Boson Approach to the Structure of $\mathbf{A = 62}$ nuclei.

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Gliwice 2006

Contents

Contents

Contents

Contents

- Introduction.
- Boson mapping of fermion systems.
- Results.

Contents

SOME REMARKS ON THE SHELL-MODEL

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The dimension of the shell model increases rapidly for the growing number of valence nucleons. For instance, in the case of medium heavy nuclei the number of shell model configurations is of order $10^{14} - 10^{18}$. In such a large space the simple and regular features observed in nuclei would remain hidden in several million of expansion coefficients. Therefore a realistic truncation of the shell model space has to be found.

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- Boson models, where the space is spanned by a quite small set of bosons (IBM).
- Boson-fermion models, where nuclear states are obtained from the coupling of a fermion with a boson core (IBFM).

WHAT WE ARE CONSIDERING?

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We investigate the possibility of transferring the description of even-A systems from the fermionic CPA space, described by the collective pairs, into a space described by corresponding bosons. This mapping procedure opens the way to a simplified (because only bosons are involved) but still microscopic (because no free parameters exist in the boson space) description of the low-lying collective excitations. The approach based on the boson mapping we define as the Boson Approximation (BA).

MOTIVATION

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In Ref.[M.Sambataro, Phys.Rev.**C** **52** (1995), 3378] it was shown, that in the correspondence to a generic, at most two-body fermion operator we can construct a $n-$body boson operator, which for a system of n pairs provides an exact image of the fermion operator in the boson space. Such a boson operator carries all information relating to the Pauli Exclusion Principle. Truncating this boson operator at a lower order induces a violation of this principle, whose effects are difficult to predict.

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The main motivation of this work is to investigate in details how the truncation of the boson Hamiltonian at two-body terms affects the spectrum of a three boson system. Answering this question will allow us to shed some light on the effectiveness of the BA formalism in realistic systems.

Formalism. Boson mapping of fermion systems.

Let us call F^n the CPA space spanned by the states

$$\big|i\big\rangle = \hat{A}^{\dagger}_{\nu_1\Gamma_1\Gamma'_1}\hat{A}^{\dagger}_{\nu_2\Gamma_2\Gamma'_2}\ldots\hat{A}^{\dagger}_{\nu_n\Gamma_n\Gamma'_n}\big|0\big\rangle\ , \qquad (1)$$

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where the operator $\hat{A}^{\dagger}_{\nu\Gamma\Gamma'}$ creates a collective pair of multipolarity $\Gamma = (J, T)$ and projection $\Gamma' = (J', T')$

$$\hat{A}^{\dagger}_{\nu\Gamma\Gamma'} = \sum_{\lambda_1\lambda_2} C^{\nu}_{\Gamma}(\lambda_1\lambda_2)[a^{\dagger}(\lambda_1)\times a^{\dagger}(\lambda_2)]_{\Gamma\Gamma'}. \tag{2}$$

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$$\hat{A}^{\dagger}_{\nu\Gamma\Gamma'} = \sum_{\lambda_1\lambda_2} C^{\nu}_{\Gamma}(\lambda_1\lambda_2)[a^{\dagger}(\lambda_1) \times a^{\dagger}(\lambda_2)]_{\Gamma\Gamma'}. \tag{2}$$

In equation (2) the operator $a^{\dagger}(\lambda_i)$ $(i = 1, 2)$ creates a nucleon occupying orbital λ_i and the coefficients $C^{\nu}_{\Gamma}(\lambda_1\lambda_2)$ determine the structure of collective pairs in the state $\nu\Gamma\Gamma'$, where the quantum number ν is used to distinguish states with the same angular momenta and their projection quantum numbers $\Gamma\Gamma'$.

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Let us call B^n the boson space spanned by the states

$$\left|i\right) = b^{\dagger}_{\nu_1\Gamma_1\Gamma'_1} b^{\dagger}_{\nu_2\Gamma_2\Gamma'_2} \ldots b^{\dagger}_{\nu_n\Gamma_n\Gamma'_n} \left|0\right) \ . \tag{3}$$

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States (3) are formally obtained from states (1) by replacing pair creation operators $\hat{A}^{\dagger}_{\nu\Gamma\Gamma'}$ with boson creation operators $b^{\dagger}_{\nu\Gamma\Gamma'}$ and replacing the fermion vacuum state $\big|0\big\rangle$ with the boson vacuum state $\big|0\big)$.

Formalism. Boson mapping of fermion systems.

Since states (3) are orthogonal, then according to the mapping procedure by M. Sambataro [Phys.Rev. **C 52** (1995), 3378] we can find the boson image $\hat{H}_b$ of the fermion Hamiltonian $\hat{H}_f$, such that energy spectrum of $\hat{H}_b$ in the boson space (3) exactly reproduces the energy spectrum of $\hat{H}_f$ in the fermion space.

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As a general result of the mapping procedure we obtain

$$\hat{H}_b = \hat{H}_b^1 + \hat{H}_b^2 + \cdots + \hat{H}_b^n. \tag{4}$$

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$$\hat{H}_b = \hat{H}_b^1 + \hat{H}_b^2 + \cdots + \hat{H}_b^n. \tag{4}$$

In this case, by definition, its eigenvalues are the same as those of the operator $\hat{H}_f$.

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An important question is if terms of $\hat{H}_b$ higher than two-body can be considered as negligible higher order contributions when studying systems made of more than two bosons.

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In order to investigate this problem we have employed the image operator containing only one- and two- body terms, i.e.

$$\hat{H}_b \cong \hat{H}_b^1 + \hat{H}_b^2, \tag{5}$$

Formalism. Boson mapping of fermion systems.

where

$$\hat{H}_b^1 = \sum_{\nu\Gamma\Gamma'} \varepsilon_\nu b^\dagger_{\nu\Gamma\Gamma'} b_{\nu\Gamma\Gamma'} \qquad (6)$$

and

$$\begin{aligned}
\hat{H}_b^2 &= \frac{1}{4} \sum_{\Gamma\Gamma'\nu_1\Gamma_1\Gamma_1'\nu_2\Gamma_2\Gamma_2} \sum_{'\nu_3\Gamma_3\Gamma_3'\nu_4\Gamma_4\Gamma_4'} \left(1+\delta_{\nu_1\Gamma_1\nu_2\Gamma_2}\right)\left(1+\delta_{\nu_3\Gamma_3\nu_4\Gamma_4}\right) \\
&\times \; E_\Gamma\left(\nu_1\Gamma_1\nu_2\Gamma_2;\nu_3\Gamma_3\nu_4\Gamma_4\right)\left(\Gamma_1\Gamma_1'\Gamma_2\Gamma_2'\right)\middle|\Gamma\Gamma'\right)\left(\Gamma_3\Gamma_3'\Gamma_4\Gamma_4'\right)\middle|\Gamma\Gamma'\right) \\
&\times \; b^\dagger_{\nu_1\Gamma_1\Gamma_1'} b^\dagger_{\nu_2\Gamma_2\Gamma_2'} b_{\nu_3\Gamma_3\Gamma_3'} b_{\nu_4\Gamma_4\Gamma_4'}.
\end{aligned}$$

Formalism. Boson mapping of fermion systems.

In this work we deal with three boson systems. Therefore, to expand three boson space, we introduce states

$$\big|i\big) = \big|[(b^{\dagger}_{\nu_1\Gamma_1} \times b^{\dagger}_{\nu_2\Gamma_2})_{\Gamma_{12}} \times b^{\dagger}_{\nu_3\Gamma_3}]_{\Lambda\Lambda'}\big|0\big). \qquad (7)$$

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Since states (7) are neither orthogonal nor linearly independent we have first diagonalized the overlap matrix $\big(\bar{i}\big|i\big)$ to obtain an orthonormal set of states

$$\big|\alpha\big) = \big(N_\alpha\big)^{-\frac{1}{2}} \sum_{i=1}^{N} C_{i\alpha}\big|i\big), \quad \alpha = 1, 2, \ldots, \overline{N} \tag{8}$$

expanding the three-boson space.

Formalism. Boson mapping of fermion systems.

In the next step we have found the matrix representation of the boson Hamiltonian (5) in the boson space spanned by the states (8). These matrix elements can be written as

$$\left(\overline{\alpha}\middle|\hat{H}_b\middle|\alpha\right) = \left(N_\alpha N_{\overline{\alpha}}\right)^{-\frac{1}{2}} \sum_{i\,\overline{i}} C^*_{i\alpha} C_{\overline{i}\overline{\alpha}} \left(\left(\overline{i}\middle|\hat{H}^1_b\middle|i\right) + \left(\overline{i}\middle|\hat{H}^2_b\middle|i\right)\right). \quad (9)$$

The explicit expression of the matrix elements $\left(\overline{i}\middle|\hat{H}^1_b\middle|i\right)$ and $\left(\overline{i}\middle|\hat{H}^2_b\middle|i\right)$ are given in: E.Kwasniewicz, E.Hetmaniok, J.Brzostowski, F.Catara and M. Sambataro, CEJP **4** (2003), 606.

Results. A=22 systems.

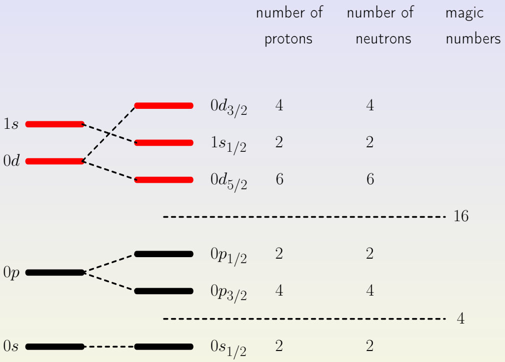

Figure 1: Single-particle energy level sequence.

In the $1s0d$ shell one can form 28 two-nucleon states: 14 $T = 0$ and 14 $T = 1$ with values of the total angular momentum J ranging from 0 up to 5.

Pairs $T = 1$ with $J = 0, 1, 2, 3, 4$ are denoted as S, P, D, F, G. Pairs $T = 0$ are denoted as Θ_J.

Corresponding bosons are denoted as s, p, d, f, g and θ_J , respectively.

J	T=0 (MeV)	J	T=1 (MeV)	J	T=2 (MeV)	J	T=3 (MeV)
3	-60.380	0	-59.788	4	-44.908	0	-35.402
1	-60.059	2	-58.278	3	-44.686	2	-31.864
4	-59.415	4	-56.238	2	-44.191	0	-30.566
5	-58.866	2	-55.188	5	-43.455	3	-30.311
1	-58.864	2	-54.679	2	-43.203	2	-28.555
3	-58.544	1	-54.048	1	-43.188	4	-28.210
2	-57.639	4	-54.038	3	-43.187	4	-27.853
3	-57.546	3	-53.997	1	-42.656	2	-27.258
6	-56.520	2	-53.277	0	-42.603	3	-26.833
1	-56.348	0	-53.155	4	-42.225	0	-26.227

Table 1: The lowest 10 shell model eigenstates of A=22 nuclei for all values of the total isospin T. States are also labeled with their angular momentum J.

J	T=0 (MeV)	J	T=1 (MeV)	J	T=2 (MeV)	J	T=3 (MeV)
3	-60.380	0	-59.788	4	-44.908	0	-35.402
1	-60.059	2	-58.278	3	-44.686	2	-31.864
4	-59.415	4	-56.238	2	-44.191	0	-30.566
5	-58.866	2	-55.188	5	-43.455	3	-30.311
1	-58.864	2	-54.679	2	-43.203	2	-28.555
3	-58.544	1	-54.048	1	-43.188	4	-28.210
2	-57.639	4	-54.038	3	-43.187	4	-27.853
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Interactions:B.A.Brown, W.A.Richter, R.E.Julies, B.H.Wildenthal, Ann.Phys.(NY), **182** *(1998), 191.*

In Table (1) are shown energies and angular momenta of the lowest 10 eigenstates for each T. $T = 0$ and $T = 1$ states are in the same range of energy while large gaps occur between the $T = 1$ and $T = 2$ states as well as between the $T = 2$ and $T = 3$ states.

$T = 0$ states only refer to a nucleus with an equal number of protons and neutrons. For the six particles systems under study, this means 3 protons and 3 neutrons, namely an odd-odd nucleus.

The spectrum of a system with 2 protons and 4 neutrons (or viceversa), being characterized by an isospin projection $T_z = -1(+1)$, includes all eigenstates with $T = 1, 2, 3$.

Due to the energy distribution evidenced in Table (1), however, we can state that the lowest eigenstates of these even-even systems are all $T = 1$ states. Going on in this way, it is easy to argue that all $T = 2$ states of Table (1) are the lowest eigenstates of systems with $T_z = \pm 2$ (1 proton and 5 neutrons or viceversa, i.e. odd-odd systems) while all $T = 3$ states of the same table are the lowest eigenstates of systems with $T_z = \pm 3$ (0 protons and 6 neutrons or viceversa, i.e. even-even systems).

We will begin our analysis by discussing the $T = 1$ and $T = 3$ cases (even-even systems) and proceed, then, with the remaining cases (odd-odd systems).

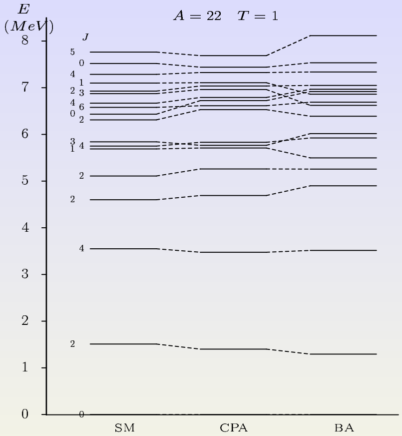

Figure 2: Comparison of the $T = 1$ low-lying shell model spectrum (SM) of $A = 22$ nuclei with the CPA spectrum calculated in the space spanned by the $T = 1$, S, S', P, D, D', F, G pairs and with the boson approximation spectrum (BA) calculated in the corresponding space.

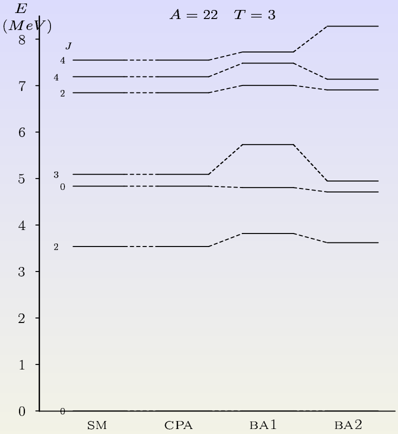

Figure 3: Comparison of the $T = 3$ low-lying shell model spectrum (SM) of $A = 22$ nuclei with the CPA spectrum calculated in the space spanned by the (1) $T = 1$, S, S', D, D', G pairs or by the (2) $T = 1$, S, S', P, D, D', F, G pairs (CPA spectrum is the same) and with the BA spectrum calculated in the corresponding space.

Results. A=62 systems.

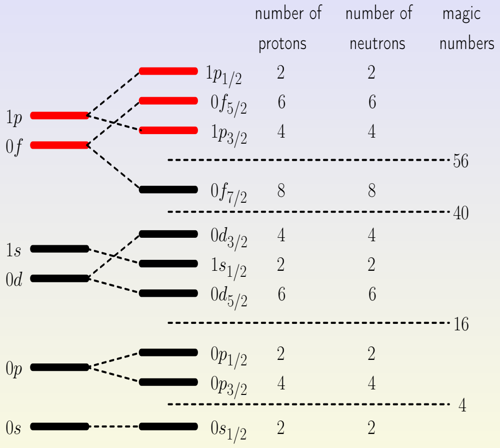

Figure 4: Single-particle energy level sequence.

J	T=0 (MeV)	J	T=1 (MeV)	J	T=2 (MeV)	J	T=3 (MeV)
1	-75.300	0	-74.718	1	-67.849	0	-62.977
3	-74.867	2	-74.132	2	-67.712	2	-62.053
2	-74.699	2	-73.462	3	-67.514	0	-61.326
1	-74.603	4	-73.218	0	-67.315	2	-61.240
3	-74.422	3	-73.009	2	-67.237	4	-61.030
1	-74.417	2	-72.987	1	-67.168	3	-60.850
2	-74.365	1	-72.807	2	-67.154	2	-60.595
4	-74.277	3	-72.701	1	-67.079	4	-60.555
5	-74.123	4	-72.606	3	-67.076	0	-60.554
3	-74.039	0	-72.560	2	-67.072	5	-60.441

Table 2: The lowest 10 shell model eigenstates of A=62 nuclei for all values of the total isospin T. States are also labeled with their angular momentum J.

J	T=0 (MeV)	J	T=1 (MeV)	J	T=2 (MeV)	J	T=3 (MeV)
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3	-74.422	3	-73.009	2	-67.237	4	-61.030
1	-74.417	2	-72.987	1	-67.168	3	-60.850
2	-74.365	1	-72.807	2	-67.154	2	-60.595
4	-74.277	3	-72.701	1	-67.079	4	-60.555
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Interactions: J.E.Koops and P.W.M Glaudemans, Z.Phys. **C 280** *(1977), 181.*

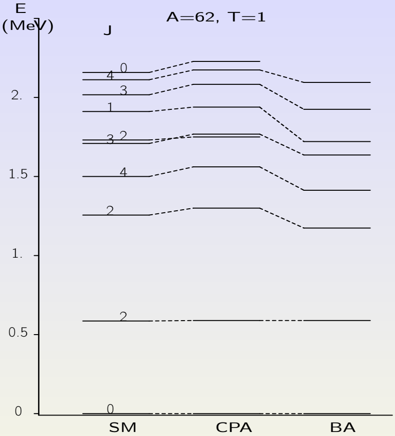

Figure 5: Comparison of the $T = 1$ low-lying shell model spectrum (SM) of $A = 62$ nuclei with the CPA spectrum calculated in the space spanned by the $T = 1$, S, S', D, D', D'', G pairs and with the boson approximation spectrum (BA) calculated in the corresponding space.

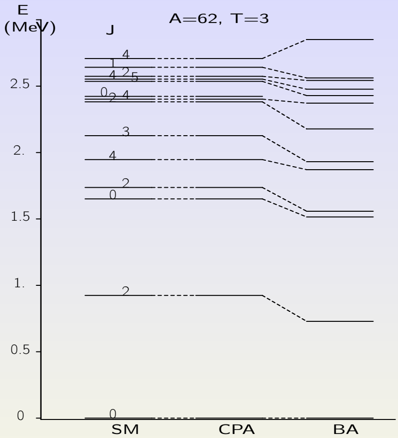

Figure 6: Comparison of the $T = 3$ low-lying shell model spectrum (SM) of $A = 62$ nuclei with the CPA spectrum calculated in the space spanned by the $T = 1$, S, S', D, D', D'', G pairs and with the boson approximation spectrum (BA) calculated in the corresponding space.

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- We have applied a procedure to transfer the description of a fermion system from a subspace of the full shell model space built in terms of collective pairs onto a space of corresponding bosons.
  We have performed exact SM calculations and compared them with calculations in the CPA and BA formalism for systems of six nucleons in the $1s0d$ $(A = 22)$ and $1p0f$ $(A = 62)$ shells.

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  We have performed exact SM calculations and compared them with calculations in the CPA and BA formalism for systems of six nucleons in the $1s0d$ ($A = 22$) and $1p0f$ ($A = 62$) shells.
- In the case of BA we have constructed a two-body boson Hamiltonian in correspondance to the fermion one.
  For both six-nucleons systems in the 1s0d and 1p0f shells the agreement between CPA and BA results has been found reasonably good for the lowest eigenstates with total isospin $T = 1$ and $T = 3$ (corresponding to even-even systems) and rather bad for $T = 0$ and $T = 2$ eigenstates (corresponding to odd-odd systems).

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- The dependence of presented results on the choice of the interaction and on the definition of the collective pairs might be possible.