

# Quasilocal density functional theory in nuclei including pairing correlations

X. Viñas <sup>a</sup>

V.I. Tselyaev <sup>b</sup>, V.B. Soubbotin <sup>b</sup>

S. Krewald <sup>c</sup>

- V. B. Soubbotin and X. Viñas, *Nucl. Phys.* **A665**, 291 (2000).
- V. B. Soubbotin, V. I. Tselyaev and X. Viñas, *Phys. Rev. C* **67**, 014324 (2003).
- S. Krewald, V. B. Soubbotin, V. I. Tselyaev and X. Viñas, *nucl-th* 04120818.
- X. Viñas, V.I. Tselyaev, S. Krewald and V.B. Soubbotin, *Physics of Atomic Nuclei* **69**, 1207 (2006).

---

<sup>a</sup>Departament d'Estructura i Constituents de la Matèria, Universitat de Barcelona, Barcelona, E-08028, Spain

<sup>b</sup>Nuclear Physics Department, V.A. Fock Institute of Physics, St.Petersburg State University, RU-198504 St.Petersburg, Russia

<sup>c</sup>Institut für Kernphysik, Forschungszentrum, Jülich, Germany

## Local Density Functional Theory

- 

$$H = T + \sum_{i \neq j} v_{ij}^{NN} + \sum_{i \neq j} v_{ij}^{Coul} + \dots,$$

$$T = - \sum_i \frac{\hbar^2}{2m} \Delta_i$$

$$E_{HK}[n] = \inf_{\Psi \rightarrow n} \langle \Psi | H | \Psi \rangle,$$

where  $|\Psi\rangle$  is an arbitrary normalized  $N$ -particle state.

- The short notation  $\Psi \rightarrow n$  hereinafter means the many-to-one mapping of the wave function  $\Psi(x_1, \dots, x_N)$  to the local density  $n(\mathbf{r})$
- In the local DFT the minimum of the functional  $E_{HK}[n]$  is proved to be just the *true* ground-state energy  $E_{GS}$  and is attained for the *true* ground-state density  $n_{GS}$ .

# Kohn-Sham method

- E.H. Lieb, Int.J.Quantum Chem.,243 (1983)

*If  $n(\mathbf{r}) \geq 0$ ,  $\int n(\mathbf{r})d\mathbf{r} = N$ ,  $\int (\nabla \sqrt{n(\mathbf{r})})^2 d\mathbf{r} < \infty$ , then there exists an  $N$ -particle Slater-determinant wave function  $\Psi_0$  built up from an orthonormal set of  $N$  single-particle wave functions  $\varphi_i$ :*

$$\Psi_0(x_1, \dots, x_N) = (N!)^{-1/2} \det\{\varphi_i(x_j)\},$$

*such that  $\Psi_0 \rightarrow n(\mathbf{r})$ . In other words there is a many-to-one mapping of  $N$ -particle Slater  $\Psi_0$  determinant wave functions onto the local particle density  $n(\mathbf{r})$ .*

- This theorem enables one to define the kinetic-energy functional  $T_0[n]$  for a system of non-interacting particles:

$$T_0[n] = \inf_{\Psi_0 \rightarrow n} \langle \Psi_0 | T | \Psi_0 \rangle,$$

and to divide the HK functional  $E_{HK}[n]$  as:

$$E_{HK}[n] = T_0[n] + W[n],$$

where the energy functional  $W[n]$  contains the potential energy as well as the correlation part of the kinetic energy.

- As the density  $n$  is produced by some Slater-determinant wave function, we have

$$n_q(\mathbf{r}) = \sum_{i=1}^N \sum_{\sigma} |\varphi_i(\mathbf{r}, \sigma, q)|^2 .$$

and

$$T_0[n] = \sum_{i=1}^N \frac{\hbar^2}{2m} \sum_{\sigma, q} \int |\nabla \varphi_i(\mathbf{r}, \sigma, q)|^2 d\mathbf{r} .$$

- Applying the variational principle to the functional  $E_{HK}[n]$  with functions  $\varphi_i, \varphi_i^*$  as functional variables, one obtains the following KS equations:

$$h_{HK}\varphi_i = \varepsilon_i\varphi_i ,$$

with

$$h_{HK} = -\frac{\hbar^2}{2m}\Delta + U(\mathbf{r}) ,$$

where  $U(\mathbf{r}) = \delta W/\delta n$  is the local mean-field potential and  $\varepsilon_i$  are the Lagrange multipliers to ensure the normalization condition of the single-particle wave functions  $\varphi_i$ .

## DFT in nuclei

- $\tilde{H}$  is a  $N$ -particle Hamiltonian with an *effective* NN interaction:

$$\tilde{H} = T + \sum_{i \neq j} (\tilde{v}_{ij}^c + \tilde{v}_{ij}^c) + \sum_{i \neq j} v_{ij}^{Coul}.$$

where

$$\hat{v}_{ij}^c = \sum_n [w_n + b_n P_{ij}^\sigma - h_n P_{ij}^\tau - m_n P_{ij}^\sigma P_{ij}^\tau] v_n(s),$$

and

$$\hat{v}_{ij}^{so} = iW_0 (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \cdot [\mathbf{k}' \times \delta(\mathbf{r}_i - \mathbf{r}_j) \mathbf{k}],$$

- We define the quasi-local HF energy functional as

$$\mathcal{E}_0^{QL} = \int d\mathbf{r} \mathcal{H}_0(\mathbf{r}).$$

where

$$\mathcal{H}_0 = \frac{\hbar^2}{2m} (\tau_n + \tau_p) + \mathcal{H}_{Dir}^{Nucl} + \mathcal{H}_{Exch}^{Nucl} + \mathcal{H}_{Dir}^{Coul} + \mathcal{H}_{Exch}^{Coul} + \mathcal{H}^{so}.$$

- The residual correlation energy entering our energy functional is taken from the phenomenological ansatz

$$\begin{aligned} E_{RC}[\hat{n}] &= \frac{t_3}{4} \int d\mathbf{r} n^\alpha(\mathbf{r}) [(2 + x_3) n^2(\mathbf{r}) \\ &\quad - (2x_3 + 1) (n_p^2(\mathbf{r}) + n_n^2(\mathbf{r}))]. \end{aligned}$$

- ETF density matrix. For a spherical local potential it reads:

$$\begin{aligned}\tilde{\rho}_{ETF}(\vec{R}, s) &= \rho \frac{3j_1(k_F s)}{k_F s} + \frac{s^2}{72} \Delta\rho [j_0(k_F s) - 6 \frac{j_1(k_F s)}{k_F s}] \\ &- \frac{s^2}{216} \frac{(\nabla\rho)^2}{\rho} [4j_0(k_F s) - 9 \frac{j_1(k_F s)}{k_F s}]\end{aligned}$$

- Energy

$$E = \int d\vec{R} \left[ \frac{\hbar^2 \tau_{ETF}(\vec{R})}{2m} + \frac{1}{2} \rho(\vec{R}) \int d\vec{s} \rho(\vec{R}-\vec{s}) v(\vec{R}, \vec{s}) + \epsilon_{ex}^{ETF}(\vec{R}) \right]$$

- Exchange Energy

$$\begin{aligned}\epsilon_{ex}^{ETF}(\mathbf{R}) &= \epsilon_{ex,0}^{ETF}(\mathbf{R}) + \epsilon_{ex,2}^{ETF}(\mathbf{R}) = \frac{1}{2} \rho(\mathbf{R}) \int ds v(s) \frac{9j_1^2(k_F s)}{k_F^2 s^2} + \\ &\frac{\hbar^2}{2m} \left[ (f-1) (\tau_{ETF} - \frac{3}{5} k_F^2 \rho - \frac{1}{4} \Delta\rho) + k_F f_k \left( \frac{1}{27} \frac{(\nabla\rho)^2}{\rho} - \frac{1}{36} \Delta\rho \right) \right]\end{aligned}$$

- Effective Mass

$$f(\vec{R}, k) = \frac{m}{m^*(\vec{R}, k)} = 1 + \frac{m}{\hbar^2 k^2} V_{ex,k}(\vec{R}, k)$$

calculated at  $k = k_F(\vec{R})$ .

- In the Kohn y Sham approach  $\tau_{ETF}$  is changed by  $\tau_{KS}$

## Single-particle equations

- Applying the variational principle to the energy functional  $\mathcal{E}^{QL}[\rho_{QL}]$  with  $\varphi_i, \varphi_i^*$  as functional variables. The resulting single-particle equations are:

$$h_q \varphi_i = \varepsilon_i \varphi_i,$$

where

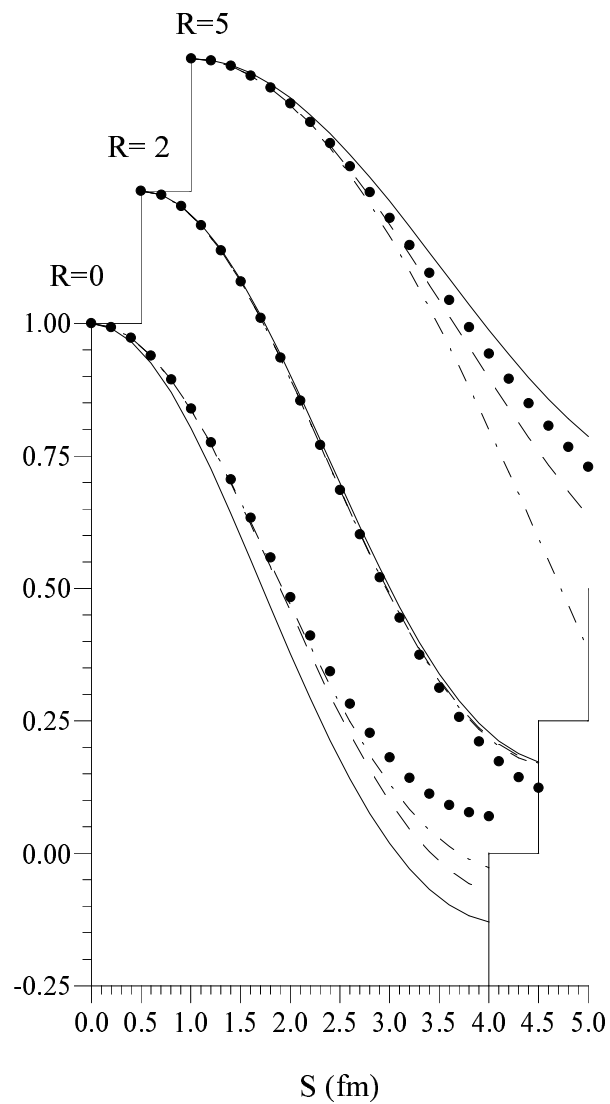
- 

$$h_q = -\nabla \frac{\hbar^2}{2m_q^*(\mathbf{r})} \nabla + U_q(\mathbf{r}) - i\mathbf{W}_q(\mathbf{r}) \cdot [\nabla \times \boldsymbol{\sigma}],$$

and

- 

$$\frac{\hbar^2}{2m_q^*(\mathbf{r})} = \frac{\delta \mathcal{E}^{QL}}{\delta \tau_q(\mathbf{r})}, \quad U_q(\mathbf{r}) = \frac{\delta \mathcal{E}^{QL}}{\delta n_q(\mathbf{r})}, \quad \mathbf{W}_q(\mathbf{r}) = \frac{\delta \mathcal{E}^{QL}}{\delta \mathbf{J}_q(\mathbf{r})}.$$



- $^{40}\text{Ca}$  HO wavefunctions
- Dots Quantal
- Solid line ETF
- Dashed line CB
- Solid-dashed line NV

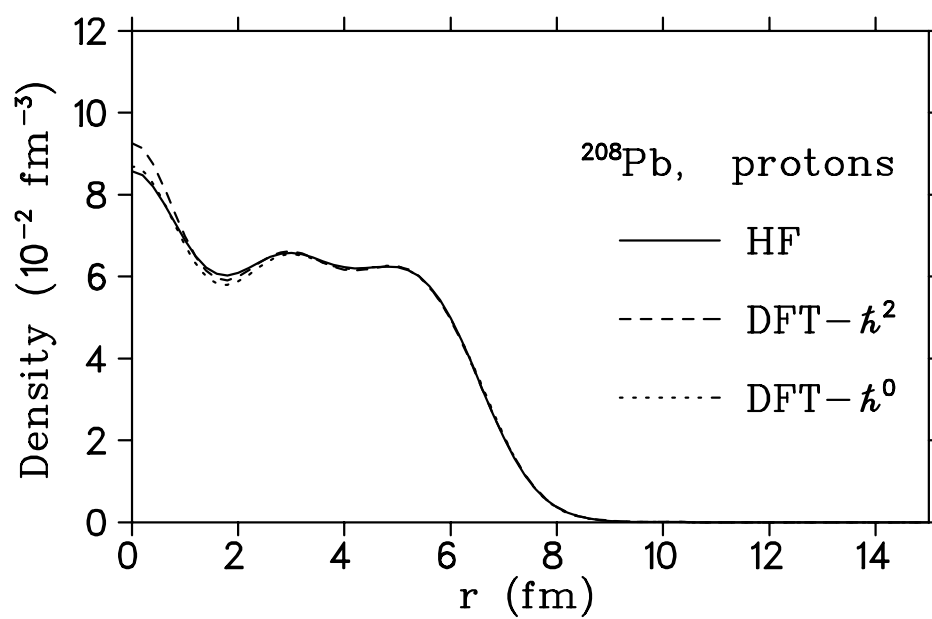
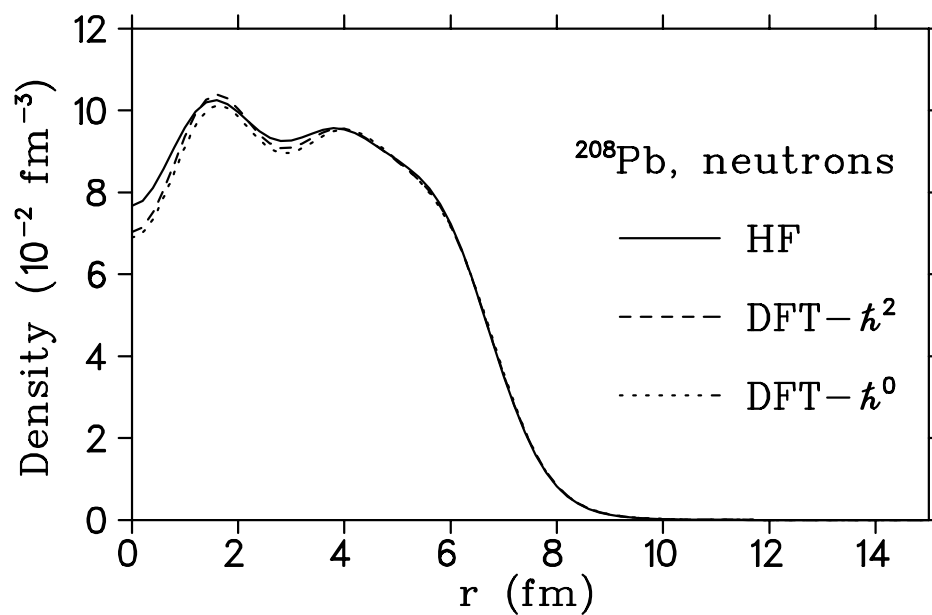


- Coulomb Exchange Energy

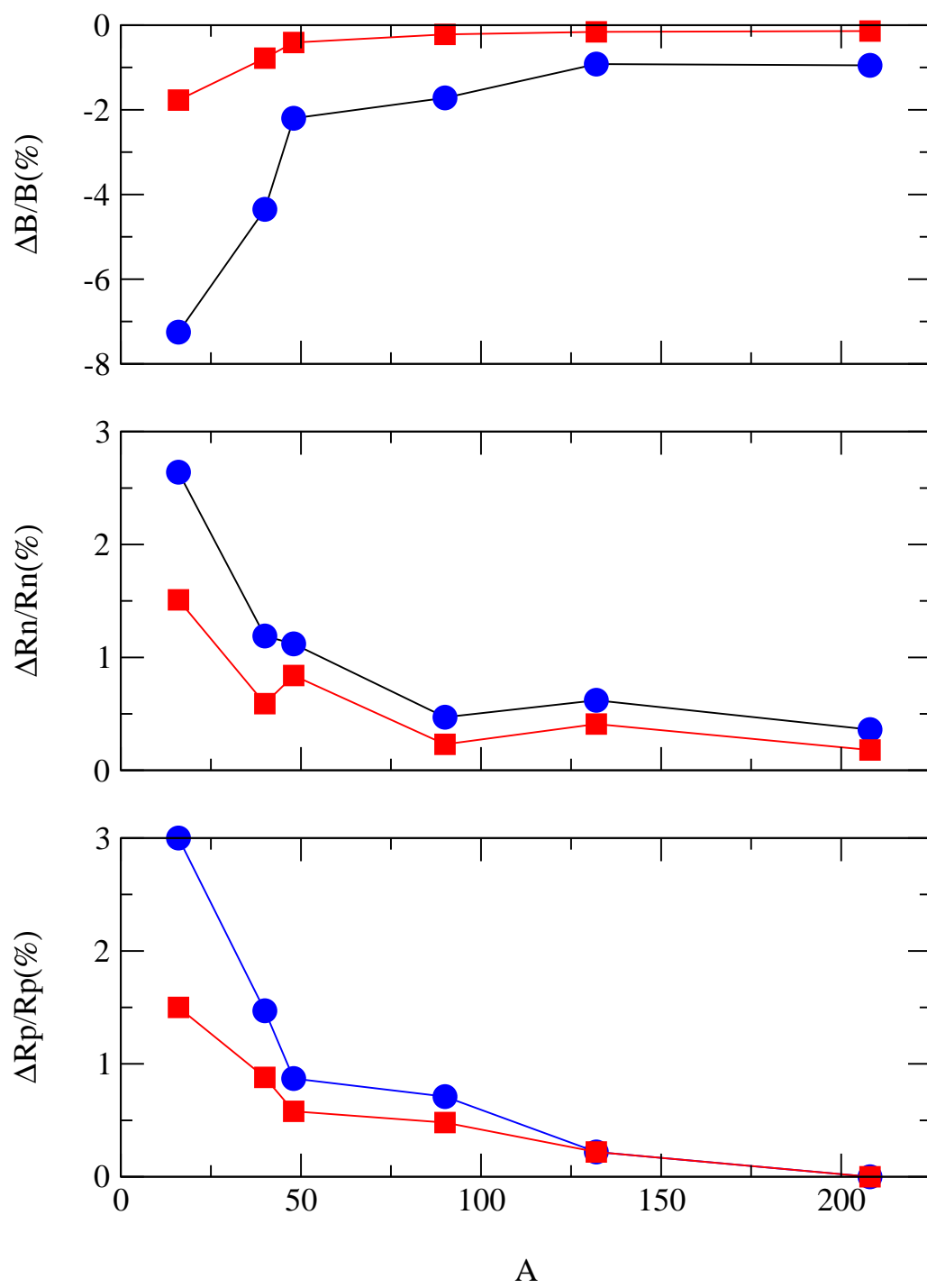
$$\varepsilon_{Coul,ex}^{ETF} = -\frac{3}{4}\left(\frac{3}{\pi}\right)^{1/3}\rho^{4/3} - \frac{7}{432\pi(3\pi^2)^{1/3}}\frac{(\nabla\rho)^2}{\rho^{4/3}}$$

	${}^4He$	${}^{16}O$	${}^{40}Ca$
QM	-0.86	-2.98	-7.46
NV	-0.47	-2.31	-6.42
CB	-0.78	-2.75	-7.03
SL	-0.74	-2.75	-7.05
ETF	-0.82	-2.89	-7.31

- QM, SL, NV y CB from  
X. Campi y A. Bouyssy, Phys. Lett. **B73** (1978)  
263.



		$^{16}\text{O}$	$^{40}\text{Ca}$	$^{48}\text{Ca}$	$^{90}\text{Zr}$	$^{208}\text{Pb}$
$B:$	DFT- $\hbar^0$	120.2	329.6	407.5	772.1	1623.3
	DFT- $\hbar^2$	127.3	341.9	415.0	783.9	1636.6
	HF	129.6	344.6	416.7	785.6	1638.9
$r_n:$	DFT- $\hbar^0$	2.72	3.41	3.62	4.29	5.59
	DFT- $\hbar^2$	2.69	3.39	3.61	4.28	5.58
	HF	2.65	3.37	3.58	4.27	5.57
$r_p:$	DFT- $\hbar^0$	2.75	3.46	3.47	4.24	5.44
	DFT- $\hbar^2$	2.71	3.44	3.46	4.23	5.44
	HF	2.67	3.41	3.44	4.21	5.44
$S_n:$	DFT- $\hbar^0$	12.20	13.21	9.31	11.87	7.45
	DFT- $\hbar^2$	14.55	15.36	9.52	12.02	8.03
	HF	15.08	16.04	9.66	11.88	7.80
$S_p:$	DFT- $\hbar^0$	8.98	6.43	14.07	7.43	8.17
	DFT- $\hbar^2$	11.24	8.45	16.51	8.25	9.29
	HF	12.53	9.27	17.09	8.36	9.51



## Including pairing correlations (BCS level)

$$h \tilde{\phi}_\lambda^* = \varepsilon_\lambda \tilde{\phi}_\lambda^*$$

$$\Delta_\lambda = - \sum_{(\lambda')} \frac{2j_{\lambda'} + 1}{4\pi} V_{(\lambda\lambda')}^{pp} \frac{\Delta_{\lambda'}}{2E_{\lambda'}}.$$

We use the pairing matrix elements

$$V_{(\lambda\lambda')}^{pp} = 2\pi \sum_{i=1}^2 \sum_L V_{i,L}^{(\lambda\lambda')} (l 0 l' 0 | L 0)^2 \left\{ W_{1,i} \right. \\ \left. + W_{2,i} \frac{8(j-l)(j'-l')}{(2j+1)(2j'+1)} [l(l+1) + l'(l'+1) - L(L+1)] \right\}, \\ W_{1,i} = W_i - B_i - H_i + M_i \quad W_{2,i} = W_i + B_i - H_i - M_i,$$

where

$$V_{i,L}^{(\lambda\lambda')} = \int_0^\infty dr_1 r_1^2 \times \\ \int_0^\infty dr_2 r_2^2 v_{i,L}(r_1, r_2) R_\lambda(r_1) R_{\lambda'}(r_1) R_\lambda(r_2) R_{\lambda'}(r_2), \\ v_{i,L}(r_1, r_2) = i_L (2 r_1 r_2 / \mu_i^2) e^{-(r_1^2 + r_2^2) / \mu_i^2},$$

and

$$V_{(\lambda\lambda')}^{pp} = \delta_{q_\lambda, q_{\lambda'}} \int_0^\infty dr r^2 R_\lambda^2(r) R_{\lambda'}^2(r) V^{pp}(n(r)). \\ V^{pp}(n(\mathbf{r})) = \frac{v_0}{2} \left[ 1 - \eta \left( \frac{n(\mathbf{r})}{n_0} \right)^\alpha \right], \quad n_0 = \frac{2}{3\pi^2} k_F^3.$$

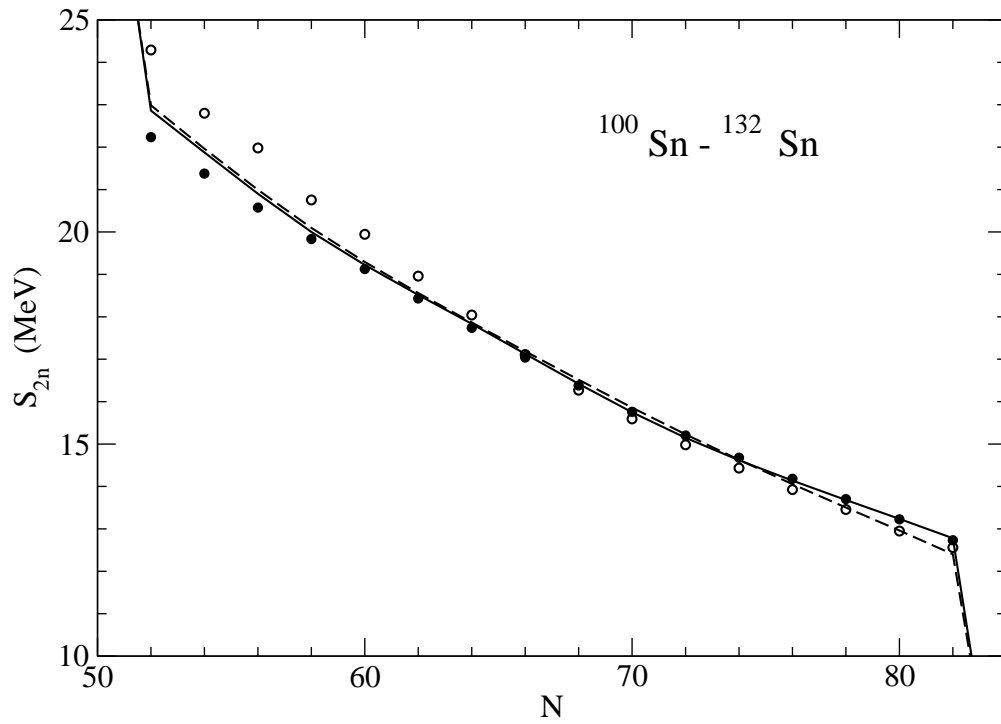
- **HF** and **HFB** from J. Dechargé and D. Gogny, Phys. Rev. C **21**, 1568 (1980).
- **QLDFT** from V. B. Soubbotin, V. I. Tselyaev, and X. Viñas, Phys. Rev. C **67**, 014324 (2003).

	HF	QLDFT
$^{112}\text{Sn}$	-948.301	-949.504
$^{114}\text{Sn}$	-968.403	-969.709
$^{116}\text{Sn}$	-985.701	-987.205
$^{118}\text{Sn}$	-1002.164	-1003.611
$^{120}\text{Sn}$	-1018.898	-1020.229
$^{122}\text{Sn}$	-1032.187	-1033.856
$^{124}\text{Sn}$	-1045.901	-1047.783

	HFB	EQLT1	EQLT2
$^{112}\text{Sn}$	-953.065	-953.064	-954.559
$^{114}\text{Sn}$	-971.434	-971.538	-972.972
$^{116}\text{Sn}$	-988.939	-989.184	-990.592
$^{118}\text{Sn}$	-1005.553	-1005.970	-1007.417
$^{120}\text{Sn}$	-1021.310	-1021.967	-1023.481
$^{122}\text{Sn}$	-1036.295	-1037.288	-1038.837
$^{124}\text{Sn}$	-1050.605	-1052.045	-1053.533

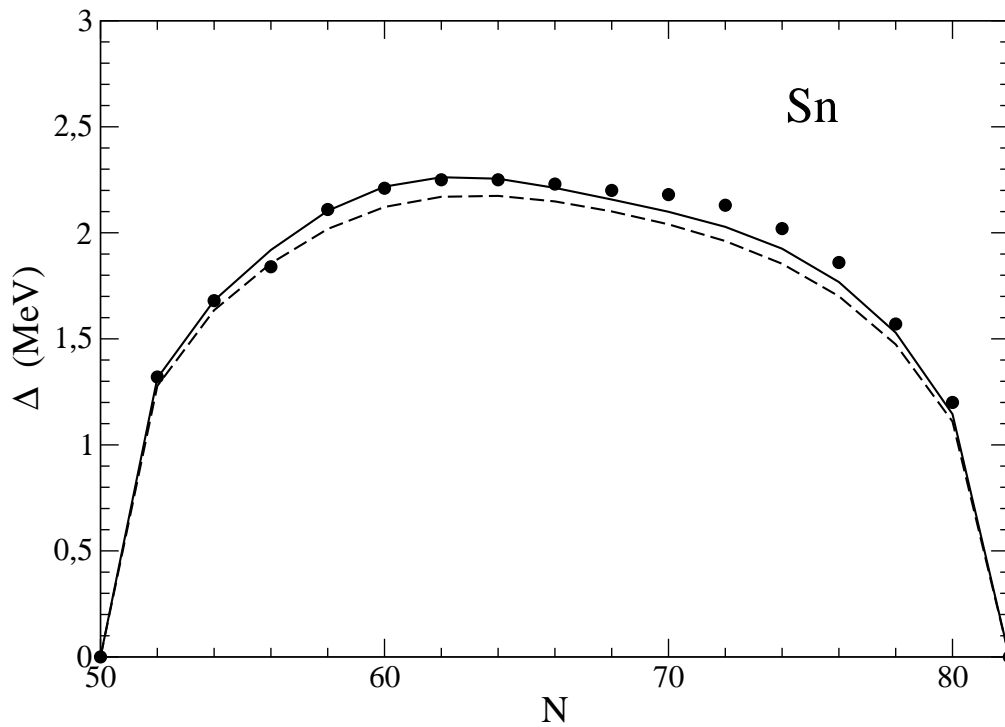
- HFB from M. Kleban, B. Nerlo-Pomorska, J. F. Berger, J. Dechargé, M. Girod, and S. Hilaire, *Phys. Rev. C* **65**, 024309 (2002).

	experiment	HFB	EQLT2	$E_{pair}^{BCS}$	$\bar{\Delta}_n$
$^{100}\text{Sn}$	-824.800	-831.276	-827.125	0.00	0.00
$^{102}\text{Sn}$	-849.090	-853.512	-849.971	-5.96	1.31
$^{104}\text{Sn}$	-871.890	-874.888	-871.850	-9.99	1.68
$^{106}\text{Sn}$	-893.870	-895.463	-892.794	-13.15	1.91
$^{108}\text{Sn}$	-914.626	-915.297	-912.862	-15.81	2.07
$^{110}\text{Sn}$	-934.571	-934.424	-932.146	-17.77	2.17
$^{112}\text{Sn}$	-953.531	-952.857	-950.712	-18.95	2.22
$^{114}\text{Sn}$	-971.574	-970.596	-968.592	-19.42	2.22
$^{116}\text{Sn}$	-988.684	-987.636	-985.794	-19.39	2.19
$^{118}\text{Sn}$	-1004.954	-1004.018	-1002.325	-19.01	2.15
$^{120}\text{Sn}$	-1020.546	-1019.778	-1018.198	-18.38	2.09
$^{122}\text{Sn}$	-1035.529	-1034.978	-1033.442	-17.41	2.01
$^{124}\text{Sn}$	-1049.963	-1049.655	-1048.090	-15.92	1.91
$^{126}\text{Sn}$	-1063.889	-1063.836	-1062.173	-13.69	1.75
$^{128}\text{Sn}$	-1077.345	-1077.538	-1075.715	-10.49	1.52
$^{130}\text{Sn}$	-1090.293	-1090.763	-1088.784	-6.42	1.18
$^{132}\text{Sn}$	-1102.851	-1103.496	-1101.182	0.00	0.00



- Two-neutron separation energies of tin isotopes in the  $^{100}\text{Sn}$ - $^{132}\text{Sn}$  range. The EQLT1 and EQLT2 plus BCS results are displayed by the solid and broken curves correspondingly. The HFB results (filled circles) as well as the experimental values (open circles) are also shown.
- HFB from M. Kleban, B. Nerlo-Pomorska, J. F. Berger, J. Dechargé, M. Girod, and S. Hilaire, Phys. Rev. C **65**, 024309 (2002).





- Average pairing gaps obtained from the EQLT2+BCS (solid line) and EQLT1+BCS calculations (broken line) compared to the HFB results.
- HFB from J. Dobaczewski, W. Nazarewicz, T. R. Werner, J. F. Berger, C. R. Chinn, and J. Dechargé, Phys. Rev. C **53**, 2809 (1996).

## The non-local generalization of the DFT

- We define the energy functional

$$\mathcal{E}_0[\rho_0] = \inf_{\Psi_0 \rightarrow \rho_0} \langle \Psi_0 | \tilde{H} | \Psi_0 \rangle ,$$

- $\Psi_0$  is any Slater-determinant wave function as the one defined previously ,  $\rho_0$  is the single-particle DM produced by  $\Psi_0$  and  $\tilde{H}$  is an *effective* many-body Hamiltonian which generally does not coincide with the microscopic Hamiltonian  $H$ .

- We define the residual correlation energy  $E_{RC}$  as

$$E_{RC}[\hat{n}] = E[\hat{n}] - E_0[\hat{n}] , \quad \hat{n} = \{n_p, n_n\} ,$$

$$E[\hat{n}] = \inf_{\Psi \rightarrow \hat{n}} \langle \Psi | H | \Psi \rangle ,$$

$$\begin{aligned} E_0[\hat{n}] &= \inf_{\Psi_0 \rightarrow \hat{n}} \langle \Psi_0 | \tilde{H} | \Psi_0 \rangle = \inf_{\rho_0 \rightarrow \hat{n}} \inf_{\Psi_0 \rightarrow \rho_0} \langle \Psi_0 | \tilde{H} | \Psi_0 \rangle \\ &= \inf_{\rho_0 \rightarrow \hat{n}} \mathcal{E}_0[\rho_0] . \end{aligned}$$

- The energy functional of the non-local DFT is given by

$$\mathcal{E}[\rho_0] = \mathcal{E}_0[\rho_0] + E_{RC}[\hat{n}] ,$$

which main property is

$$\inf_{\rho_0} \mathcal{E}[\rho_0] = \inf_{\hat{n}} \inf_{\rho_0 \rightarrow \hat{n}} \mathcal{E}[\rho_0] = \inf_{\hat{n}} E[\hat{n}] = E_{GS} ,$$

- $E_{GS}$  is the *true* ground state energy of the interacting system as in the case of the HK theory.

## Reduction to the quasilocal theory

- We introduce a reduced energy functional  $\mathcal{E}_0^{QL}$  which depends the *local* particle  $n_q$ , kinetic-energy  $\tau_q$  and spin  $\mathbf{J}_q$  *densities* for neutrons and protons:

$$n_q(\mathbf{r}) = \sum_{\sigma} \int dx' \delta(x - x') \rho_0(x, x'),$$

$$\tau_q(\mathbf{r}) = \sum_{\sigma} \int dx' \delta(x - x') (\nabla_r \nabla_{r'}) \rho_0(x, x'),$$

$$\mathbf{J}_q(\mathbf{r}) = i \sum_{\sigma} \int dx' \delta(\mathbf{r} - \mathbf{r}') \delta_{q,q'} [(\boldsymbol{\sigma})_{\sigma',\sigma} \times \nabla_r] \rho_0(x, x'),$$

where  $\delta(x - x') = \delta(\mathbf{r} - \mathbf{r}') \delta_{\sigma,\sigma'} \delta_{q,q'}$ .

- Using the notation  $\rho_{QL} \equiv \{n_p, n_n, \tau_p, \tau_n, \mathbf{J}_p, \mathbf{J}_n\}$  we define the quasi-local energy functional as:

$$\mathcal{E}^{QL}[\rho_{QL}] = \mathcal{E}_0^{QL}[\rho_{QL}] + E_{RC}[\hat{n}],$$

$$\mathcal{E}_0^{QL}[\rho_{QL}] = \inf_{\rho_0 \rightarrow \rho_{QL}} \mathcal{E}_0[\rho_0].$$

- Notice that the many-to-one mapping  $\rho_0 \rightarrow \rho_{QL}$  is established according to the previous equations, and that  $\hat{n} \in \rho_{QL}$ . Finally

$$\inf_{\rho_{QL}} \mathcal{E}^{QL}[\rho_{QL}] = E_{GS}.$$

## Generalization with account of pairing correlations

- We generalize the QLDFT starting from the extended density matrix (EDM), which in matrix form reads

$$\mathcal{R} = \begin{pmatrix} \rho & \tilde{\kappa} \\ -\tilde{\kappa}^* & 1 - \rho^* \end{pmatrix}.$$

where  $\rho(x, x') = \langle \Psi | a^\dagger(x') a(x) | \Psi \rangle$  is the normal DM,  $\tilde{\kappa}(x, x')$  is an auxiliary quantity which has a sense of an anomalous DM  $\kappa(x, x') = \langle \Psi | a(x') a(x) | \Psi \rangle$ , but which does not coincide with  $\kappa$  in the general case.

- The main condition imposed on the matrix  $\tilde{\kappa}$  is that the EDM be idempotent ( $\mathcal{R}^2 = \mathcal{R}$ ). This matrix can be easily constructed in the canonical basis where the DM  $\rho$  is diagonal. The quasiparticle-vacuum wave function  $\tilde{\Psi}$  is just the one that produces the EDM  $\mathcal{R}$  through the many-to-one mapping  $\tilde{\Psi} \rightarrow \mathcal{R}$ .
- *For an arbitrary nonlocal DM  $\rho(x, x')$ , corresponding to some interacting fermion system, there are a quasiparticle-vacuum wave function  $\tilde{\Psi}$  and a EDM  $\mathcal{R}$  such that  $\tilde{\Psi} \rightarrow \mathcal{R} \rightarrow \rho$  exists. This statement can be considered as a generalization of the Lieb theorem proved for the local particle density and the Slater-determinant wave functions.*

- To extend the DFT one defines a functional which depends only on the normal nonlocal DM  $\rho$

$$E[\rho] = \inf_{\Psi \rightarrow \rho} \langle \Psi | H | \Psi \rangle ,$$

where  $H$  is the exact non-relativistic many-body Hamiltonian and  $\Psi$  an arbitrary normalized many-fermion wave function

- As previously, we introduce an auxiliary functional

$$\tilde{\mathcal{E}}[\rho, \tilde{\kappa}, \tilde{\kappa}^*] = \tilde{\mathcal{E}}[\rho, \tilde{\kappa}, \tilde{\kappa}^*] = \inf_{\tilde{\Psi} \rightarrow \rho, \tilde{\kappa}, \tilde{\kappa}^*} \langle \tilde{\Psi} | \tilde{H} | \tilde{\Psi} \rangle ,$$

where  $\tilde{\Psi}$  are the quasiparticle-vacuum wave functions. In analogy with the discussion in the local case, we introduce the residual correlation energy as:

$$\mathcal{E}[\rho, \tilde{\kappa}, \tilde{\kappa}^*] = \tilde{\mathcal{E}}[\rho, \tilde{\kappa}, \tilde{\kappa}^*] + E_{RC}[\rho] .$$

which main property is

$$\inf_{\rho, \tilde{\kappa}, \tilde{\kappa}^*} \mathcal{E}[\rho, \tilde{\kappa}, \tilde{\kappa}^*] = \inf_{\rho} E[\rho] = E_{GS} ,$$

where  $E_{GS}$  is the exact ground-state energy.

## Reduction to the extended quasilocal theory

We can simplify the motion equations by reducing the total energy functional  $\mathcal{E}[\rho, \tilde{\kappa}, \tilde{\kappa}^*]$  to a quasilocal form. In addition to  $\rho_{QL}$  defined previously, we define the quantities  $\varkappa_q$  which have the meaning of local anomalous densities:

$$\varkappa_q(\mathbf{r}) = i \sum_{\sigma} \int dx' \delta(\mathbf{r} - \mathbf{r}') \delta_{q,q'} (\sigma_y)_{\sigma', \sigma} \tilde{\kappa}(x, x')$$

We define the following quasilocal energy functionals:

$$\mathcal{E}^{QL1}[\rho_{QL}, \tilde{\kappa}, \tilde{\kappa}^*] = \inf_{\rho \rightarrow \rho_{QL}} \mathcal{E}[\rho, \tilde{\kappa}, \tilde{\kappa}^*],$$

$$\mathcal{E}^{QL2}[\rho_{QL}, \varkappa, \varkappa^*] = \inf_{\rho \rightarrow \rho_{QL}} \inf_{\tilde{\kappa} \rightarrow \varkappa} \inf_{\tilde{\kappa}^* \rightarrow \varkappa^*} \mathcal{E}[\rho, \tilde{\kappa}, \tilde{\kappa}^*].$$

The difference between these two versions of the quasilocal approach lies in the definition of the pairing field which in QL1 reads

$$\Delta = -2\delta\mathcal{E}^{QL1}/\delta\tilde{\kappa}^*$$

as in the nonlocal theory, and

$$\Delta = -2i\delta(\mathbf{r} - \mathbf{r}') \delta_{q,q'} (\sigma_y)_{\sigma, \sigma'} \delta\mathcal{E}^{QL2}/\delta\varkappa_q^*(\mathbf{r})$$

in QL2, where the pairing field is completely local.