Pairing as Collective Mode

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Krzysztof Pomorski

Department of Theoretical Physics,

Maria Curie-Skłodowska University - Lublin

My coworkers:

This research was performed in collaboration with:

Andrzej Baran, *Andrzej Góźdź,*

Zdzisław Łojewski, *Stanisław Piłat,*

Bożena Nerlo-Pomorska, *Stanisław G. Rohoziński,*

Kamila Sieja, *Julian Srebrny,*

Andrzej Staszczak, *Krystyna Zając*

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- Pairing Hamiltonian in the mean-field approximation

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- Summary and conclusions

Mean-field approximation:

Let us assume the monopole pairing Hamiltonian

$$\hat{H} = \hat{H}_0(\text{def}) - G\hat{P}^+\hat{P} \ ,$$

where $\hat{H}_0$ is the single–particle Hamiltonian and

$$\hat{P} = \sum_{\nu>0} c_{-\nu} c_\nu \ ,$$

is the pair annihilation operator.
The mean–field Hamiltonian should be hermitian:

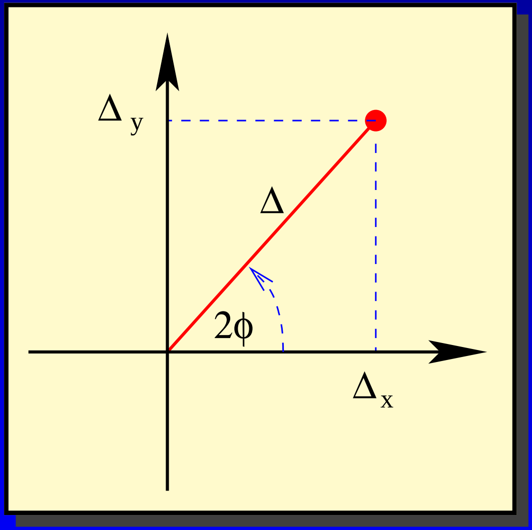
$$\hat{H}_{MF} = \hat{H}_0 - G(\langle\hat{P}^+\rangle\hat{P} + \hat{P}^+\langle\hat{P}\rangle) + G\langle\hat{P}^+\rangle\langle\hat{P}\rangle \ ,$$

what implies that in general $\Delta = \langle\hat{P}\rangle)$ is complex.

The average in $\langle \hat{P} \rangle$ is taken between the BCS wave function of the following form (D.R. Bès et al. NPA 143 (1970) 1):

$$|\Delta\Phi\rangle = e^{iN\Phi} \prod_{\nu>0} [u_\nu(\Delta) + e^{-2i\Phi} v_\nu(\Delta) c^+_\nu c_{-\nu}] |0\rangle$$

The pairing gap Δ and the gauge angle Φ are the generator coordinates and $\langle \hat{P} \rangle = \Delta e^{-2i\Phi}$:



In the following we assume the Gaussian Overlap Approximation of the BCS wave functions on (Δ_x, Δ_y) plane.

Collective Hamiltonian

Using the GCM+GOA model (or cranking) one can obtain the general form of the collective pairing Hamiltonian (A. Góźdź et al. NPA A442 (1985) 50)

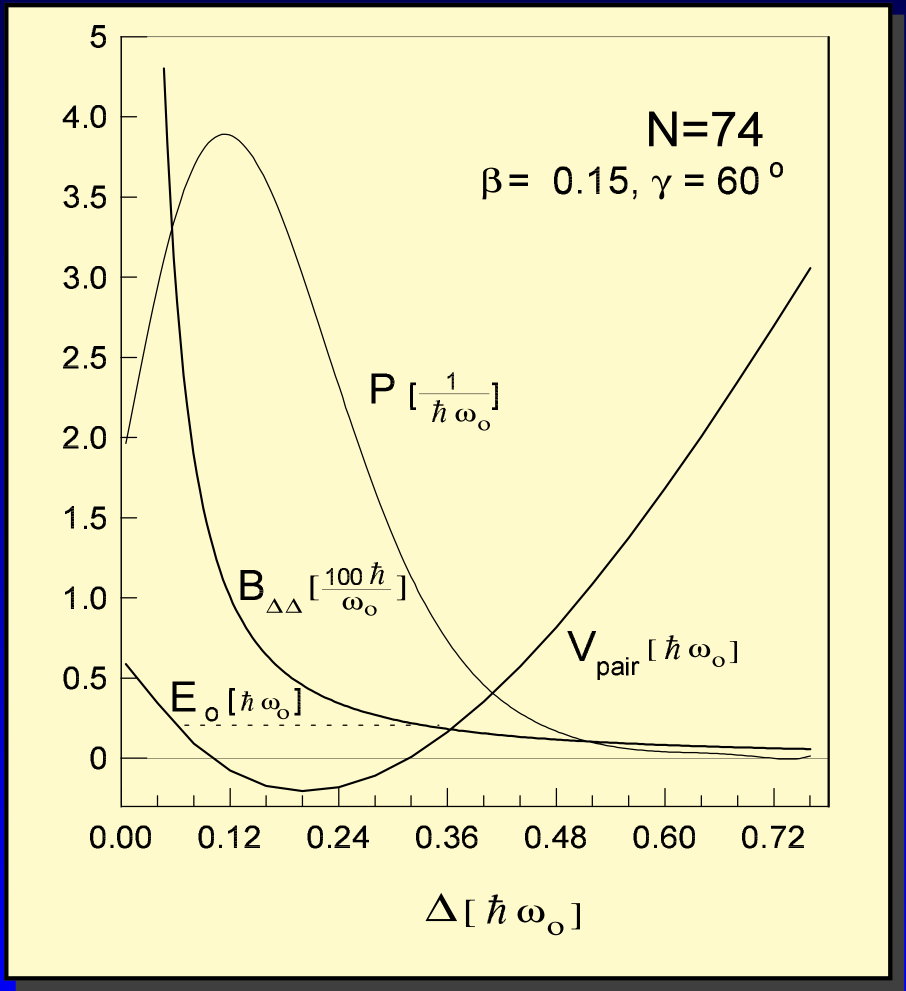
$$
\begin{aligned}
\hat{\mathcal{H}}_{coll} &= -\frac{\hbar^2}{2\sqrt{\det\gamma}}\frac{\partial}{\partial\Delta}\sqrt{\det\gamma}\,\mathcal{M}^{-1}_{\Delta\Delta}\frac{\partial}{\partial\Delta} \\
&- i\hbar\frac{\mathrm{Im} < \Delta\Phi|\frac{\partial}{\partial\Phi}\hat{H}|\Delta\Phi\rangle}{\gamma_{\Phi\Phi}}\frac{\partial}{\partial\Phi} \\
&- \frac{1}{2}\hbar^2\mathcal{M}^{-1}_{\Phi\Phi}\frac{\partial^2}{\partial\Phi^2} + V(\Delta)\ ,
\end{aligned}
$$

where $\mathcal{M}_{\Delta\Delta}$ and $\mathcal{M}_{\Phi\Phi}$ are the mass parameters and γ is the metric tensor.

Collective pairing potential

$$V = \langle \Delta\Phi | \hat{H} | \Delta\Phi \rangle - \epsilon_0 \; .$$

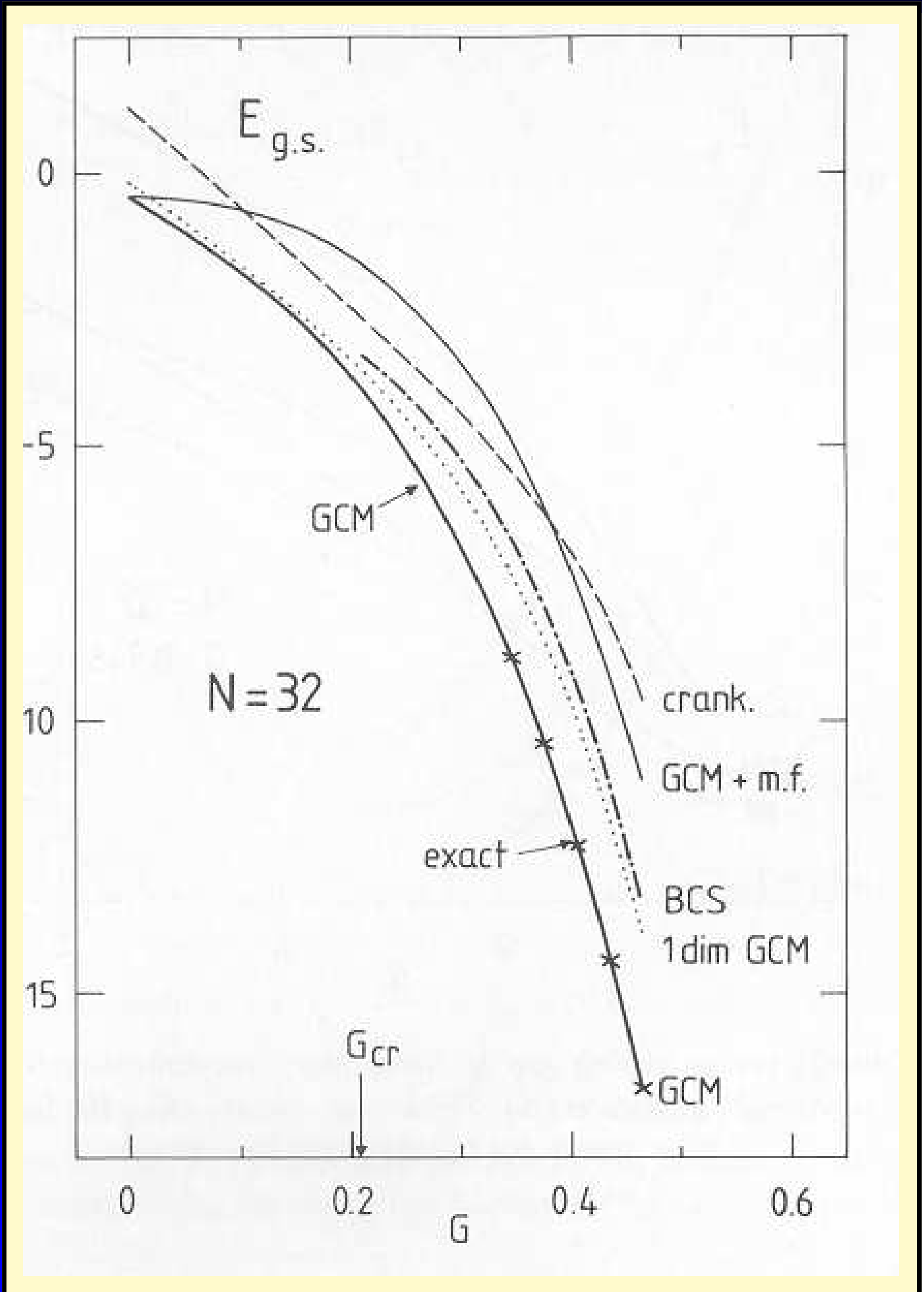
Here ϵ_0 stays for the zero-point correlation energy.



Notice:

- the strong dependence of $B_{\Delta\Delta}$ on Δ,
- the most probable Δ is smaller than Δ_{eq},
- the inertia parameter decreases significantly with growing Δ.

Comparison with Richardson Model

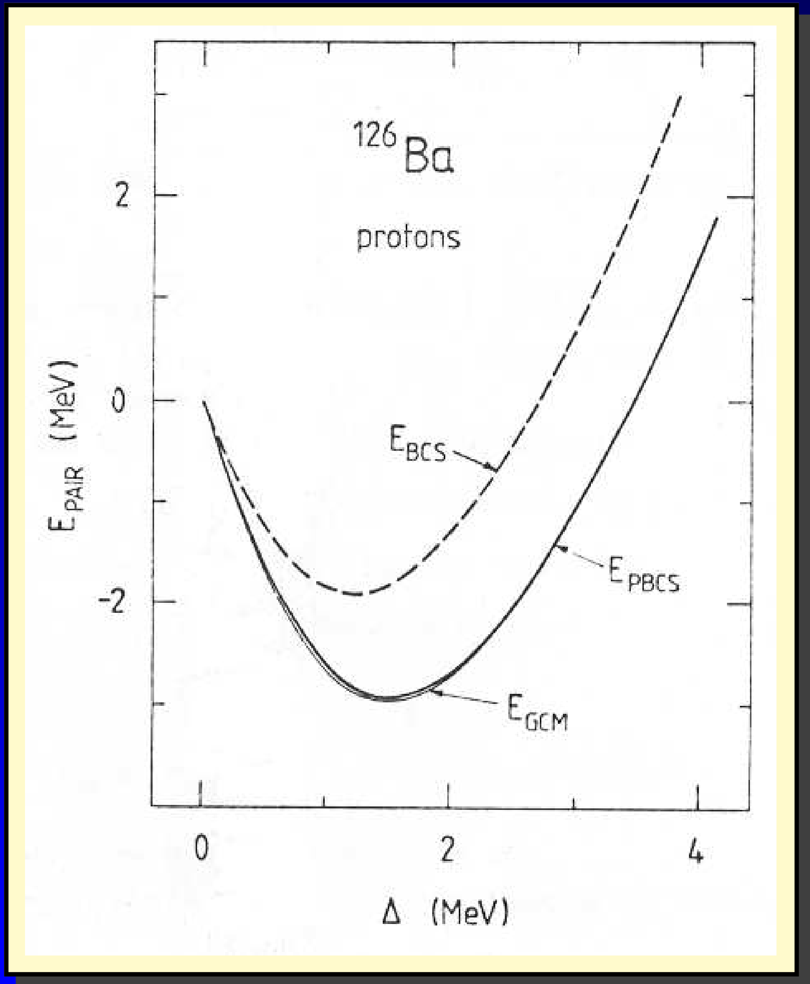


Dependence of the ground state energy of $N = 32$ particles distributed on 32 equidistant levels on the pairing strength G.

Particle number projection

Using the generator function of the following form

$$|\Phi;\Delta\rangle = e^{i(N-\hat{N})\Phi} \prod_{\nu\rangle 0} [u_\nu + v_\nu c_\nu^+ c_{-\nu}]|0\rangle$$



with Φ as the generator coordinate, one can obtain an approximate particle number projection. The projected BCS energy is equal

$$E_{\mathrm{GCM}} = E_{\mathrm{BCS}} - E_0^{\Phi} \ ,$$

where E_0^{Φ} is the zero-point energy (A. Góźdź et al. NPA 451 (1986) 1).

Pairing + surface vibrations

Taking the BCS functions for protons and neutrons is taken as the generator function $\mid q\rangle$:

$$\mid \beta_\lambda, \Delta_p, \phi_p, \Delta_n, \phi_n\rangle = \prod_{\tau=p,n} e^{-i(\hat{N}_\tau - N_\tau)\phi_\tau} \mid BCS_\tau\rangle$$

and using the GOA one can obtain the following collective Hamiltonian

$$\hat{\mathcal{H}}_{\text{coll}} = -\frac{1}{2\sqrt{\gamma}} \sum_{i,j} \frac{\partial}{\partial q_i} \sqrt{\gamma} (B^{-1})_{ij} \frac{\partial}{\partial q_j} + V(q) \ ,$$

where $\{q_i\} = \{\beta_\lambda, \Delta_p, \phi_p, \Delta_n, \phi_n\}$ and γ is the determinant of the overlap width tensor γ_{ij}.

The overlap width tensor is equal to:

$$\gamma_{ij} = \langle q \mid \frac{\overleftarrow{\partial}}{\partial q_i} \frac{\overrightarrow{\partial}}{\partial q_j} \mid q \rangle$$

and the collective inertia tensor B is given by:

$$(B^{-1})_{ij} = \frac{1}{2} \sum_{k,l} (\gamma^{-1})_{ik} \{ \langle q \mid \frac{\overleftarrow{\partial}}{\partial q_k} \hat{H} \frac{\overrightarrow{\partial}}{\partial q_l} \mid q \rangle_L - \langle q \mid \hat{H} \frac{\partial^2}{\partial q_k \partial q_l} \mid q \rangle_L \} (\gamma^{-1})_{lj} \; .$$

The GCM+GOA collective potential is defined as:

$$V(q) = E_{HFB}(q) - \frac{1}{2} \sum_{i,j} (\gamma^{-1})_{ij} \langle q \mid \frac{\overleftarrow{\partial}}{\partial q_i} \hat{H} \frac{\overrightarrow{\partial}}{\partial q_j} \mid q \rangle_L$$

Energy of the lowest 0^+ states

	GCM		CRANK.		exp.
	old	new	old	new	
^{132}Ba	3.15	2.73	2.91	1.65	1.50
^{134}Ba	3.45	2.92	3.18	1.98	1.76
^{118}Xe	2.21	1.23	1.95	0.93	0.83
^{120}Xe	2.15	1.20	1.91	0.82	0.91
^{124}Xe	2.64	1.76	2.43	1.43	1.27
^{126}Xe	2.82	1.93	2.72	1.47	1.31
^{128}Xe	3.13	2.27	2.94	1.65	1.58
^{130}Xe	3.45	2.53	3.34	2.08	1.79

Only the axial quadrupole deformation is taken here (S. Piłat et al. NPA 554 (1993) 413).

Generalized Bohr Hamiltonian

The Hamiltonian for the coupled quadrupole and pairing vibrations has the following form:

$$\hat{\mathcal{H}}_{\mathbf{CQP}} = \hat{\mathcal{H}}_{\mathbf{CQ}}(\beta, \gamma, \Omega; \Delta^p, \Delta^n) + \hat{\mathcal{H}}_{\mathbf{CP}}(\Delta^p, \Delta^n; \beta, \gamma) + \hat{\mathcal{H}}_{\mathbf{int}} \ .$$

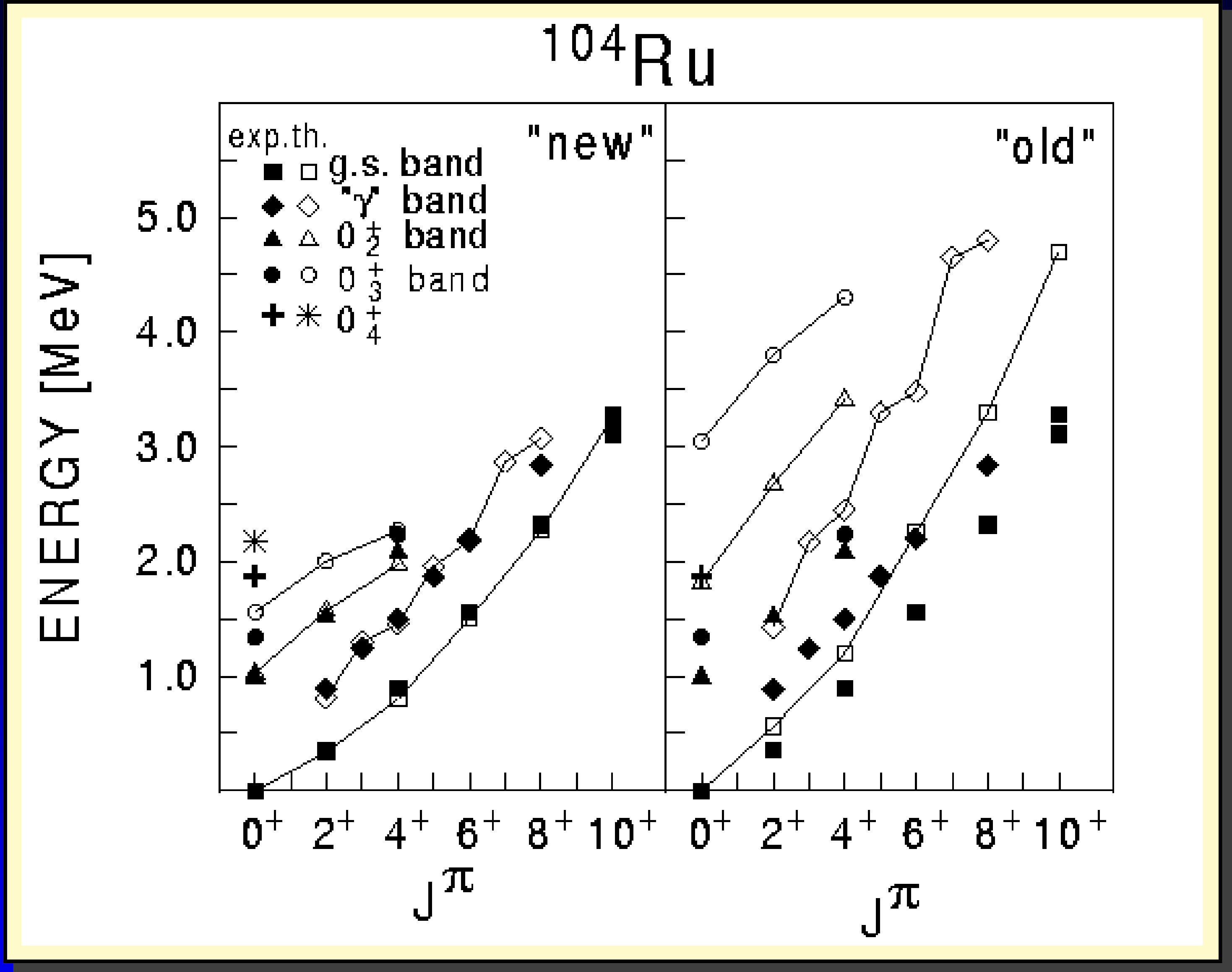
The first term is simply the Bohr Hamiltonian

$$\hat{\mathcal{H}}_{\mathbf{CQ}} = \hat{\mathcal{T}}_{\mathbf{vib}} + \hat{\mathcal{T}}_{\mathbf{rot}} + V_{\mathbf{coll}}$$

which consists of the vibrational, rotational and potential energy parts.

The operator $\hat{\mathcal{H}}_{\mathbf{CP}}$ describes the collective pairing vibrations while $\hat{\mathcal{H}}_{\mathbf{int}}$ gives the interaction between the both modes (L. Próchniak et al. NPA 648 (1999) 181).

Results obtained with the GBH



K. Zając et al. Nucl. Phys. A653, (1999) 71.

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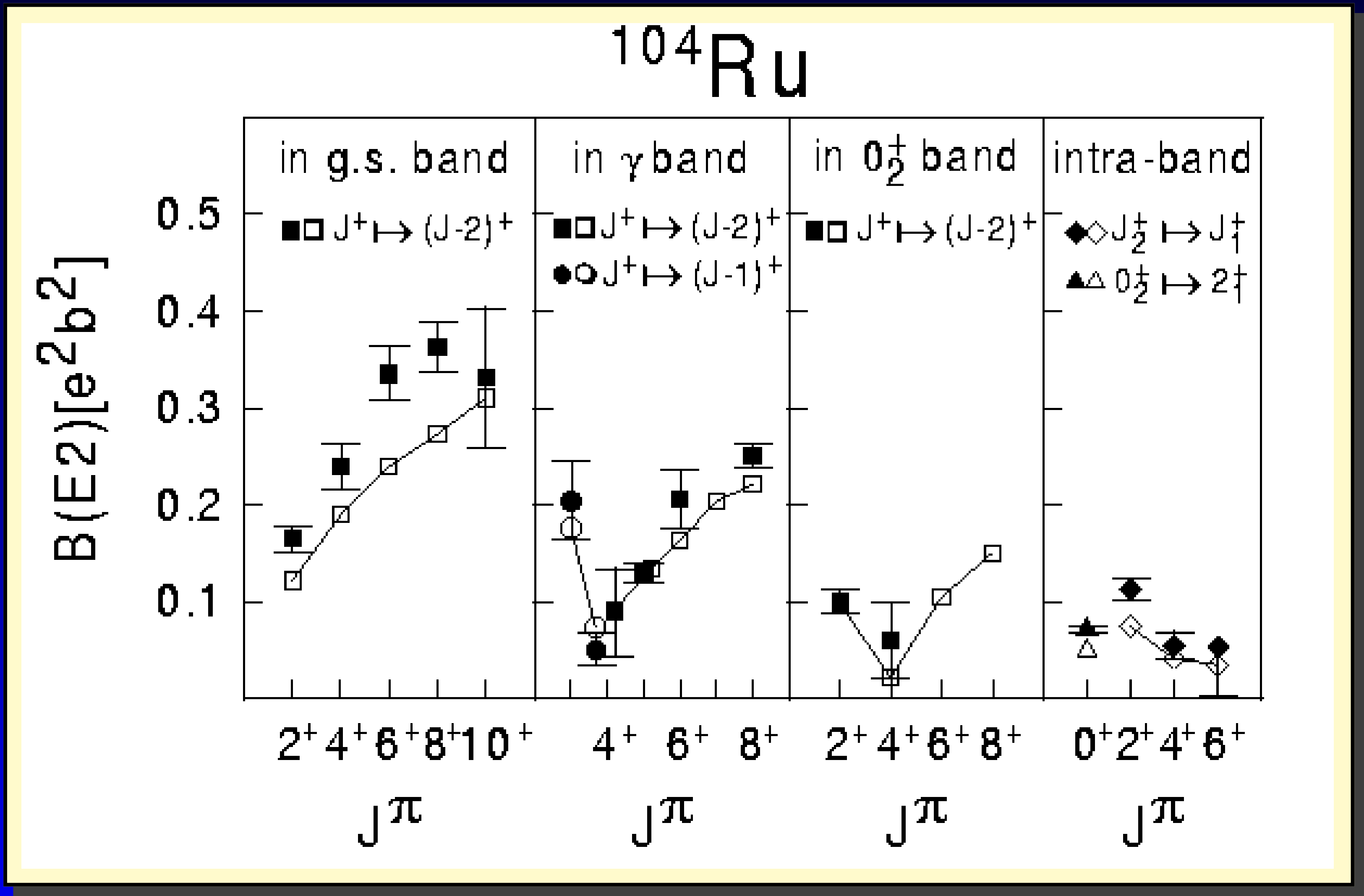

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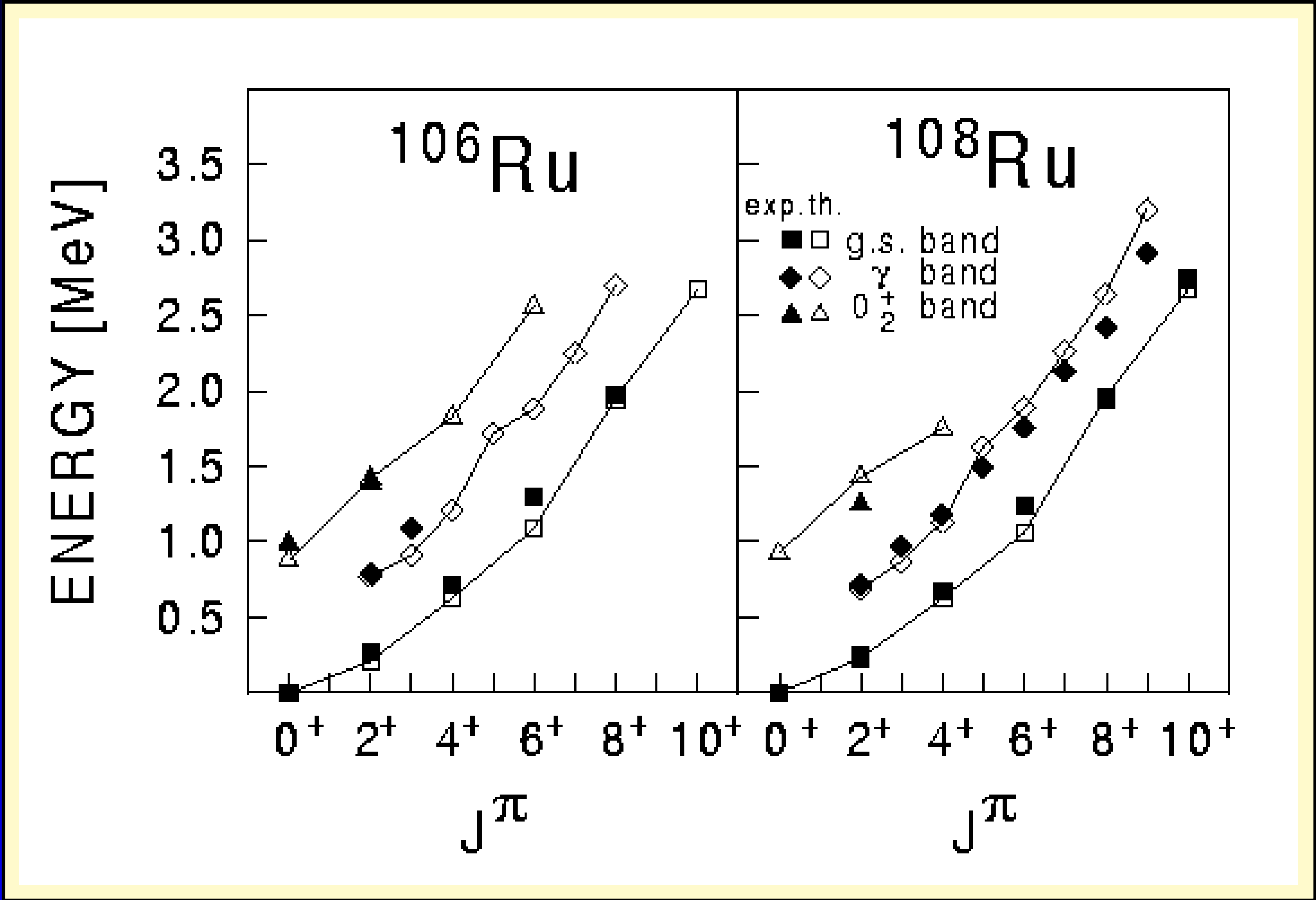

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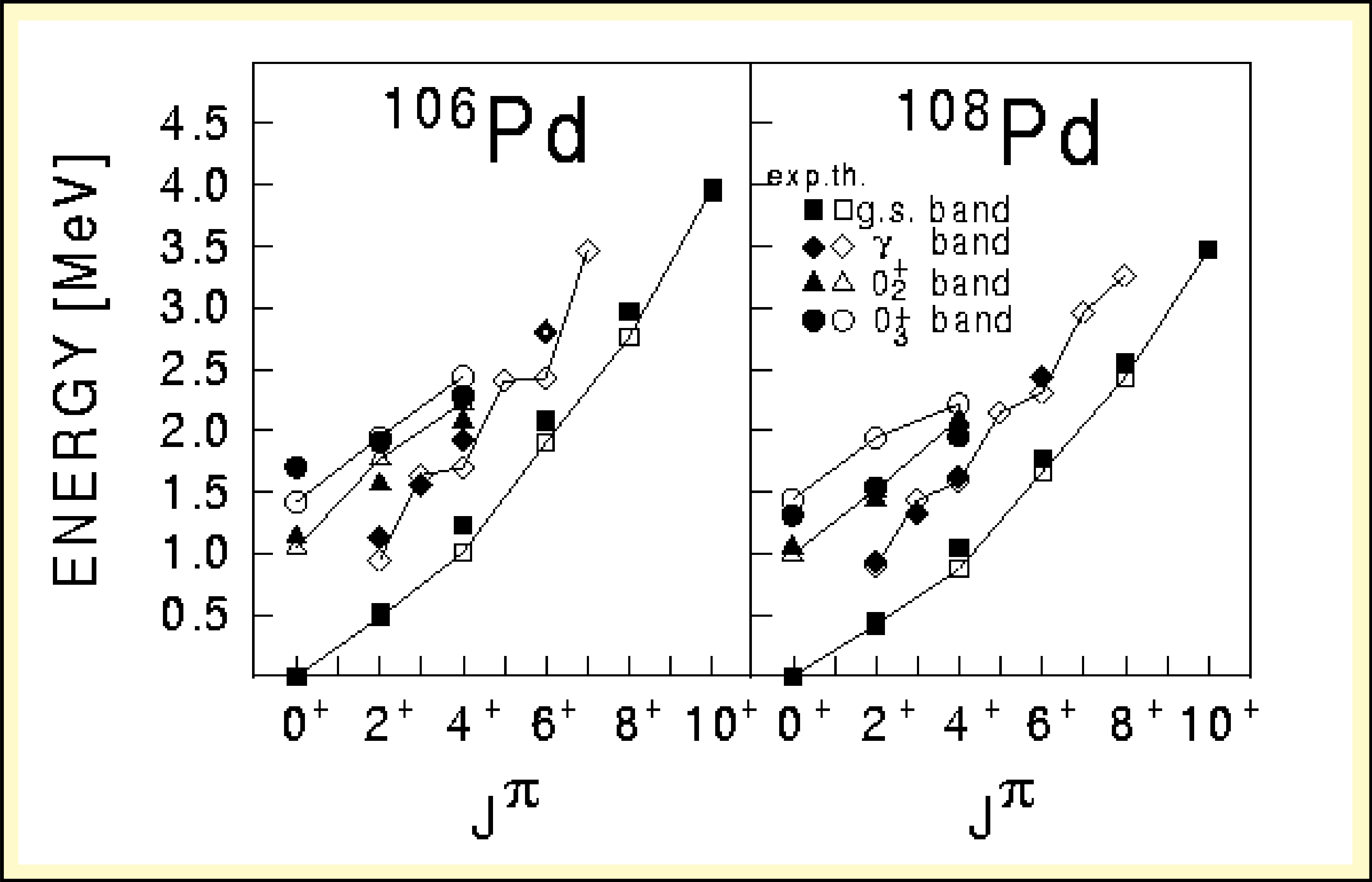
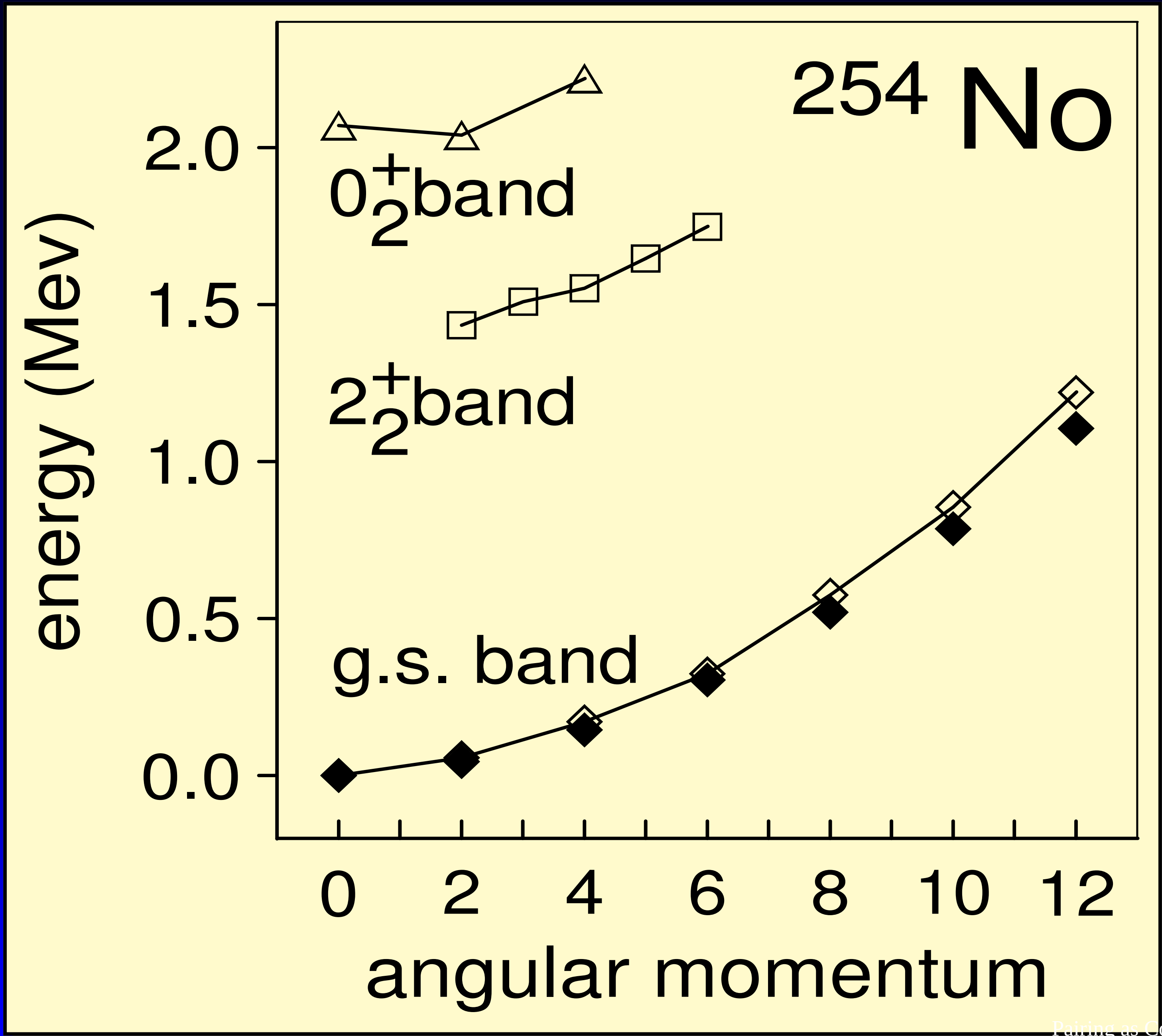
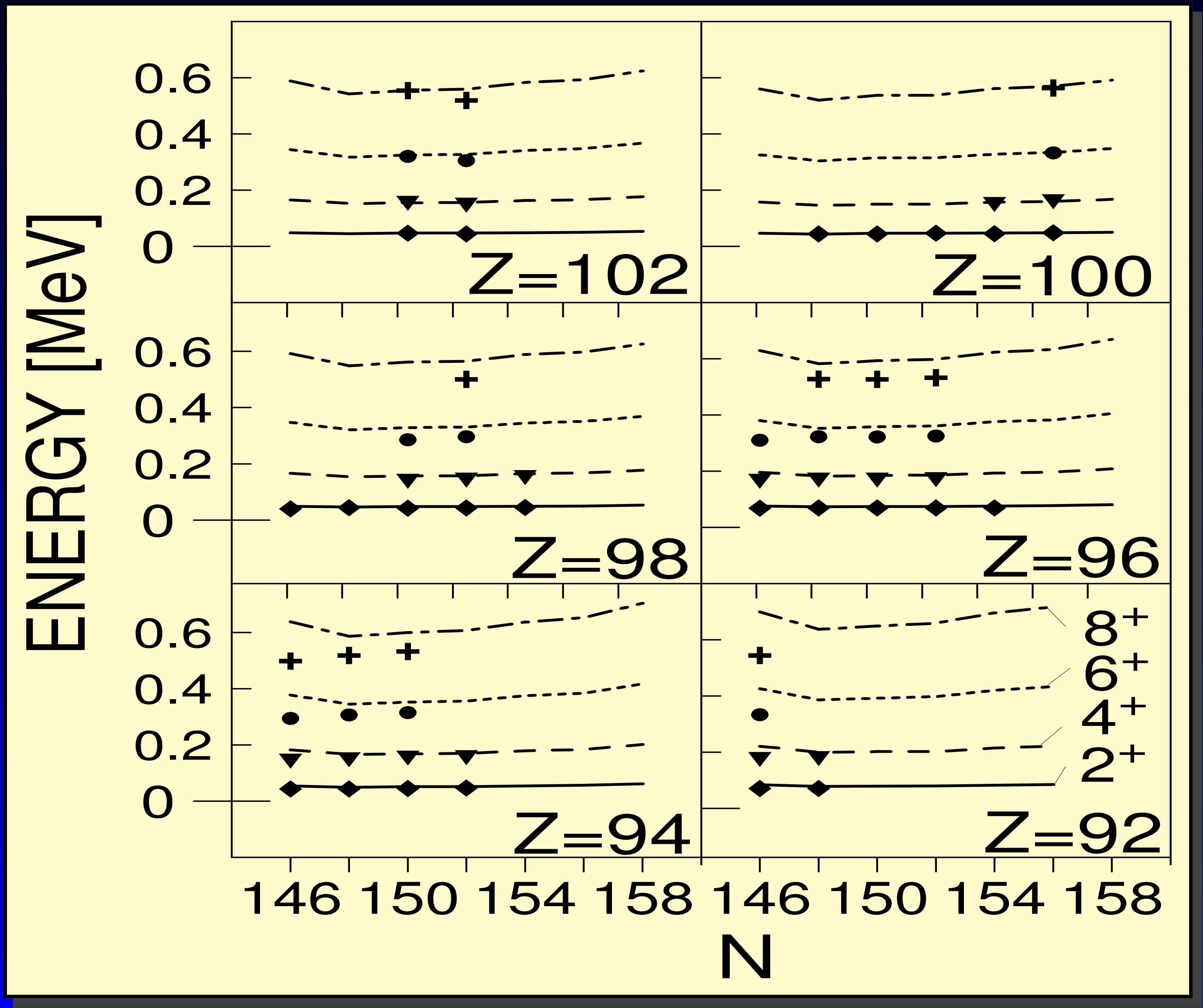


K. Zając et al. Nucl. Phys. A653, (1999) 71.

GBH results for ^{254}No



GBH results for transactinides



K. Zając et al. Acta Phys. Polon. B34 (2003) 1789.

Coupling of fission and pairing modes

The potential energy of fissioning nucleus is given by

$$V = E_{\rm macr}(\beta_\lambda) + \delta E_{\rm shell}(\beta_\lambda) + \delta E_{\rm pair}(\beta_\lambda, \Delta_p, \Delta_n)\,.$$

Within the WKB approximation the spontaneous–fission half–life is given by

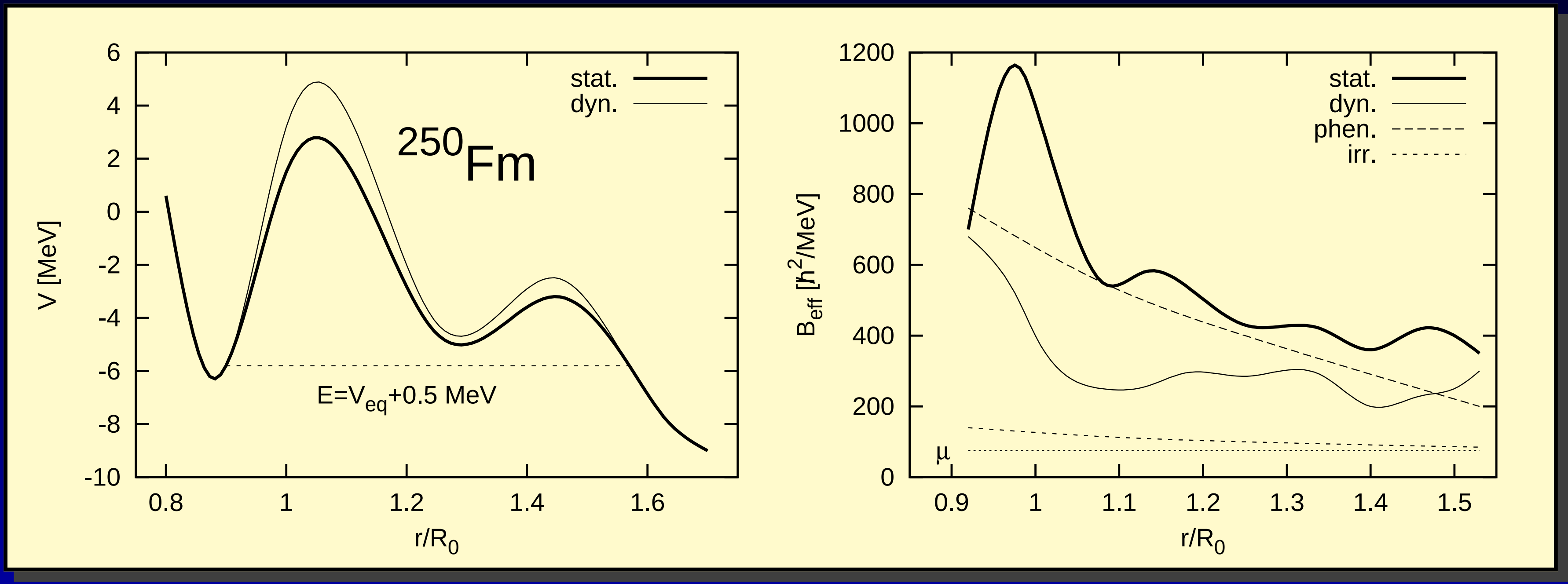
$$T_{1/2}^{\rm SF}\,[yr] = \frac{10^{-28.04}}{\hbar\omega_0}\,[1 + \exp 2S(L)]\,,$$

where $S(L)$ is the action–integral calculated along a fission path $L(s)$ in the $\{\beta_\lambda, \Delta_p, \Delta_n\}$ space

$$S(L) = \int_{s_1}^{s_2} \left\{\frac{2}{\hbar^2} B_{\rm eff}(s)[V(s) - E]\right\}^{1/2} ds\,.$$

A. Staszczak et al., Phys. Lett. 161B (1985) 227.

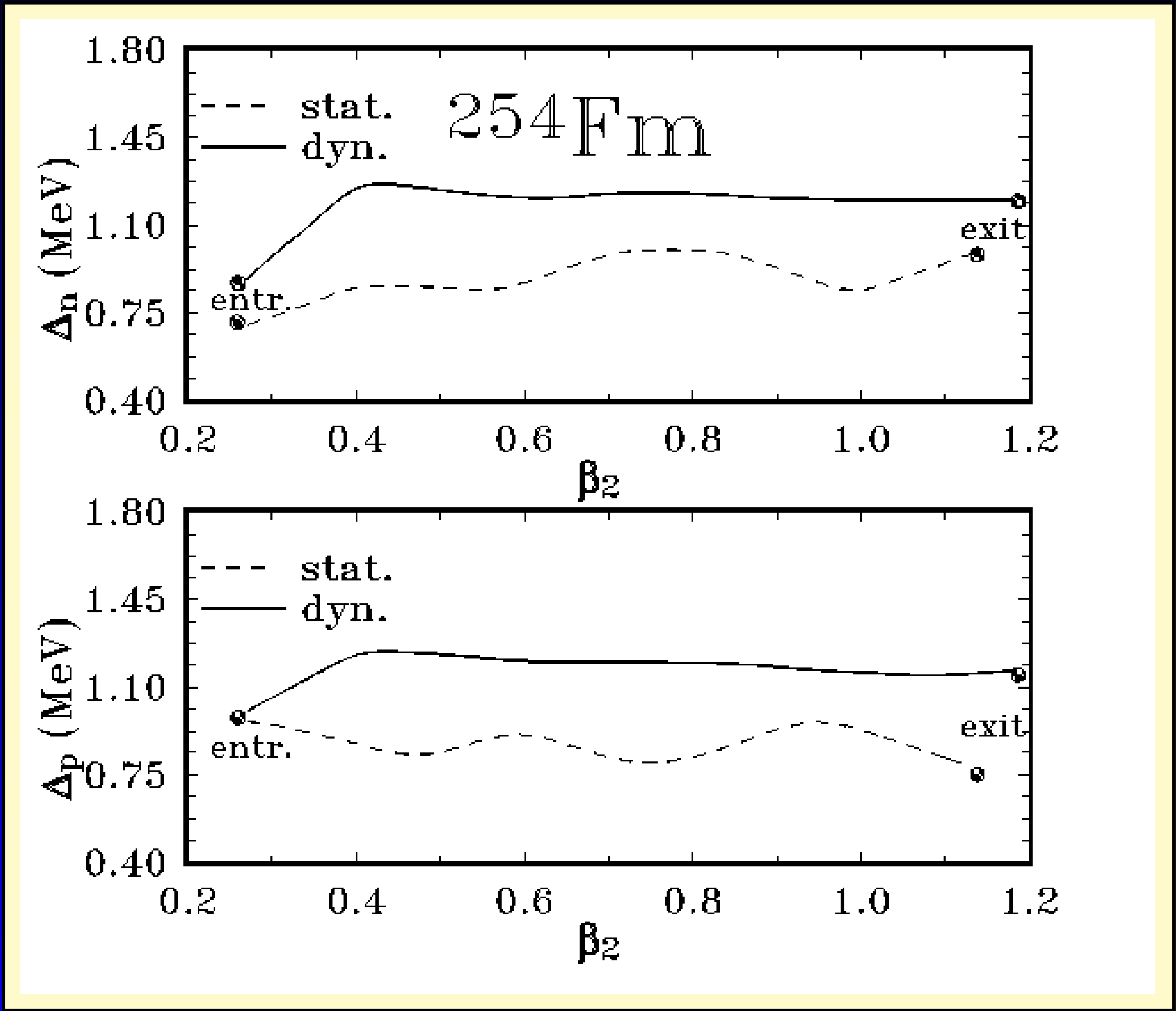
Fission barriers and inertia

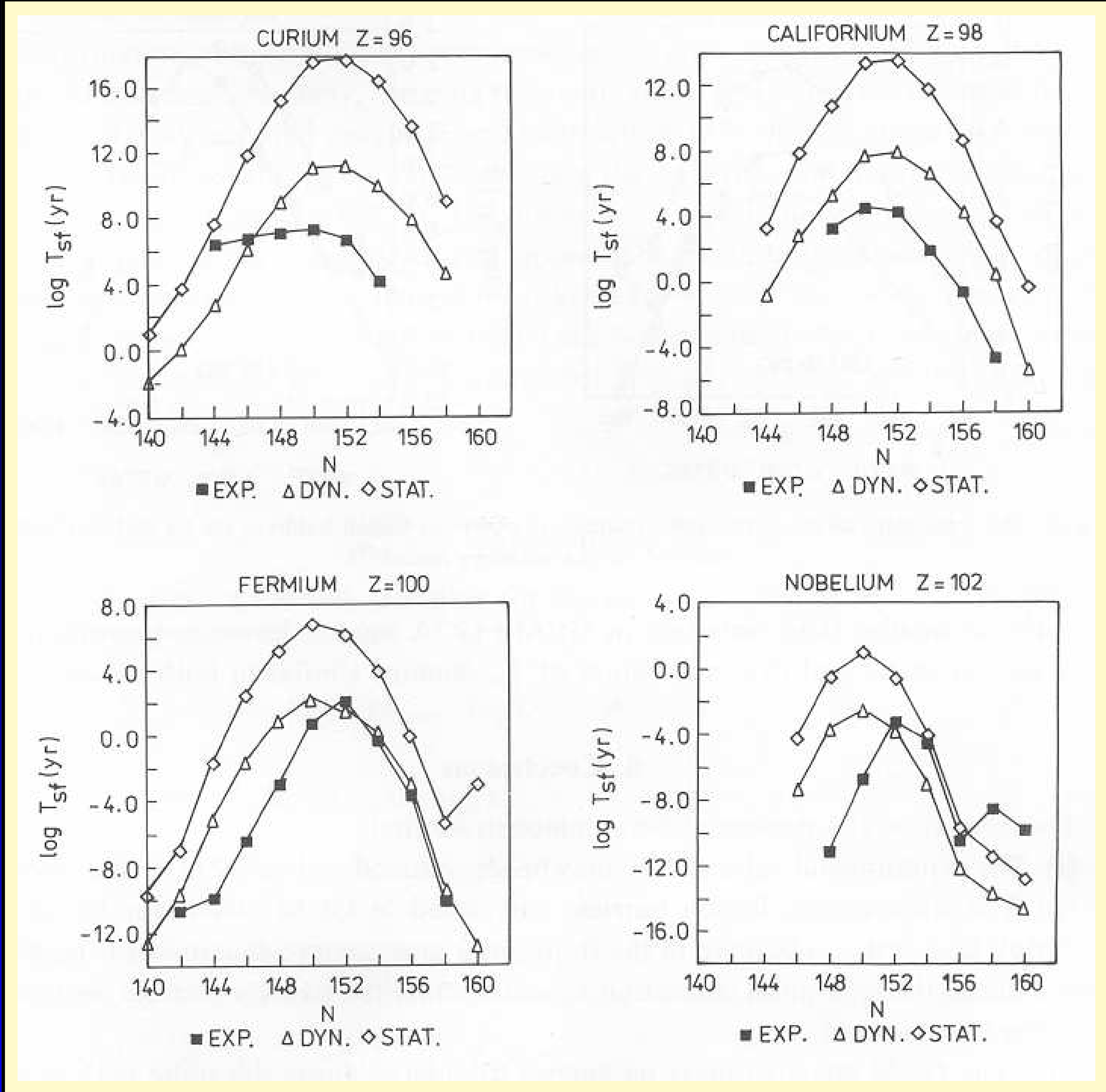


Along the least action trajectory (dynamic path) the fission barrier is higher up 2 MeV than in the static case while the effective inertia is reduced by factor 2.

These two effects reduce the estimate of T_{sf} of ^{250}Fm by about 4 orders of magnitude!

Paths to fission





A. Staszczak et al. NPA 504 (1989) 589.

State dependent pairing force

In case of the state dependent pairing Hamiltonian

$$\hat{H} = \sum_{k>0} \langle k|\hat{h}|k\rangle (c_k^\dagger c_k + c_{\bar{k}}^\dagger c_{\bar{k}}) - \sum_{k,l>0} V_{k\bar{k}l\bar{l}} c_k^\dagger c_{\bar{k}}^\dagger c_{\bar{l}} c_l$$

one can construct the monopole pairing operator

$$\hat{A} = \frac{1}{2} \sum_{k>0} \left(e^{-2i\phi} c_k^\dagger c_{\bar{k}}^\dagger + e^{2i\phi} c_{\bar{k}} c_k \right) ,$$

which mean value $\alpha = \langle \hat{A} \rangle = \sum_{k>0} u_k v_k$
represents the pair condensate in the BCS function

$$|\alpha\phi\rangle = e^{iN\phi} \prod_{k>0} \left(u_k + v_k e^{-2i\phi} c_k^\dagger c_{\bar{k}}^\dagger \right) |0\rangle .$$

(K. Sieja et al. EPJA 20 (2004) 413.)

$\hat{H}_{coll}$ for the s.d. pairing force

Using α and ϕ as the generator coordinates and the BCS function

$$|\alpha\phi\rangle = e^{iN\phi}\prod_{k>0}\left(u_k + v_k e^{-2i\phi}c_k^\dagger c_{\bar{k}}^\dagger\right)|0\rangle\,,$$

as the generator function one can derive the collective Hamiltonian in a form analogues to the monopole pairing case:

$$\begin{aligned}\hat{H}_{coll} \;&=\; -\frac{\hbar^2}{2\sqrt{\det\gamma}}\frac{\partial}{\partial\alpha}\sqrt{\det\gamma}\mathcal{M}^{-1}_{\alpha\alpha}\frac{\partial}{\partial\alpha} - \frac{\hbar^2}{2}\mathcal{M}^{-1}_{\phi\phi}\frac{\partial^2}{\partial\phi^2}\\ &-i\hbar\frac{\mathrm{Im}\langle\alpha\phi|\overleftarrow{\frac{\partial}{\partial\phi}}\hat{H}|\alpha\phi\rangle}{\gamma_{\phi\phi}}\frac{\partial}{\partial\phi} + V(\alpha)\,,\end{aligned}$$

Conclusions:

- The ground state of the collective pairing Hamiltonian approximates well the energy of the exact solution of the pairing eigenproblem.
- Using of the GCM+GOA in the $(\boldsymbol{\Delta}, \boldsymbol{\phi})$ space approximates well the effect of particle number projection of the BCS wave function.
- Taking into account the coupling of the pairing and quadrupole vibrations improves significantly predictive power of the Bohr Hamiltonian.
- Inclusion of the pairing degrees of freedom $\boldsymbol{\Delta_p}$ and $\boldsymbol{\Delta_n}$ reduce theoretical estimates of the spontaneous fission half–lives by about 1-6 orders and bring them towards experimental data.

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