Correlations and chaotic motion

in nuclear masses

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The Nuclear Mass

$$\mathcal{M}(Z,N) = ZM_{\rm H} + NM_{\rm n} - \mathcal{B}(Z,N)$$

$\mathcal{B}$ - Binding energy

$$\mathcal{M}(Z,N) = E_{\rm macro}(Z,N) + E_{\rm micro}(Z,N)$$

Macroscopic model: Liquid drop

$$E_{\rm LD} = ZM_{\rm H} + NM_{\rm n} + E_{\rm vol} + E_{\rm surf} + E_{\rm coul} + \bar{\Delta}_{\rm oe} + \ldots$$

Microscopic model:

$$E_{\rm shell} + (E_{\rm chaotic}??)$$

Chaotic Energy in the Nuclear Ground-State

Highly excited nuclei
Level statistics for neutron resonances suggests GOE.
(Chaotic time-reversal invariant system)

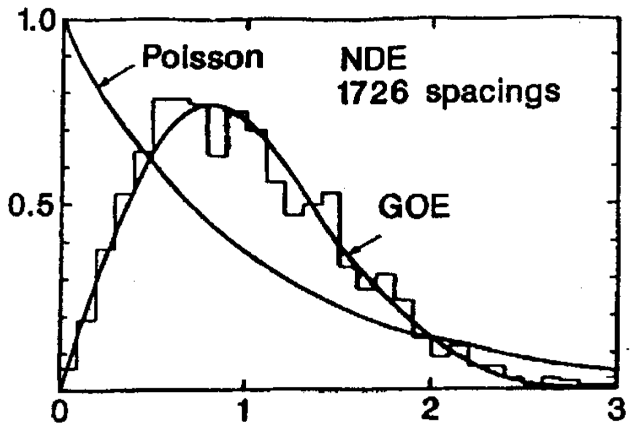
Ground-state region
Level statistics suggests Poisson
(Regular system)

$$\widetilde{U} = \mathcal{M}_{\rm exp} - \mathcal{M}_{\rm model}$$

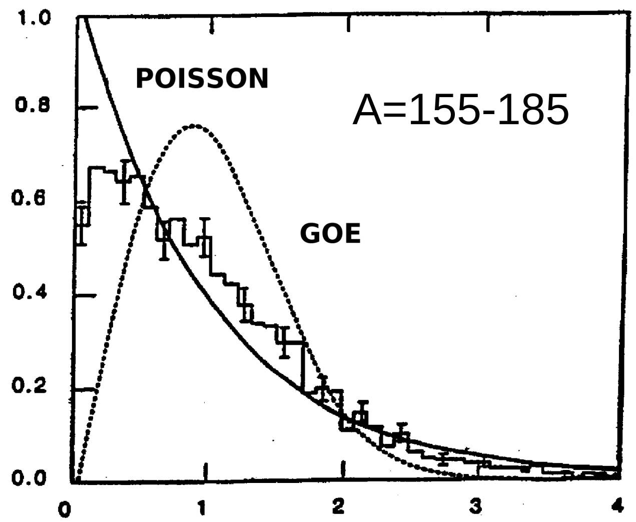
$$\sqrt{\widetilde{U}^2} \sim 0.7 \text{ MeV}$$

Typical chaotic energy??

Neutron Resonances



Ground-State region



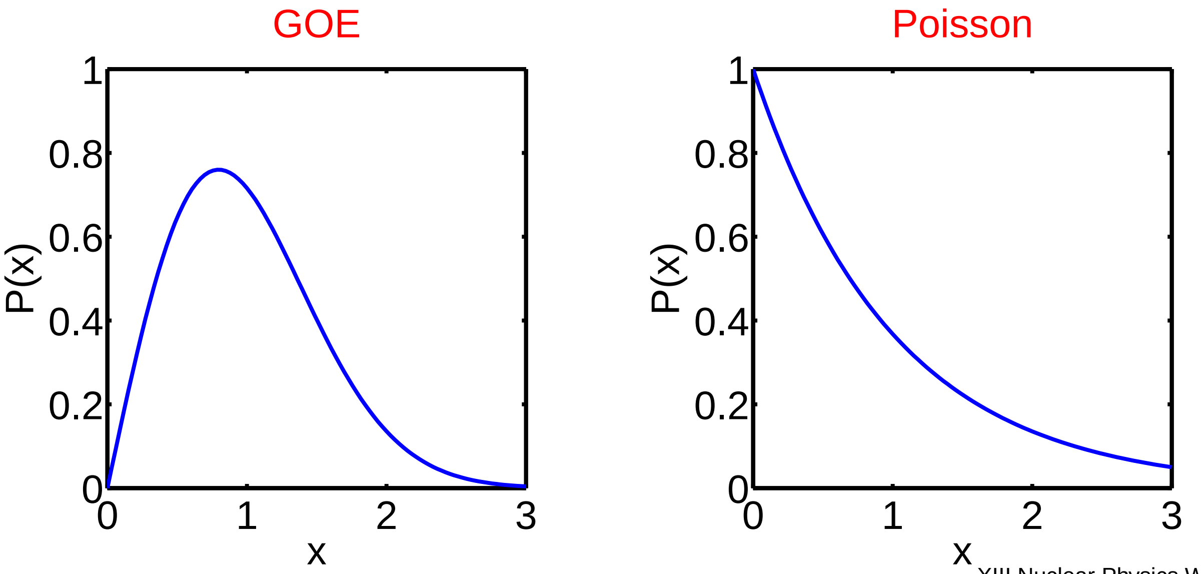
Quantum Chaos

Bohigas-Giannoni-Schmit Conjecture

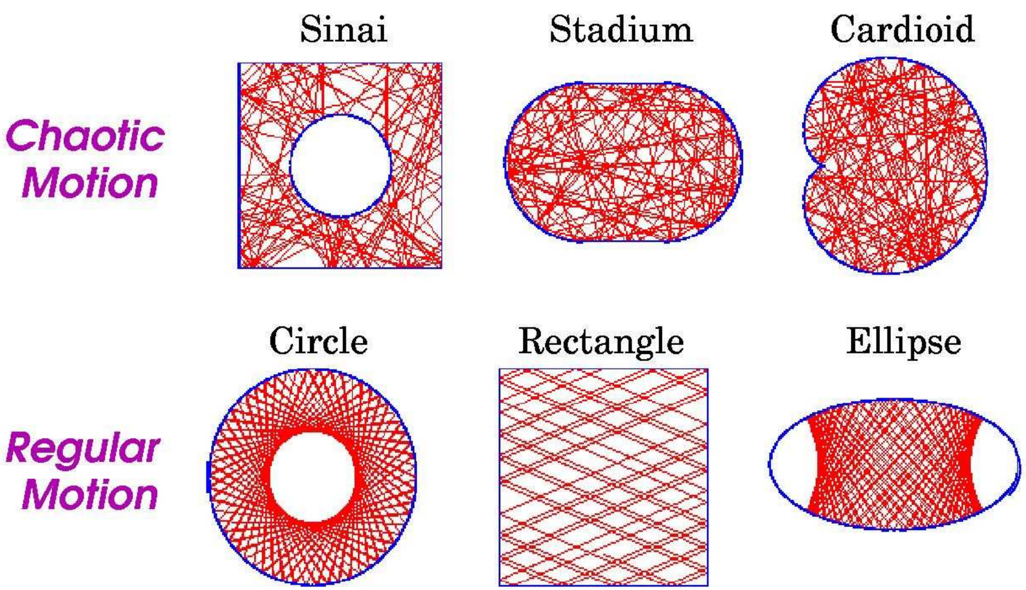
Spectral fluctuations agree with

- GOE if the corresponding classical system is chaotic and obeys time-reversal symmetry.
- Poisson if the corresponding classical system is integrable.

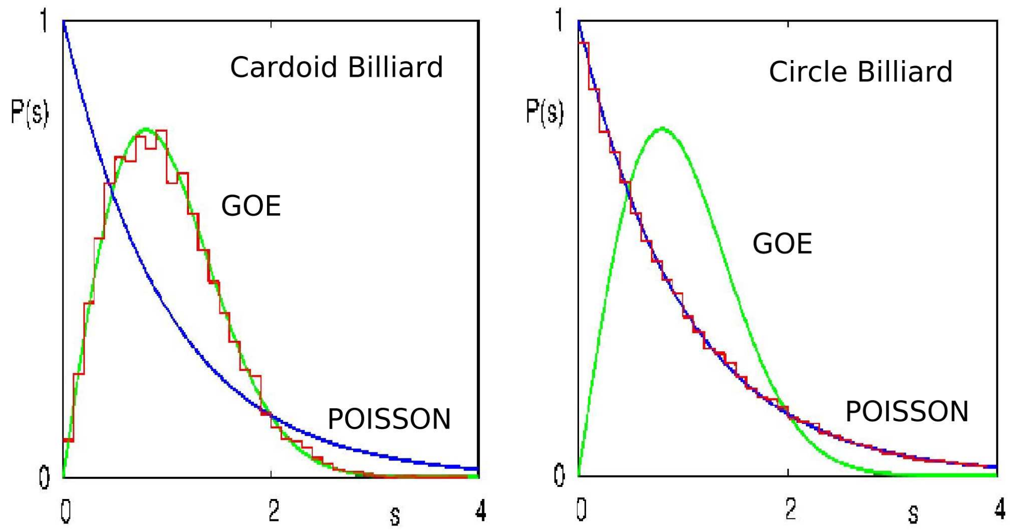
Nearest neighbor distribution



Chaotic Motion inside 2D cavities



Area filling orbits



Periodic Orbit Theory

Random Matrix theory describe short energy range fluctuations of the order of the mean level spacing.

Heisenberg time: $\tau_{\mathrm{H}} = h\rho_{\mathrm{mean}}$

Long energy range fluctuations are described by semi-classics. Energy scale is defined by the shortest periodic orbit.

$$E_c = \frac{h}{\tau_{\mathrm{min}}}$$

In a ballistic system this correspond to the time of flight across the system.

Periodic Orbit Theory (II)

Mean field theory connects the quantum mechanical many-body problem to classical mechanics.

Reduces the fully interacting many-body problem to single-particle motion in a self-consistent potential.

The basic object is the single-particle level density

$$\rho(e,x) = \sum_i \delta(e - e_i(x))$$

In a semiclassical $\hbar$-expansion

$$\rho(e,x) = \rho_{\mathrm{mean}}(e,x) + \rho_{\mathrm{fluct}}(e,x)$$

$$E(e,x) = E_{\mathrm{mean}}(e,x) + E_{\mathrm{shell}}(e,x)$$

The Trace Formula

Leading order in a $\hbar$-expansion gives a sum over classical periodic orbits

$$E_{\text{shell}}(x) = 2\hbar^2 \sum_{p,r} \frac{A_{p,r}(x)}{r^2 \tau_p^2} \cos\left[\frac{r S_p(x)}{\hbar} + \nu_{p,r}\right]$$

Stability amplitude $A_{p,r}$

Classical action $S_p = \oint p dq$

Maslov index $\nu_{p,r}$

The second moment may be written using the spectral form factor $K(\tau)$

$$H_{\text{QM}}(x) \leftrightarrow H_{\text{CM}}(q,p)$$

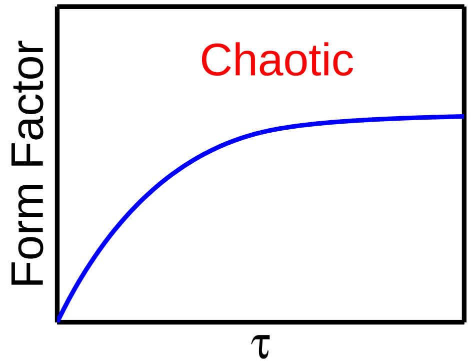
$$\langle E_{\text{shell}}^2 \rangle \approx \frac{\hbar^2}{2\pi^2} \int_0^\infty \frac{d\tau}{\tau^4} K(\tau)$$

The Spectral Form Factor

The form factor is the Fourier transform of the two-point correlation function.

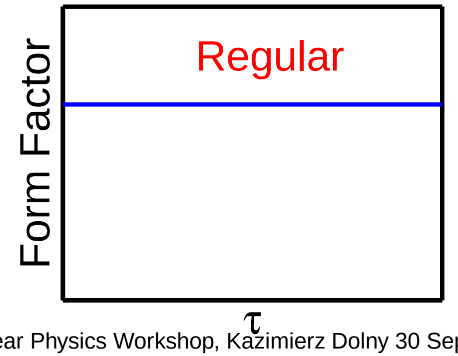
Depending on the character of the dynamics the form factor can be described by Random Matrix Theory.

Chaotic GOE: $K(\tau) = 2\tau, \quad \tau \ll \tau_{\mathrm{H}}$



Regular: $K(\tau) = \tau_{\mathrm{H}}$

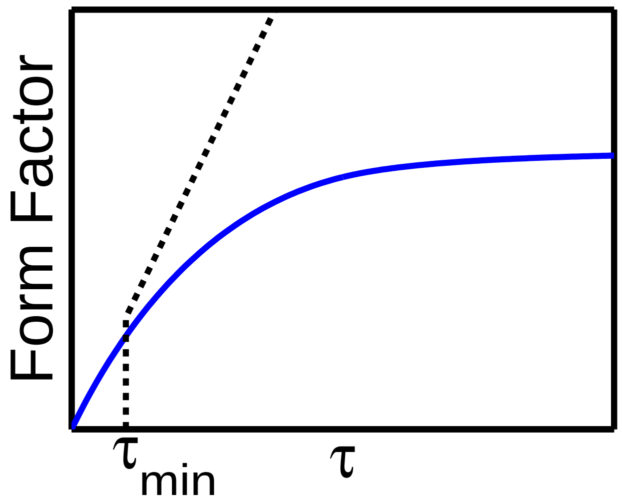
$\tau_{\mathrm{H}} = h\rho_{\mathrm{mean}}$



$\tau_{\min}$ Approximation

Incorporates

- Shortest periodic orbit
  (System dependent)
- Linear growth
  (Universal chaotic dependence)
  No τ dependence
  (Universal regular dependence)



$$\langle E_{\text{shell}}^2 \rangle \approx \frac{\hbar^2}{2\pi^2} \int_0^\infty \frac{d\tau}{\tau^4} K(\tau)$$

Independent of any detailed information of the many-body problem. Only parameter is $\tau_{\min}$, which is related to the size of the nucleus.

$$\tau_{\min} \sim \frac{4R}{v_F} \sim A^{1/3}$$

Typical Chaotic Energy

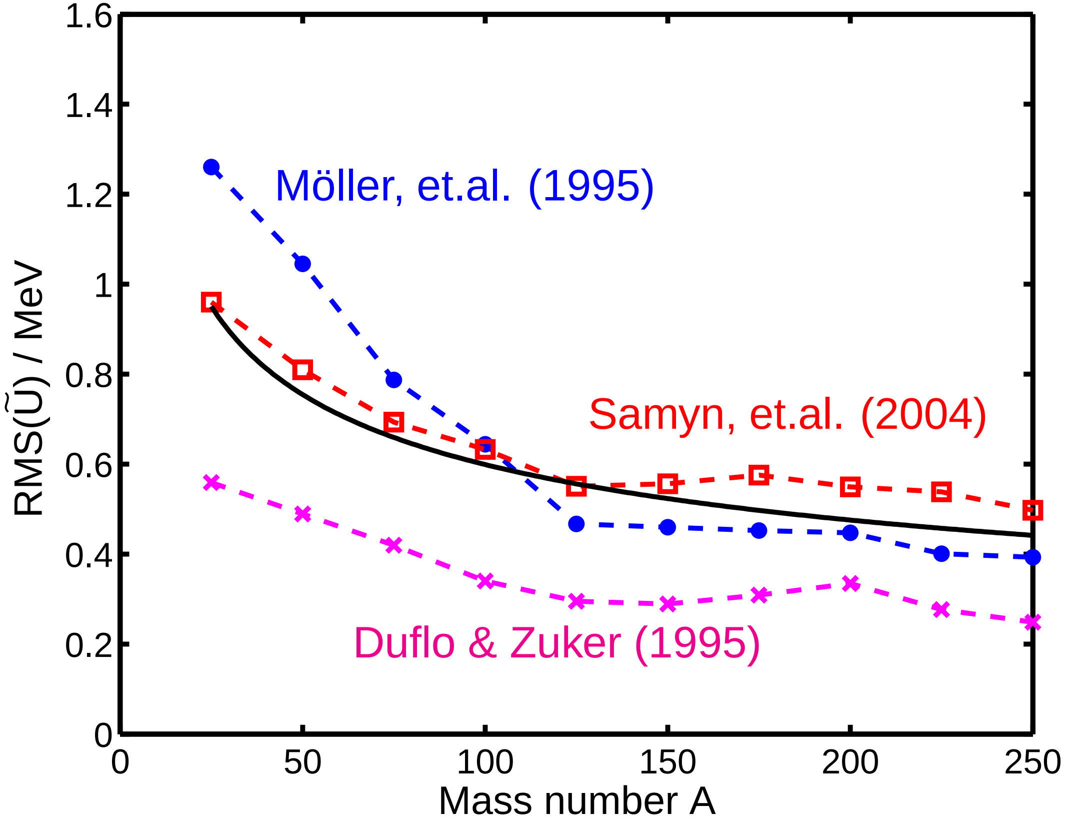
$$\sigma_{\text{ch}} = \sqrt{\langle E_{\text{shell}}^{\text{chaotic }2} \rangle} =$$

$$= \frac{2.8}{A^{1/3}} \text{ MeV}$$

Mass models

$$\widetilde{U} = \mathcal{M}_{\text{exp.}} - \mathcal{M}_{\text{model}}$$

- Möller, et.al. (1995)
- Samyn, et.al. (2004)
- Duflo & Zuker (1995)



Order of magnitude estimate

Autocorrelation function between neighboring nuclei

When the external parameter x is varied, the autocorrelation function is defined

$$C(x) = \langle E_{\text{shell}}(x_0 - x/2) E_{\text{shell}}(x_0 + x/2) \rangle_{x_0}$$

Leads to a double sum with interferent terms between different periodic orbits. The main contribution comes from the shortest orbits.

Diagonal Approximation

$$C(x) = 2\hbar^4 \sum_{p,r} \frac{A_{p,r}^2}{r^4 \tau_p^4} \cos\left[\frac{rQ_p}{\hbar} x\right]$$ where $Q_p = \dfrac{\partial S_p}{\partial x}$

Using the spectral form factor

$$C(x) = \frac{\hbar^2}{2\pi^2} \int_0^\infty \frac{d\tau}{\tau^4} \left\langle \cos\left(\frac{Q_p x}{\hbar}\right)\right\rangle_\tau K(\tau)$$

Valid for regular or chaotic motion

Autocorrelation function

Assuming chaotic dynamics

$$C_N(\zeta) = \left(1 - \frac{\zeta^2}{4}\right) e^{-\zeta^2/4} + \frac{\zeta^4}{16}\Gamma(0, \zeta^2/4)$$

$$\Gamma(a, z) = \int_z^\infty t^{a-1} e^{-t} dt$$

Universal function since all the system dependent features are hidden in the unfolding.

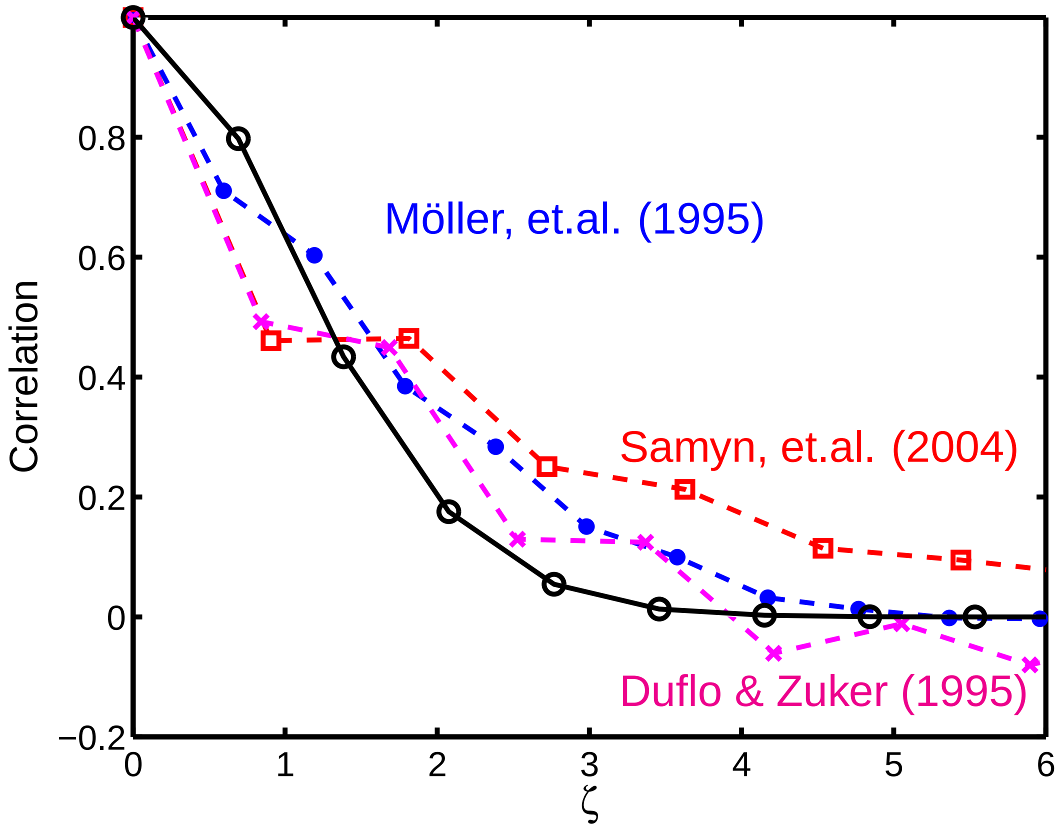
$$\zeta = \frac{\sqrt{2\alpha\tau_{\mathrm{min}}}}{\hbar} x$$

$$\zeta = \sqrt{\frac{\left\langle \left(\partial_x E_{\mathrm{shell}}(x)\right)^2 \right\rangle}{\langle E_{\mathrm{shell}}(x)^2 \rangle}} x$$

Comparison with Mass Models

$$C(dN) = \left\langle \frac{\left\langle \widetilde{U}(Z,N)\widetilde{U}(Z,N+dN)\right\rangle_N}{\left\langle \widetilde{U}^2\right\rangle_N} \right\rangle_Z$$

$$\zeta = \sqrt{\frac{\left\langle \left(\partial_N \widetilde{U}\right)^2 \right\rangle_N}{\left\langle \partial_N \widetilde{U}^2 \right\rangle_N} dN}$$



Mass models

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Conclusions on Chaotic Nuclear Masses

- Periodic Orbit Theory describe the regular and the chaotic part of the shell energy on equal footing.
- The second moment of the chaotic shell energy agree well with the typical size of the error in nuclear mass formulas.
- The correlations for the error between neighboring nuclei agree well with estimates from periodic orbit theory assuming chaotic dynamics.
- Periodic Orbit Theory predicts definite non-random fluctuations. It may be a difficult task to compute the chaotic contributions for each nucleus, but periodic orbit theory puts NO á priori physical barrier to the accuracy of theoretical mass calculations.

List of Papers

- Paper 1

  H. Olofsson, S. Åberg, O. Bohigas and P. Leboeuf
  Correlations in Nuclear Masses
  Phys. Rev. Lett. 96, 042502 (2006)

- Paper 2

  H. Olofsson, S. Åberg, O. Bohigas and P. Leboeuf
  Correlations and Chaotic Motion in Nuclear Masses
  Phys. Scr. T125, 162-166 (2006)