Collective pairing hamiltonian in a self-consistent framework

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S. T. Belyaev, Nucl. Phys. 64 (1965) 17

> ”In eq.(10.1), apart from the set α_μ, the formally independent parameter is also Δ. However, from physical considerations it is obvious that excitations connected with Δ are essentially single-particel excitations having high energies. Therefore, when considering low-energy collective excitations, Δ can be determined from the static consistency condition (10.9)” p.44

M. Baranger and K. Kumar, Nucl. Phys. A 122 (1968) 241

> ”Thus far, D_M and Δ have been considered as six independent collective variables. [..] Collective motion in the Δ direction represents the pairing vibrations which have recently received much attention. [..]
> However we prefer not to include pairing vibrations together with collective quadrupole motion in our theory. We do this for simplicity, and also because the two phenomena usually manifest themselves in different nuclei and at different energies.” p. 260–261

D. R. Bes, R. E. Broglia, Nucl. Phys. 80 (1966) 289

D. R. Bes, R. E. Broglia, R. P. J. Perazzo, K. Kumar Nucl. Phys. A 143 (1970) 1

Collective variables, collective hamiltonian

Adiabatic Time Dependent HFB (cranking)

Generating Coordinate Method

Variables used

1. Δ, ϕ

$$R(\Delta,\phi) = \begin{pmatrix} \rho & e^{-2i\phi}\kappa \\ -e^{2i\phi}\kappa^* & \rho \end{pmatrix} \leftrightarrow |\Phi\rangle = \prod_{\mu>0} (u_\mu(\Delta) + s_\mu v_{\bar{\mu}}(\Delta) e^{2i\phi} c^+_{\bar{\mu}} c^+_\mu)\,|0\rangle$$

$v_i \to v_i(\Delta, \lambda)$ as in the BCS theory, λ from $\langle N \rangle = N_0$

2. $\alpha = \sum_{\mu>0} u_\mu v_\mu\ ,\ \phi$ (more natural for δ pairing)

$$\sum_{\mu>0} u_\mu v_\mu = \langle P \rangle; \quad P = \frac{1}{2}(\sum_{\mu>0} e^{-2i\phi} s_{\bar{\mu}} c^+_\mu c^+_{\bar{\mu}} + \text{h.c.})$$

Condition

$$\delta\langle H - \lambda\hat{N} - 2xP + \beta Q \rangle = 0$$

leads to equations

$$2\sum_{\mu>0} v_\mu^2 = N$$
$$\Delta_\mu = \sum_{\mu'>0} V_{\mu\mu'} u_{\mu'} v_{\mu'}$$
$$\sum_{\mu>0} u_\mu v_\mu = \alpha$$

where we denoted

$$2v_\mu^2(\Delta_\mu, x, \lambda) = 1 - \varepsilon_\mu/E_\mu, \quad E_\mu = \sqrt{\varepsilon_\mu^2 + d_\mu^2}$$
$$d_\mu = \Delta_\mu + x$$
$$\varepsilon_\mu = e_\mu - \lambda \quad -V_{\mu\mu} v_\mu^2$$

$$u_\mu^2 - v_\mu^2 = \varepsilon_\mu/E_m$$
$$2u_\mu v_\mu = d_\mu/E_\mu$$

Quantum collective hamiltonian

$$H_{\rm coll} = -\frac{1}{2\sqrt{\det g}} \sum_{ij} \frac{\partial}{\partial q^i} \sqrt{\det g}\,(B^{-1})^{ij} \frac{\partial}{\partial q^j} + V$$

Both ATDHFB and GCM methods require derivatives

$$\begin{pmatrix} 0 & f_k \\ -f_k^* & 0 \end{pmatrix} = \left(\frac{\partial R_0}{\partial q_k} \right)_{qp}$$

$$f_{k,\mu\nu} = \langle \Phi | \beta_\nu \beta_\mu | \partial_k \Phi \rangle$$

In our case

$$f_{x,\mu\nu} = s_\nu \delta_{\mu\bar{\nu}} \frac{\partial_x \lambda d_\mu + \varepsilon_\mu}{2E_\mu^2} =: s_\nu \delta_{\mu\bar{\nu}} X_\mu \qquad x \text{ is } x \text{ or } \Delta$$

$$f_{\phi,\mu\nu} = i s_\nu \delta_{\mu\bar{\nu}} \frac{d_\mu}{E_\mu} =: i s_\nu \delta_{\mu\bar{\nu}} F_\mu$$

Moreover

$$\partial_x \lambda = -\sum_\mu \frac{d_\mu \varepsilon_\mu}{E_\mu^3} / \sum_\mu \frac{d_\mu^2}{E_\mu^3}; \qquad \partial_x \alpha = \sum_{\mu>0} \frac{\varepsilon_\mu(\varepsilon_\mu + \partial_x \lambda d_\mu)}{2E_\mu^3} \quad \text{(if needed)}$$

ATDHFB

$$B_{kj} = \frac{1}{2} \sum_{\mu\nu} \frac{f_{j,\mu\nu} f^*_{k,\mu\nu} + f^*_{j,\mu\nu} f_{k,\mu\nu}}{(E_\mu + E_\nu)}$$

$$B_{xx} = \sum_{\mu>0} X_\mu^2 / E_\mu$$

$$B_{x\phi} = \sum_{\mu>0} F_\mu^2 / E_\mu$$

$$B_{x\phi} = 0$$

$$g = B, \quad \det g = \det B$$

$$V = \langle H \rangle$$

GCM

$$
\begin{aligned}
N(a,a') &:= (\Phi(a),\Phi(a')) \\
H(a,a') &:= (\Phi(a),\hat{H}\Phi(a')) \\
h(a,a') &:= H(a,a')/N(a,a') \\
(B^{-1})_{kj} &= \frac{1}{2}(\mathrm{Re}\,h_{12}-\mathrm{Re}\,h_{11})_{kj} \\
h_{12,mn} &= D_{a_m}D_{a'_n}h(a,a')|_{a=a'=q} \\
g_{kj} &= D_{a_m}D_{a'_j}N(a,a')|_{a=a'=q} \\
V &= \langle H\rangle-\frac{1}{2}\sum_{kj}g^{kj}(B^{-1})_{kj}
\end{aligned}
$$

Collective pairing: g and B^{-1} are diagonal

$$g_{xx} = \sum_{\mu>0} X_\mu^2, \qquad g_{\phi\phi} = \sum_{\mu>0} F_\mu^2$$

$$(B^{-1})_{xx} = \sum_{\mu>0} X_\mu^2 W_\mu + b_{xx}$$
$$(B^{-1})_{\phi\phi} = \sum_{\mu>0} F_\mu^2 W_\mu + b_{\phi\phi}$$
$$V = \langle H\rangle - ((B^{-1})_{xx}/g_{xx} + (B^{-1})_{\phi\phi}/g_{\phi\phi})/2$$

and

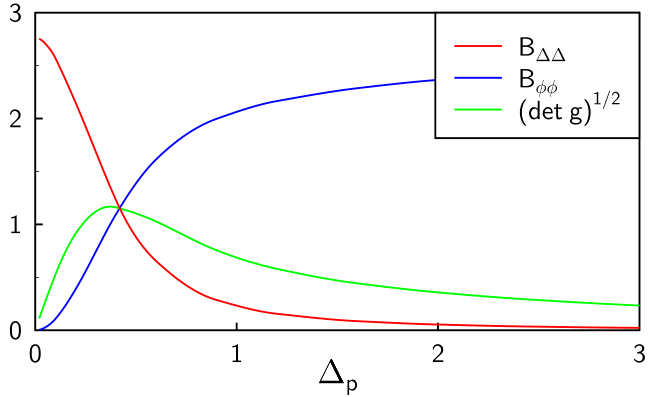
$$W_\mu = E_\mu - x d_\mu / E_\mu$$
$$b_{xx} = \frac{1}{2}\Big(\sum_{\mu\nu>0} X_\mu X_\nu V_{\mu\nu} + \partial_x \ln g_{xx} \sum_{\mu>0} X_\mu \varepsilon_\mu x/E_\mu\Big)$$
$$b_{\phi\phi} = \frac{1}{2}\Big(\sum_{\mu\nu>0} F_\mu F_\nu (v_\mu^2 - u_\mu^2)(v_\nu^2 - u_\nu^2) V_{\mu\nu} +$$
$$+4\sum_{\mu>0} F_\mu^2 V_{\mu\mu} u_\mu^2 v_\mu^2 - g_{xx}^{-1} \partial_x g_{\phi\phi} \sum_{\mu>0} F_\mu \varepsilon_\mu x/E_\mu\Big)$$

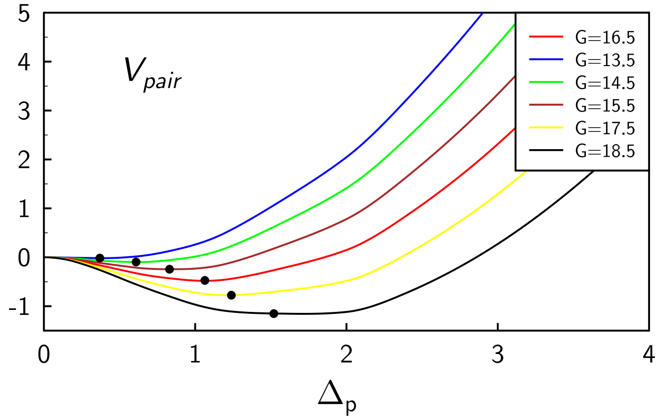
Some examples

ATDHFB, Skyrme interactions, Δ, ϕ variables

$$\left(-\frac{1}{\sqrt{\det g}}\frac{\partial}{\partial\Delta}\sqrt{\det g}B^{-1}_{\Delta\Delta}\frac{\partial}{\partial\Delta}+V\right)f=Ef,\qquad \det g=\sqrt{B_{\Delta\Delta}B_{\phi\phi}}$$

^{128}Xe, SIII interaction, $\beta=0.2$, $\gamma=18^\circ$, only Δ_p





If $\varepsilon_\mu = e_\mu - \lambda - V_{\mu\mu} v_\mu^2$

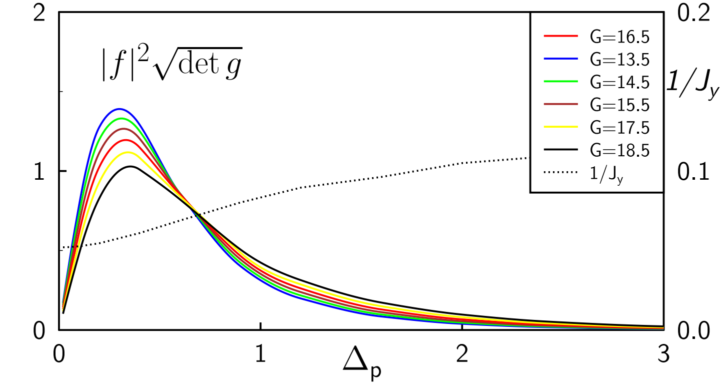
$$f_{\Delta,\mu\nu} \to f_{\Delta,\mu\nu}/w_\mu$$

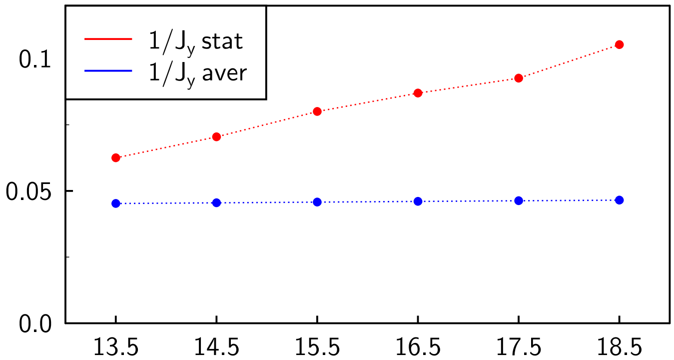
where

$$w_\mu := 1 - \Delta^2 V_{\mu\mu}/2E_\mu^3$$

and

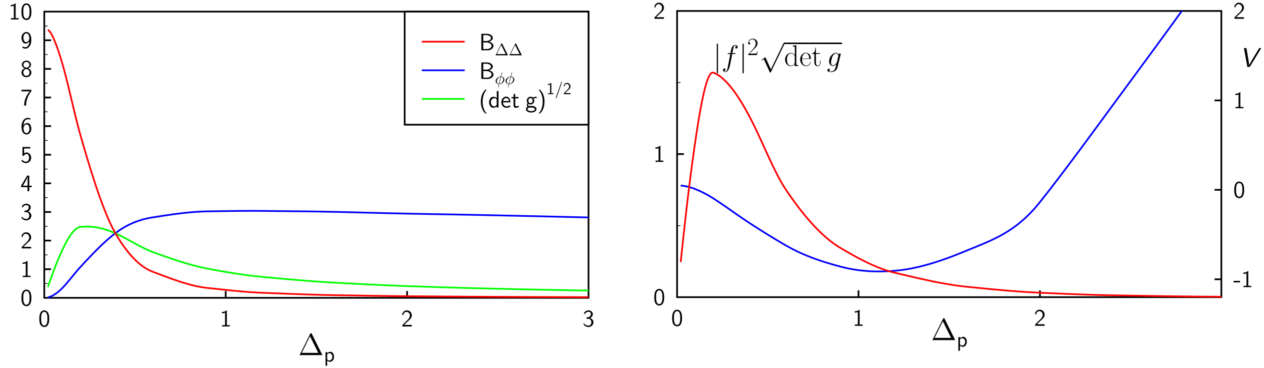
$$\Delta\partial_\Delta\lambda = -\left(\sum_\mu \frac{\epsilon_\mu}{w_\mu E_\mu^3}\right) \Big/ \sum_\mu \frac{1}{w_\mu E_\mu^3}$$


$|f|^2\sqrt{\det g}$
G=16.5
G=13.5
G=14.5
G=15.5
G=17.5
G=18.5
$1/J_y$
$1/J_y$
Δ_p
0
1
2
3
0.0
0.1
0.2

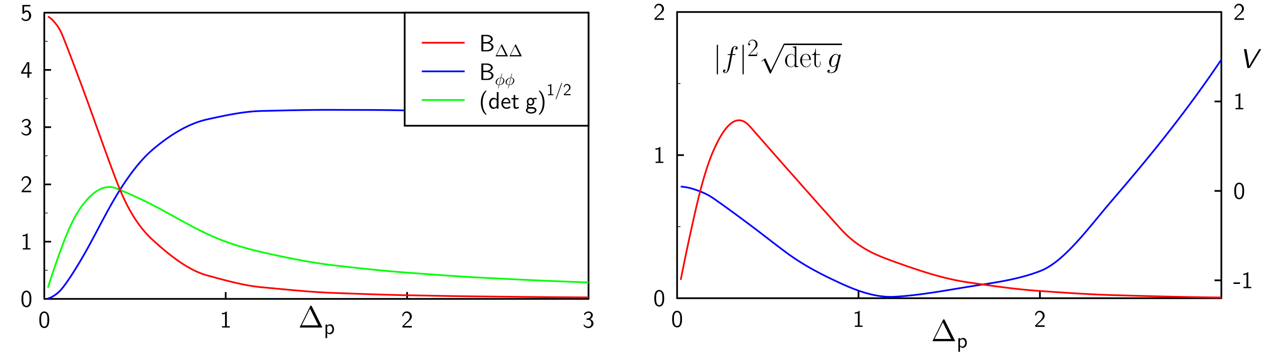

$1/J_y$ stat
$1/J_y$ aver
0.0
0.05
0.1
13.5
14.5
15.5
16.5
17.5
18.5

Some problems

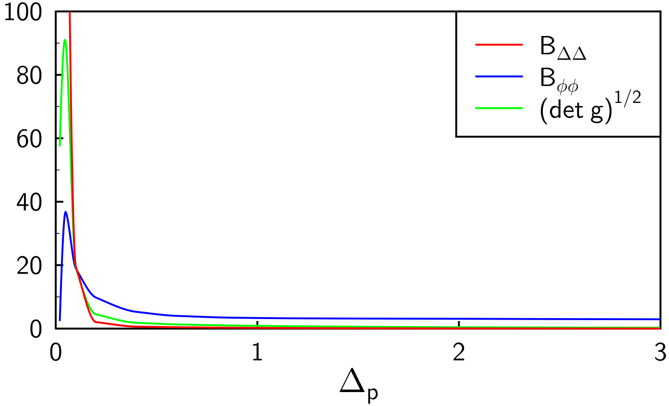
^{152}Sm, $\beta = 0.3$, $\gamma = 0$



^{156}Dy, $\beta = 0.3$, $\gamma = 0$



^{154}Gd, $\beta = 0.3$, $\gamma = 0$



$|e_\mu - \lambda|_{\min}$ is important for $\Delta \to 0$, $B_{\Delta\Delta} \sim |e_\mu - \lambda|^{-3}$