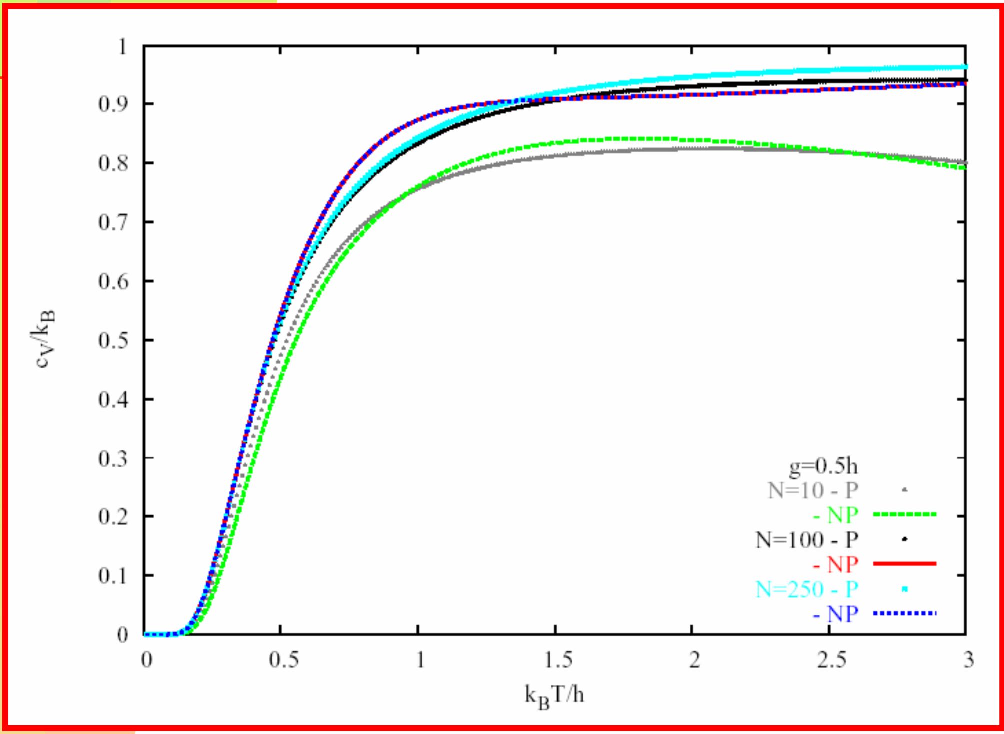
$$Z(T) = \sum_{\nu} e^{-\beta E_{\nu}} \sim\sim\sim\sim\sim\sim\sim\sim\sim \beta = 1/k_B T$$

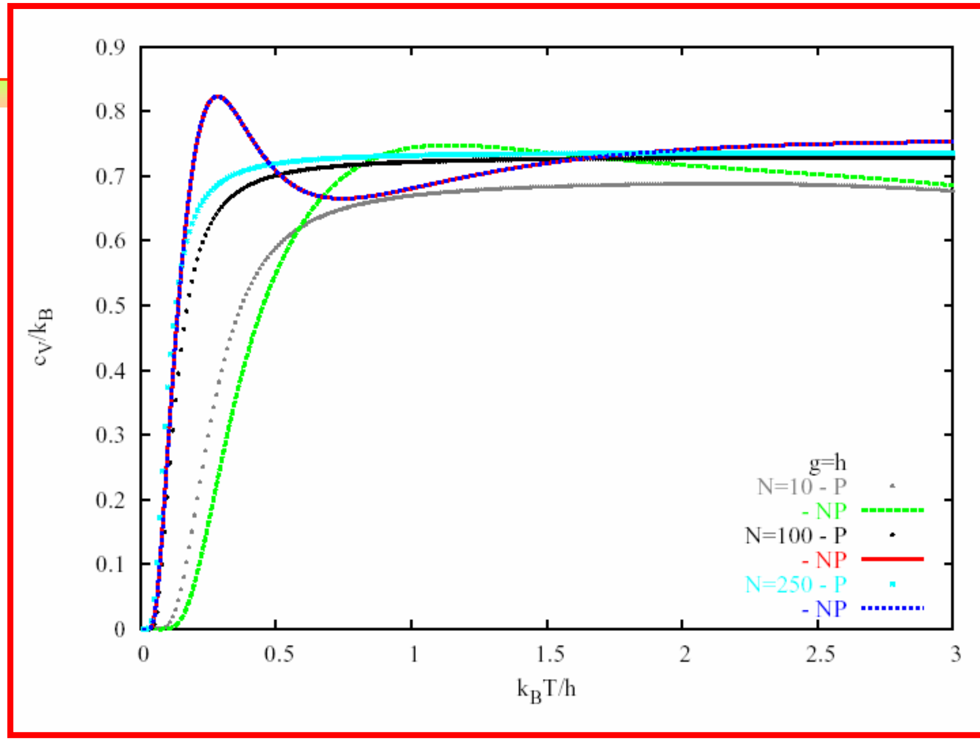
- But:
  - we know the solution at T=0K only
  - some of the energies are complex (numerical problem!) with each complex energy is accompanied by the conjugate one
- Is it possible to formulate the thermodynamics? What is the dependence of the specific heat on temperature? The pair susceptibility and superconducting transition temperature of the small system? Coherence length?

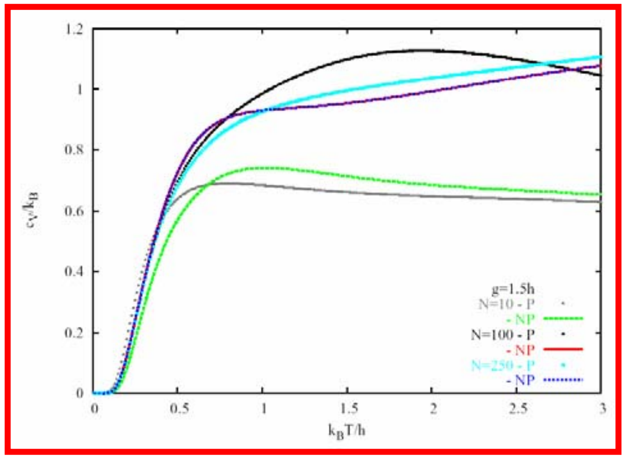
Two Level Model: energies



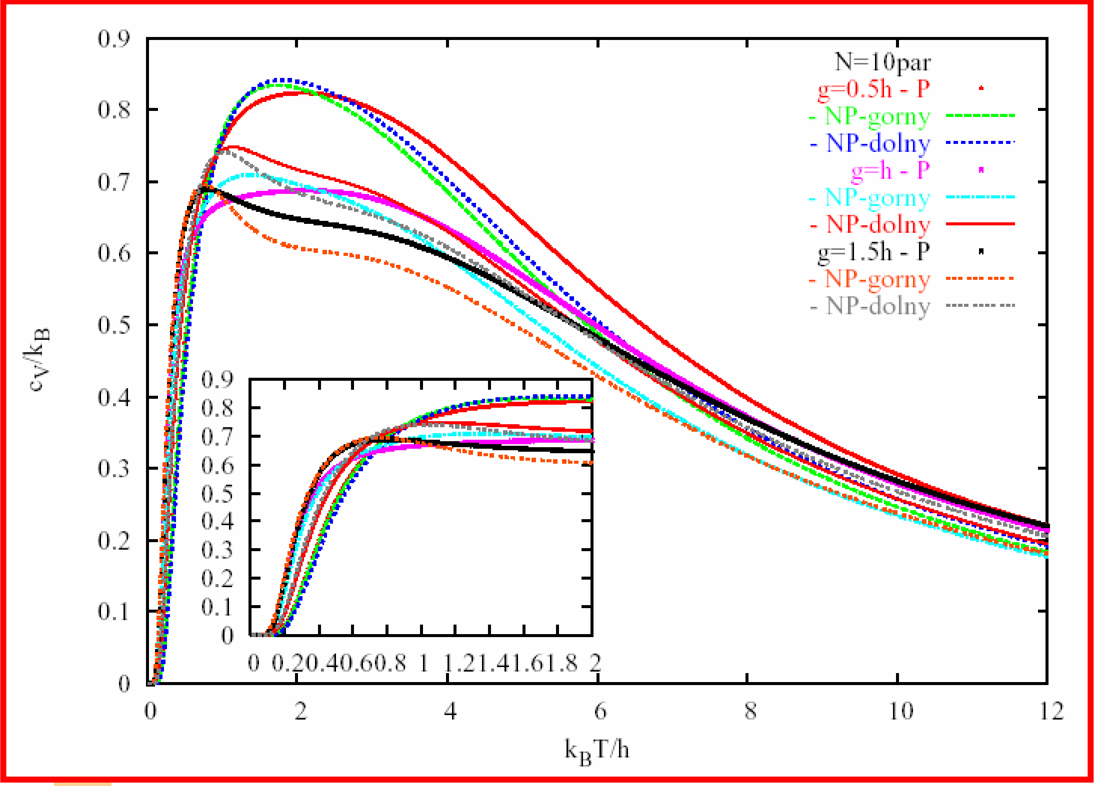
Two Level Model: specific heat

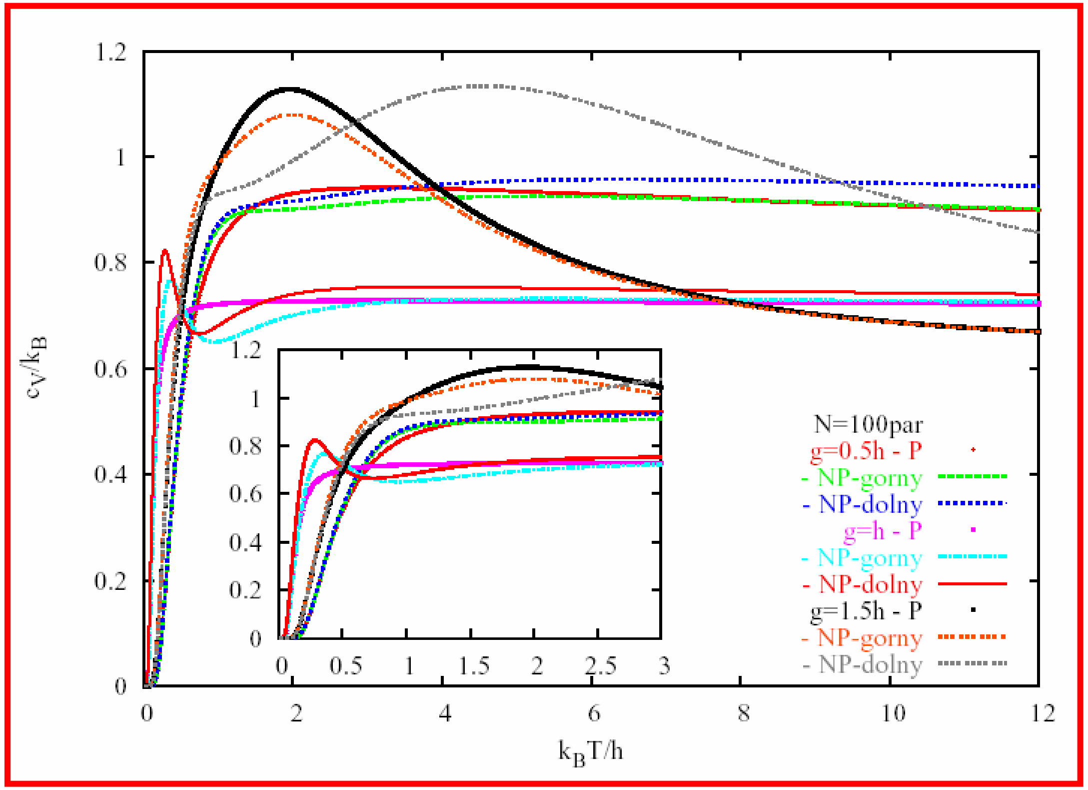






Two Level Model: specific heat





Heat capacity: Monte Carlo vs. Richardson

PHYSICAL REVIEW B **73**, 132509 (2006)

K. Van Houcke, S. M. A. Rombouts, L. Pollet

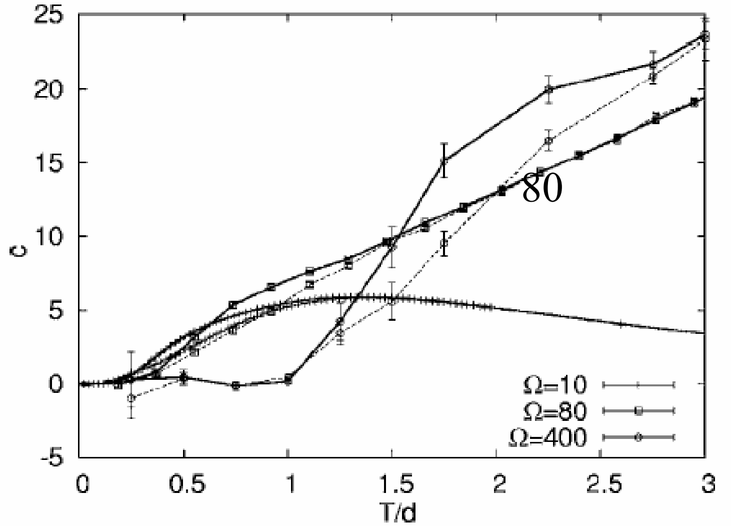
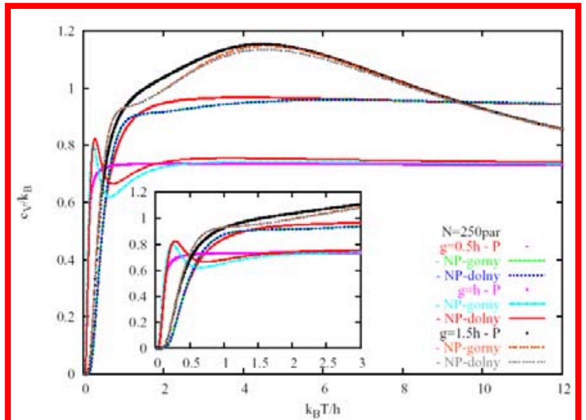
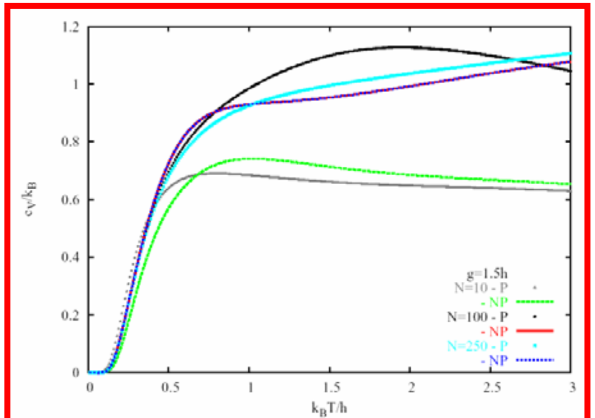


FIG. 5. The heat capacity $c=\partial\langle H\rangle/\partial T$ as a function of T/d for system sizes $\Omega=10$, 80, and 400. Even (odd) grain data points are connected by a solid (dashed) line. Around temperatures $T\approx 0.5d$ for $\Omega=10$, 80, and $T\approx d$ for $\Omega=400$ the heat capacity of the even grain (with $N=\Omega$ electrons) exceeds the odd ($N=\Omega+1$) specific capacity.





Conclusions

- *Many superconductors show small scale structures – superconducting nanograins*
- *The exact Richardson solution exhibits quantum phase transition (at T=0K) from normal metal to BCS superconductors*
- *Leading order corrections to the N=∞ limit: $1/\text{volume}=1/N$; $1/N^{4/3}=(1/\text{surface})^2$*
- *Open issue:*
  - *Properly formulate finite temperature theory! (how to define T-dependent coherence length?)*