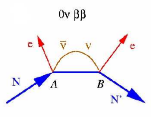
$\beta\beta$ decay and nuclear structure



F. Nowacki[1]

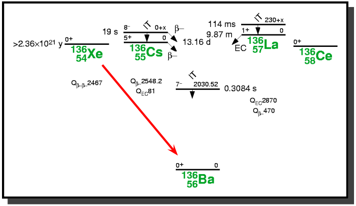
XIII Nuclear Physics Workshop
Kazimierz Dolny
September 27-October 1, 2006

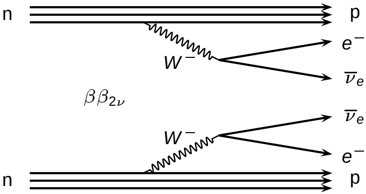
[1]Strasbourg-Madrid Shell-Model collaboration; OF :(

Outline

- $\beta\beta$ decay
- Shell model
- 2ν calculations
- 0ν calculations
- summary

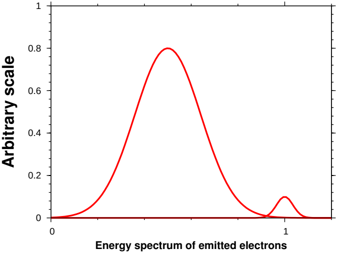
$\beta\beta$ decay





authorized by SM

Transition	$Q_{\beta\beta}$ (keV)	Ab.$(^{232}Th = 100)$
$^{110}Pd \to ^{110}Cd$	2013	12
$^{76}Ge \to ^{76}Se$	2040	8
$^{124}Sn \to ^{124}Te$	2288	6
$^{136}Xe \to ^{136}Ba$	2479	9
$^{130}Te \to ^{130}Xe$	2533	34
$^{116}Cd \to ^{116}Sn$	2802	7
$^{82}Se \to ^{82}Kr$	2995	9
$^{100}Mo \to ^{100}Ru$	3034	10
$^{96}Zr \to ^{96}Mo$	3350	3
$^{150}Nd \to ^{150}Sm$	3667	6
$^{48}Ca \to ^{48}Ti$	4271	0.2



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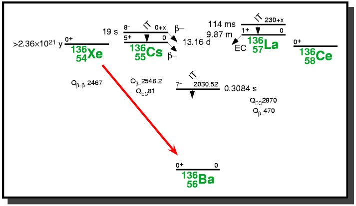

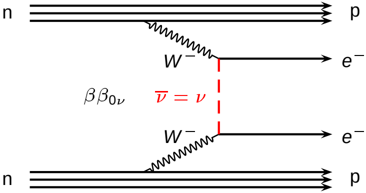

violation of leptonic number conservation

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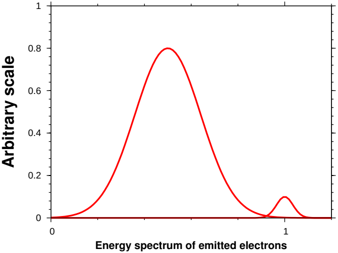
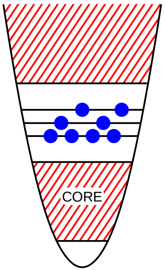


Shell Model Problem



- Define a valence space
- Derive an effective interaction

$$\mathcal{H}\Psi = E\Psi \rightarrow \mathcal{H}_{\text{eff}}\Psi_{\text{eff}} = E\Psi_{\text{eff}}$$

- Build and diagonalize the Hamiltonian matrix.

In principle, all the spectroscopic properties are described simultaneously (Rotational band AND β decay half-life).

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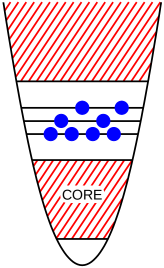

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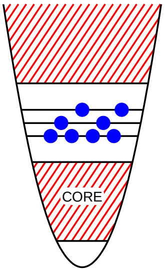

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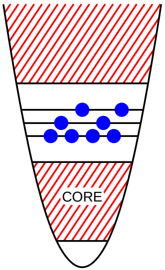

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- hamiltonian: $\mathcal{H}\Psi = E\Psi$
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G matrix: M. Hjorth-Jensen, T.T.S. Kuo and E. Osnes, Realistic effective interactions for nuclear systems, Physics Reports 261 (1995) 125-270

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- energies of states of (semi) magic nuclei
- systematics of B(E2) transitions ➤ effective charge; GT transitions ➤ quenching factor

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Specificity of $(\beta\beta)_{0\nu}$:

NO EXPERIMENTAL DATA !!!

prediction for m_ν very difficult
easier for m_ν(A)/m_ν(A')

What is the best isotope to observe $(\beta\beta)_{0\nu}$ decay ?

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Two neutrinos mode

The theoretical expression of the half-life of the 2ν mode can be written as:

$$[T^{2\nu}_{1/2}]^{-1} = G_{2\nu}|M^{2\nu}_{GT}|^2,$$

with

$$M^{2\nu}_{GT} = \sum_m \frac{\langle 0^+_f||\vec{\sigma}t_-||1^+_m\rangle\langle 1^+_m||\vec{\sigma}t_-||0^+_i\rangle}{E_m + E_0}$$

- $G_{2\nu}$ contains the phase space factors and the axial coupling constant g_A
- summation over intermediate states
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Lanczos Algorithm

In the first step we write:

$$\mathcal{H}|\mathbf{1}\rangle = E_{11}|\mathbf{1}\rangle + E_{12}|\mathbf{2}\rangle$$

there E_{11} is just $\langle\mathbf{1}|\mathcal{H}|\mathbf{1}\rangle$ $\langle\mathcal{H}\rangle$, the mean value of $\mathcal{H}$.
In the second step:

$$\mathcal{H}|\mathbf{2}\rangle = E_{21}|\mathbf{1}\rangle + E_{22}|\mathbf{2}\rangle + E_{23}|\mathbf{3}\rangle$$

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At rank N, the following relations hold:

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Lanczos Algorithm

It is explicit that we have built a tridiagonal matrix

$$\langle I|\mathcal{H}|J\rangle = \langle J|\mathcal{H}|I\rangle = 0 \text{ if } |I-J| > 1$$

$$\begin{pmatrix} E_{11} & E_{12} & 0 & 0 & 0 & 0 \\ E_{12} & E_{22} & E_{23} & 0 & 0 & 0 \\ 0 & E_{32} & E_{33} & E_{34} & 0 & 0 \\ 0 & 0 & E_{43} & E_{44} & E_{45} & 0 \\ & & & & & \\ & & & & & \end{pmatrix}$$

Lanczos Strength Function

Let Ω be an operator acting on some initial state $|\Phi_{ini}\rangle$, we obtain the state $\Omega|\Phi_{ini}\rangle$ whose norm is the sum rule of the operator Ω in the initial state:

$$S = ||\Omega|\Phi_{ini}\rangle|| = \sqrt{\langle\Phi_{ini}|\Omega^2|\Phi_{ini}\rangle}$$

Depending on the nature of the operator Ω,
the state $\Omega|\Phi_{ini}\rangle$ belongs to the same nucleus (if Ω is a e.m transition operator) or to another (Gamow-Teller, nucleon transfer, $a_j^\dagger/\tilde{a}_j$, $\beta\beta$, ...)

If the operator Ω does not commute with $\mathcal{H}$, $\Omega|\Phi_{ini}\rangle$ is not necessarily an eigenvector of the system,
BUT it can be developped in energy eigenstates:

$\Omega|\Phi_{ini}\rangle = \sum_i S(E_i)|E_i\rangle$ and $\langle\Phi_{ini}|\Omega^2|\Phi_{ini}\rangle = \sum_i S^2(E_i)$
where $S^2(E_i)$ is the strength function (or structure function)
$(S(E_i) = \langle E_i|\Omega|\Phi_{ini}\rangle)$

Lanczos Structure Function

If we carry on the Lanczos procedure
using $|\Sigma\rangle = \Omega|\Phi_{ini}\rangle$ as initial pivot.
then H is again diagonalized to obtain the eigenvalues $|E_i\rangle$

U is the unitary matrix that diagonalizes $\mathcal{H}$ and gives the expression of the eigenvectors in terms of the Lanczos vectors:

$$U = \begin{array}{c} |\Sigma\rangle \\ |2\rangle \\ |3\rangle \\ \vdots \\ |N\rangle \end{array} \overset{\begin{array}{ccccc} |E_1 & |E_2\rangle & |E_3\rangle & \dots & |E_N\rangle \end{array}}{\left(\begin{array}{c} \\ \\ \\ \\ \\ \end{array} \right)}$$

$$S(E_i) = U(1,i)$$

How good is the Strength function obtained at iteration N compared to the exact one?

Lanczos Structure Function

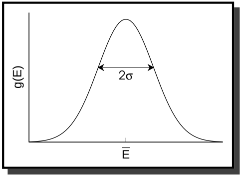
Any distribution can be characterized by the moments of the distribution.

$$\bar{E} = \langle\Omega|\boldsymbol{H}|\Omega\rangle = \sum_i E_i|\langle E_i|\boldsymbol{\Omega}|\Phi_{ini}\rangle|^2$$

$$m_n = \langle\Omega|(\boldsymbol{H}-\bar{E})^n|\Omega\rangle = \sum_i (E_i-\bar{E})^n|\langle E_i|\boldsymbol{\Omega}|\Phi_{ini}\rangle|^2$$

Gaussian distribution characterized by two moments $(\bar{E}, \sigma^2 = m_2)$

$g(E) = \frac{1}{\sigma\sqrt{2\pi}}\exp(-\frac{(E-\bar{E})^2}{2\sigma^2})$



Lanczos Structure Function

Lanczos method provides a natural way of determining the basis $|\alpha\rangle$.

Initial vector $|\mathbf{1}\rangle = \frac{|\Omega\rangle}{\sqrt{\langle\Omega|\Omega\rangle}}$.

$$
\begin{aligned}
E_{12}|\mathbf{2}\rangle &= (\boldsymbol{H} - E_{11})|\mathbf{1}\rangle \\
E_{23}|\mathbf{3}\rangle &= (\boldsymbol{H} - E_{22})|\mathbf{2}\rangle - E_{12}|\mathbf{1}\rangle \\
&\dots \\
E_{NN+1}|\mathbf{N}+\mathbf{1}\rangle &= (\boldsymbol{H} - E_{NN})|\mathbf{N}\rangle \\
&\quad - E_{N-1N}|\mathbf{N}-\mathbf{1}\rangle
\end{aligned}
$$

where

$$E_{NN} = \langle\mathbf{N}|\boldsymbol{H}|\mathbf{N}\rangle, \quad E_{NN+1} = E_{N+1N}$$

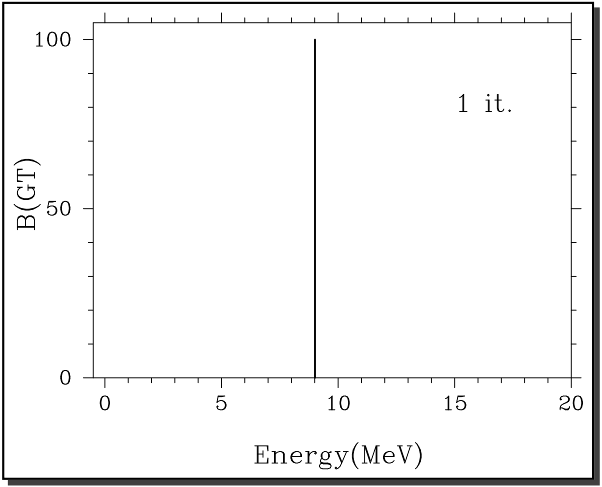
Each Lanczos iteration gives information about two new moments of the distribution.

$$
\begin{aligned}
E_{11} &= \langle\mathbf{1}|\boldsymbol{H}|\mathbf{1}\rangle = \bar{E} \\
E_{12}^2 &= \langle\Omega|(\boldsymbol{H} - E_{11})^2|\Omega\rangle = m_2 \\
E_{22} &= \frac{m_3}{m_2} + \bar{E} \\
E_{23}^2 &= \frac{m_4}{m_2} - \frac{m_3^2}{m_2^2} - m_2
\end{aligned}
$$

Diagonalizing Lanczos matrix after N iterations gives an approximation to the distribution with the same lowest $2N$ moments.

Evolution of Strength Distribution

GT Strength on ^{48}Sc



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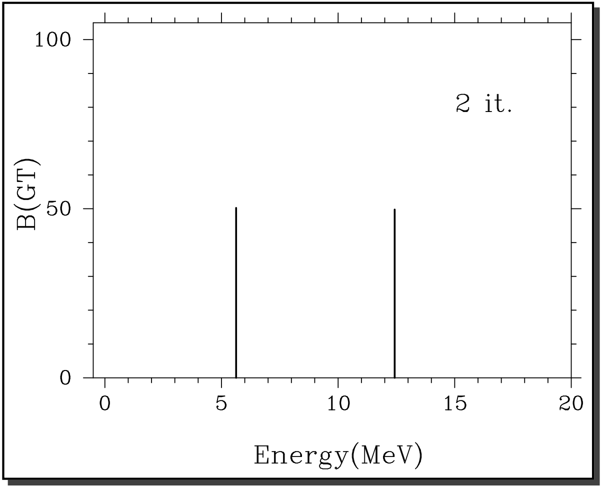

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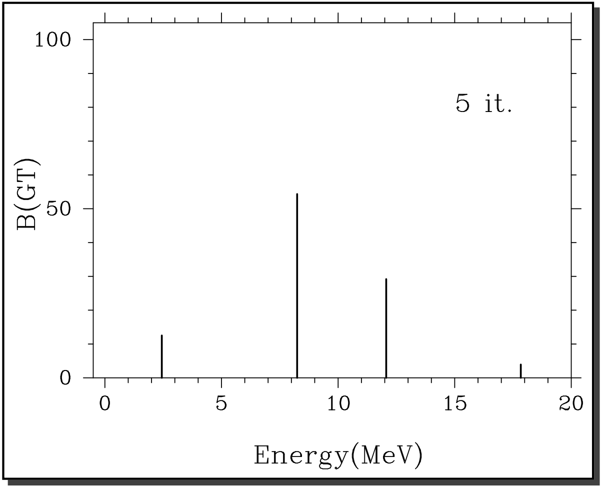

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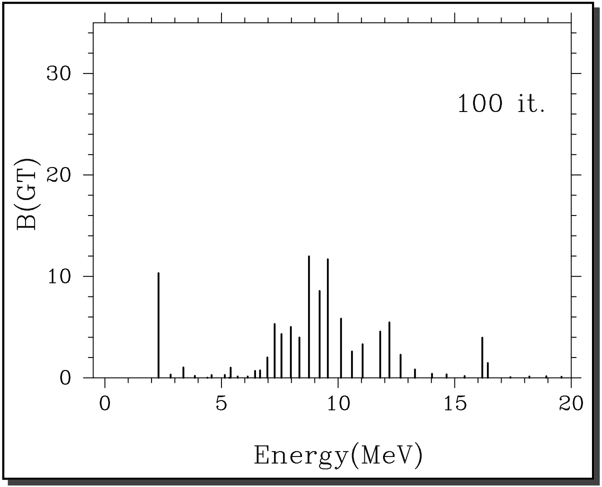

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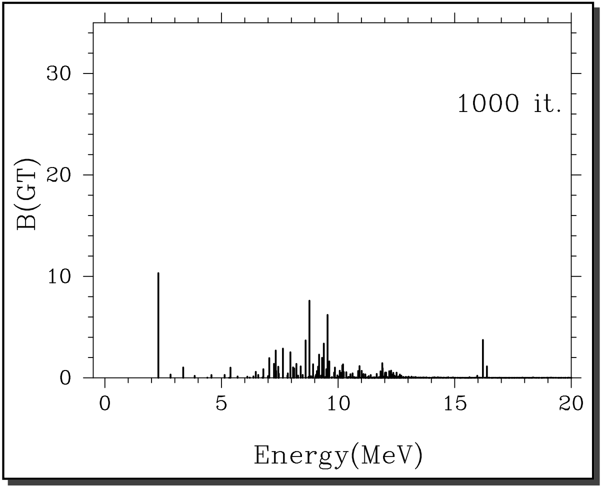

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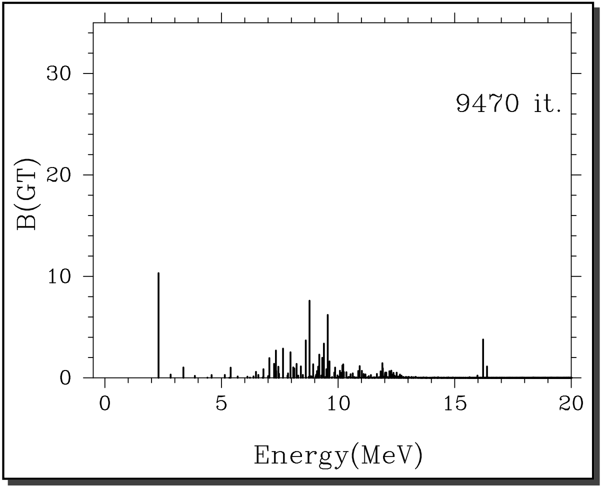

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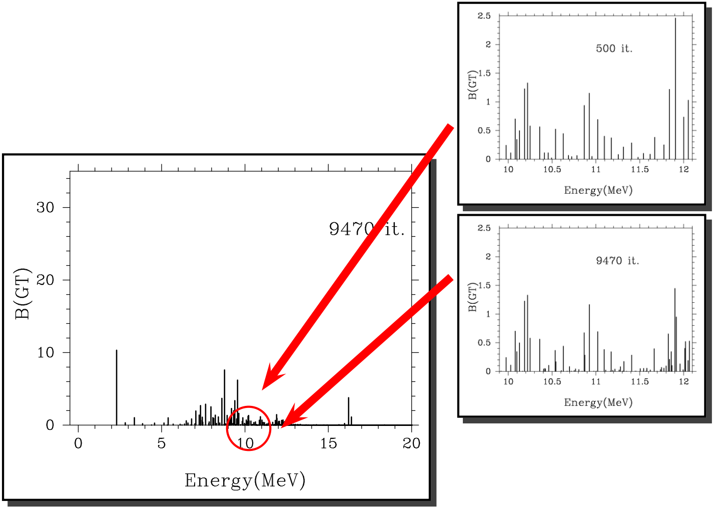

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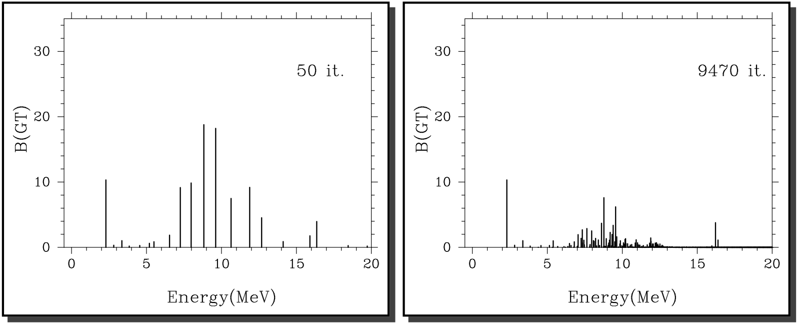

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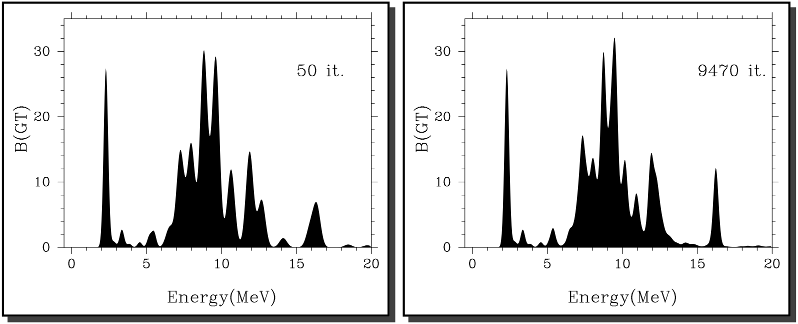
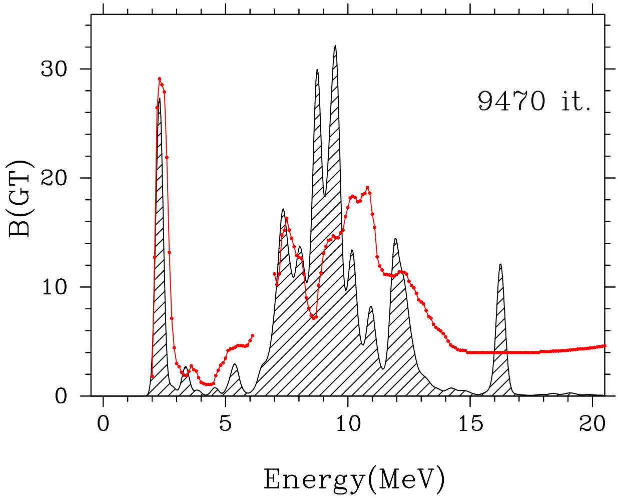


^{48}Ca(p,n)^{48}Sc Strength Function



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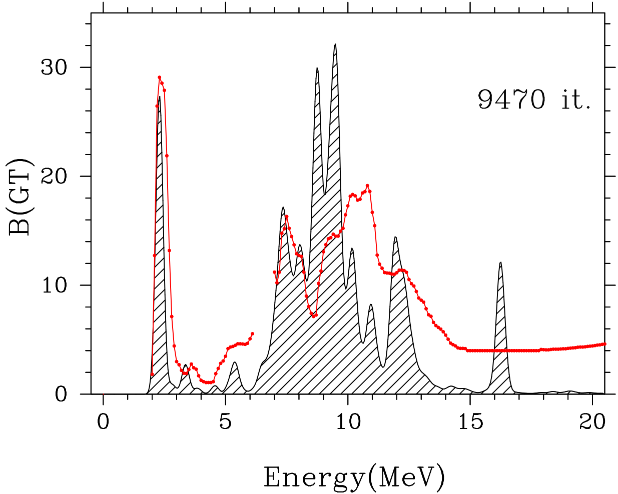
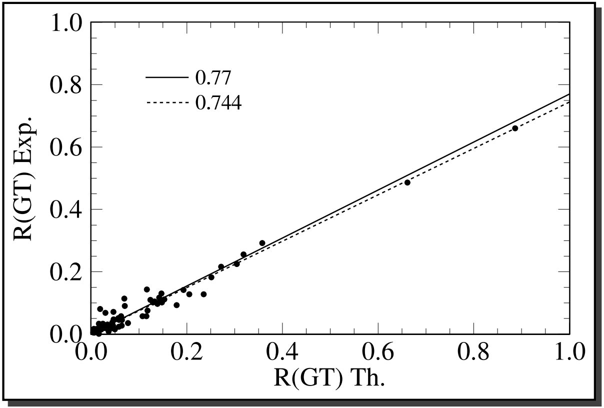


Quenching of GT strength in the *pf*-shell



Quenching of GT operator

If we write

$$|\hat{i}\rangle = \alpha|0\hbar\omega\rangle + \sum_{n\neq 0} \beta_n |n\hbar\omega\rangle,$$

$$|\hat{f}\rangle = \alpha'|0\hbar\omega\rangle + \sum_{n\neq 0} \beta'_n |n\hbar\omega\rangle$$

then

$$\langle \hat{f} \parallel \mathcal{T} \parallel \hat{i}\rangle^2 = \left(\alpha\alpha' T_0 + \sum_{n\neq 0} \beta_n\beta'_n T_n\right)^2,$$

- $n \neq 0$ contributions negligible
- $\alpha \approx \alpha'$

→ projection of the physical wavefunction in the $0\hbar\omega$ space is $Q \approx \alpha^2$

→ transition quenched by Q^2

$(\beta\beta)_{2\nu}$ structure function

$$M_{GT}^{2\nu} = \sum_m \frac{\langle \; 0_f^+ \; ||\vec{\sigma}t_-||1_m^+\rangle\langle 1_m^+||\vec{\sigma}t_-|| \; 0_i^+ \; \rangle}{E_m + E_0}$$

$(\beta\beta)_{2\nu}$ structure function

$$M^{2\nu}_{GT} = \sum_m \frac{\langle 0^+_f ||\vec{\sigma}t_-||1^+_m\rangle\langle 1^+_m||\vec{\sigma}t_-|| 0^+_i \rangle}{E_m + E_0}$$

Calculation in three steps:

- calculate the final and initial states

$(\beta\beta)_{2\nu}$ structure function

$$M^{2\nu}_{GT} = \sum_m \frac{\langle 0^+_f || \vec{\sigma} t_- || 1^+_m \rangle \langle 1^+_m || \vec{\sigma} t_- || 0^+_i \rangle}{E_m + E_0}$$

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Calculation in three steps:

- calculate the final and initial states
- generate the doorway states $\vec{\sigma}t_-|0^+_i\rangle$ and $\vec{\sigma}t_+|0^+_f\rangle$

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Calculation in three steps:

- calculate the final and initial states
- generate the doorway states $\vec{\sigma}t_-|0^+_i\rangle$ and $\vec{\sigma}t_+|0^+_f\rangle$
- take a doorway and use of Lanczos Strength Function method:

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- at iteration N, N 1^+ states in the intermediate nucleus, with excitation energies E_m

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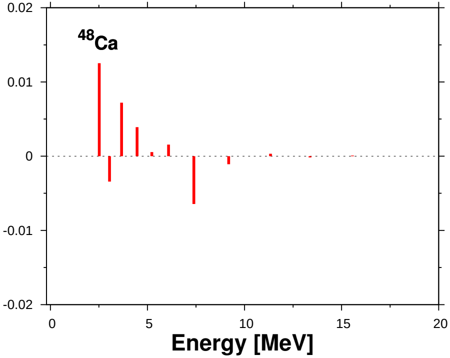
Calculation in three steps:

- calculate the final and initial states
- generate the doorway states $\vec{\sigma}t_-|0_i^+\rangle$ and $\vec{\sigma}t_+|0_f^+\rangle$
- take a doorway and use of Lanczos Strength Function method:

• at iteration N, N 1^+ states in the intermediate nucleus, with excitation energies E_m

• overlap with the other doorway, enter energy denominators and add up the N contributions

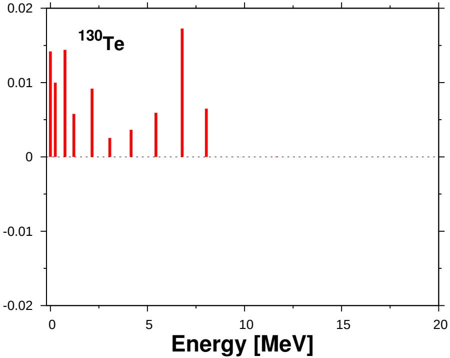
2ν half-lifes



2ν strength function in ^{48}Ca, ^{130}Te and ^{136}Xe

Parent nuclei	^{48}Ca	^{76}Ge	^{82}Se	^{130}Te	^{136}Xe
$T^{2\nu}_{1/2}(g.s.)$ th.	$3.7E19$	$1.15E21$	$3.4E19$	$4E20$	$6E20$
$T^{2\nu}_{1/2}(g.s.)$ exp	$4.2E19$	$1.4E21$	$8.3E19$	$2.7E21$	$> 8.1E20$

2ν half-lifes



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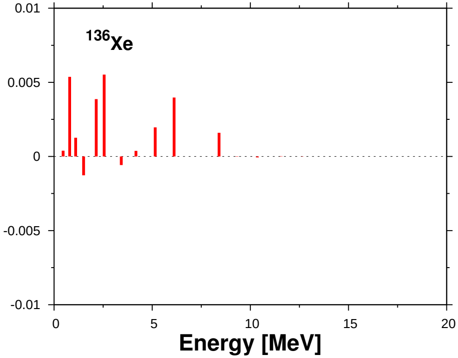

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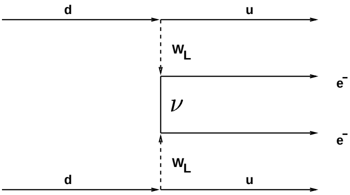
Parent nuclei	^{48}Ca	^{76}Ge	^{82}Se	^{130}Te	^{136}Xe
$T_{1/2}^{2\nu}(g.s.)$ th.	$3.7E19$	$1.15E21$	$3.4E19$	$4E20$	$6E20$
$T_{1/2}^{2\nu}(g.s.)$ exp	$4.2E19$	$1.4E21$	$8.3E19$	$2.7E21$	$> 8.1E20$

Neutrinoless mode:

Exchange of a light neutrino, only left-handed currents



The theoretical expression of the half-life of the 0ν mode can be written as:

$$[T^{0\nu}_{1/2}(0^+ \rightarrow 0^+)]^{-1} = G_{0\nu}|M^{0\nu}|^2\langle m_\nu\rangle^2$$

Neutrinoless mode:

CLOSURE APPROXIMATION then

$$\langle \Psi_f || \mathcal{O}^{(K)} || \Psi_i \rangle \quad \text{with} \quad \mathcal{O}^{(K)} = \sum_{ijkl} W_{ijkl}^{\lambda,K} \left[(a_i^\dagger a_j^\dagger)^\lambda (\tilde{a}_k \tilde{a}_l)^\lambda \right]^K$$

We are left with a "standard" nuclear structure problem

$$M_{(0\nu)} = M_{GT}^{(0\nu)} - \frac{g_V^2}{g_A^2} M_F^{(0\nu)} = \langle 0_f^+ | \sum_{n,m} h(\sigma_n.\sigma_m) t_{n-} t_{m-} | 0_i^+ \rangle - \frac{g_V^2}{g_A^2} \langle 0_f^+ | \sum_{n,m} h t_{n-} t_{m-} | 0_i^+ \rangle$$

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two-body operator

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Update of 0ν results

	$\langle m_\nu \rangle$ for $T_{\frac{1}{2}} = 10^{25}$ y.	$M^{GT}_{0\nu}$	1-χ_F
^{48}Ca	0.85	0.67	1.14
^{76}Ge	0.90	2.35	1.10
^{82}Se	0.42	2.35	1.10
^{96}Zr			
^{100}Mo			
^{110}Pd	0.67	2.52	1.16
^{116}Cd	0.24	2.59	1.19
^{124}Sn	0.45	2.11	1.13
^{128}Te	1.92	2.36	1.13
^{130}Te	0.35	2.13	1.13
^{136}Xe	0.41	1.77	1.13
^{150}Nd	**hopeless**	**for**	**SM !**

Dependance on the effective interaction

The results depend only weakly on the effective interactions provided they are compatible with the spectroscopy of the region.
For the lower *pf* shell we have three interactions that work properly, KB3, FPD6 and GXPF1. Their predictions for the 2ν and the neutrinoless modes are quite close to each other

	KB3	FPD6	GXPF1
$M_{GT}(2\nu)$	0.083	0.104	0.107
$M_{GT}(0\nu)$	0.667	0.726	0.621

Similarly, in the $r3g$ and $r4h$ spaces, the variations among the predictions of spectroscopically tested interactions is small (10-20%)

Influence of deformation

Changing adequately the effective interaction we can increase or decrease the deformation of parent, grand-daughter or both, and so gauge its effect on the decays. A mismatch of deformation can reduce the $\beta\beta$ matrix elements by factors 2-3. In fact the fictitious decay Ti-Cr, using the same energetics that in Ca-Ti, has matrix elements more than twice larger. If we increase the deformation in both Ti and Cr nothing happens. On the contrary, if we reduce the deformation of Ti, the matrix elements are severely quenched. The effect of deformation is therefore quite important and cannot be overlooked

	$^{48}Ca \rightarrow ^{48}Ti$	$^{48}Ti \rightarrow ^{48}Cr$
$M_{GT}(2\nu)$	0.083	0.213
$M_{GT}(0\nu)$	0.667	1.298

Influence of the spin-orbit partner

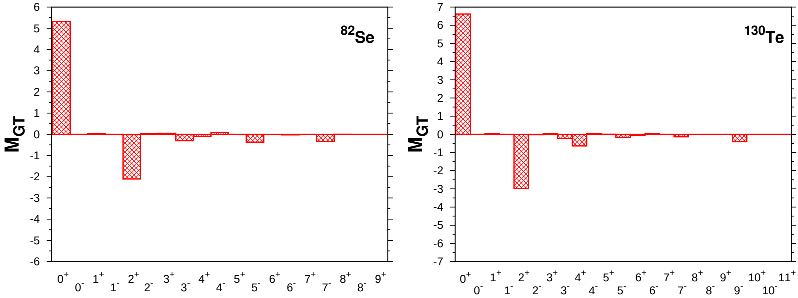
Similarly, we can increase artificially the excitation energy of the spin-orbit partner of the intruder orbit. Surprisingly enough, this affects very little the values of the matrix elements, particularly in the neutrinoless case. Even removing the spin-orbit partner completely produces minor changes

	$^{48}Ca \rightarrow ^{48}Ti$	$^{48}Ti \rightarrow ^{48}Cr$
$M_{GT}(2\nu)$	0.083	0.213
$M_{GT}(0\nu)$	0.667	1.298

Without spin-orbit partner

	$^{48}Ca \rightarrow ^{48}Ti$	$^{48}Ti \rightarrow ^{48}Cr$
$M_{GT}(2\nu)$	0.049	0.274
$M_{GT}(0\nu)$	0.518	1.386

$\beta\beta$ decay



J components (particle-particle representation)

Summary I

- Large scale shell model calculations with high quality effective interactions are available or will be in the immediate future for all but one of the neutrinoless double beta emitters
- The theoretical spread of the values of the nuclear matrix elements entering in the lifetime calculations is greatly reduced if the ingredients of each calculation are examined critically and only those fulfilling a set of quality criteria are retained
- A concerted effort of benchmarking between LSSM and QRPA practitioners would be of utmost importance to increase the reliability and precision of the nuclear structure input for the double beta decay processes

Summary II

- The most favorable case appears when both nuclei (emitter and daugther) are spherical (superfluid)

  ➡ not far from semi-magic:

  ^{82}Se, ^{116}Cd, ^{124}Sn, ^{130}Te, ^{136}Xe

- With the present work, ^{116}Cd is the best case BUT ...
  - uncertainties with the interaction
  - what happens when we enlarge the valence space ?
  - region of ^{96}Zr/^{100}Mo remains to be studied more carefully

$(\beta\beta)_{0\nu}$ matrix elements

$$M^{(0\nu)}_{GT} = \langle 0^+_f \| \sum_{n,m} h(\sigma_n.\sigma_m) t_{n-} t_{m-} \| 0^+_i \rangle, \qquad \chi_F = \langle 0^+_f \| \sum_{n,m} h t_{n-} t_{m-} \| 0^+_i \rangle \left(\frac{g_V}{g_A}\right)^2 / M^{(0\nu)}_{GT},$$

$$\chi'_{GT} = \langle 0^+_f \| \sum_{n,m} h' (\sigma_n.\sigma_m) t_{n-} t_{m-} \| 0^+_i \rangle / M^{(0\nu)}_{GT}, \qquad \chi'_F = \langle 0^+_f \| \sum_{n,m} h' t_{n-} t_{m-} \| 0^+_i \rangle \left(\frac{g_V}{g_A}\right)^2 / M^{(0\nu)}_{GT},$$

$$\chi^{\omega}_{GT} = \langle 0^+_f \| \sum_{n,m} h_{\omega} (\sigma_n.\sigma_m) t_{n-} t_{m-} \| 0^+_i \rangle / M^{(0\nu)}_{GT}, \qquad \chi^{\omega}_F = \langle 0^+_f \| \sum_{n,m} h_{\omega} t_{n-} t_{m-} \| 0^+_i \rangle \left(\frac{g_V}{g_A}\right)^2 / M^{(0\nu)}_{GT},$$

$$\chi_T = \langle 0^+_f \| \sum_{n,m} h' [(\sigma_n.\hat{r}_{n,m})(\sigma_m.\hat{r}_{n,m}) - \frac{1}{3}\sigma_n.\sigma_m] t_{n-} t_{m-} \| 0^+_i \rangle / M^{(0\nu)}_{GT},$$

$$\chi_P = \langle 0^+_f \| i \sum_{n,m} h' \left(\frac{r_{+n,m}}{2r_{n,m}}\right) [(\sigma_n - \sigma_m).(\hat{r}_{n,m} \times \hat{r}_{+n,m})] t_{n-} t_{m-} \| 0^+_i \rangle \frac{g_V}{g_A} / M^{(0\nu)}_{GT},$$

$$\chi_R = \frac{1}{6}(g^s_{-\frac{1}{2}} - g^s_{\frac{1}{2}}) \langle 0^+_f \| \sum_{n,m} h_R (\sigma_n.\sigma_m) t_{n-} t_{m-} \| 0^+_i \rangle \frac{g_V}{g_A} / M^{(0\nu)}_{GT}.$$

back

$(\beta\beta)_{0\nu}$ matrix elements

$$
\begin{aligned}
h(r,\langle\mu\rangle) &= \frac{R_0}{r}\phi(\langle\mu\rangle m_e r), \\
h'(r,\langle\mu\rangle) &= h + \langle\mu\rangle m_e R_0 h_0(\langle\mu\rangle r), \\
h_\omega(r,\langle\mu\rangle) &= h - \langle\mu\rangle m_e R_0 h_0(\langle\mu\rangle r), \\
h_R(r,\langle\mu\rangle) &= -\frac{\langle\mu\rangle m_e}{M_i}(\frac{2}{\pi}\left(\frac{R_0}{r}\right)^2 - \langle\mu\rangle m_e R_0 h) + \frac{4\pi R_0^2}{M_p}\delta(r), \\
h_0(x) &= -\frac{d\phi}{dx}(x), \\
\phi(x) &= \frac{2}{\pi}[\sin(x)C_{int}(x) - \cos(x)S_{int}(x)], \\
\frac{d\phi}{dx} &= \frac{2}{\pi}[\sin(x)C_{int}(x) + \cos(x)S_{int}(x)].
\end{aligned}
$$

$S_{int}(x)$ and $C_{int}(x)$ being the integral sinus and cosinus functions,

$$
S_{int}(x) = -\int_x^\infty \frac{\sin(\zeta)}{\zeta}d\zeta, \quad C_{int}(x) = -\int_x^\infty \frac{\cos(\zeta)}{\zeta}d\zeta
$$

back