Vibrational state contribution to nuclear level density

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Nuclear level density is one of the basic quantities for description of high-excited nuclear states and analysis of their nature. It also determines the characteristics of nuclear decay. The collective states can strongly effect on the level density, specifically, at low excitation energies.

The simplest method to estimate effect of the vibrational states on level densities is calculation of collective enhancement factor

$$K = \rho / \rho_0$$

ρ, ρ_0 - level densities with and without allowing for collective states

Problems with estimation of K

1. Uncertainties in value

Closed-form approach

Ignatyuk A.V. Statistical properties of excited nuclei. 1983; Yad. Fiz. 21(1975) 20 ; Izv.AN SSSR 38 (1974) 2612 (RPA approach); Ignatyuk A.V., Weil J.L., Raman S., Kahane S. PRC. 47 (1993) 1504

$$K_{vibr}(S_n) \sim 15 \div 30 \quad (A \sim 100); \ \ RIPL-2(2003)$$

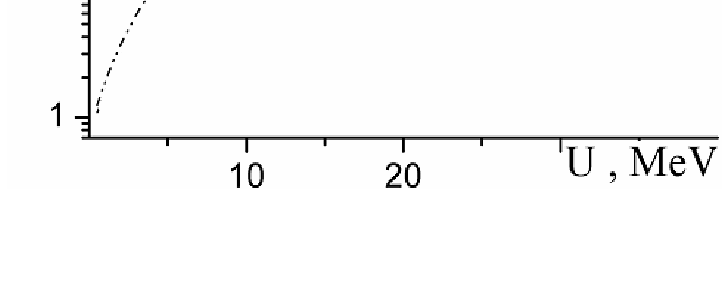
Microscopic calculations within quasiparticle-quasiphonon model

Soloviev V.G., Stoyanov Ch., Vdovin A.I. NPA224 (1974) 411; Voronov V.V., Malov L.A., Soloviev V.G. Yad.Fiz. 21 (1975) 40; Malov L.A., Soloviev V.G., Voronov V.V. Phys.Lett. B55 (1975) 17; Vdovin A.I., Voronov V.V., Malov L.A. Soloviev V.G., Stoyanov Ch. Fiz.El.Chas.At.Yad. 7 (1976) 952

$$K_{vibr}(S_n) \sim 2 \div 5 \quad (A \sim 100)$$

2. Energy dependence

Unrealistic dependence of vibrational enhancement factor on excitation energy without collective state damping



Main goals of the studies:

1) To develop and test the theory supported practical method for the calculation of vibrational enhancement factor with allowance for damping collective states.

2) To estimate contribution of 2+&3- vibrational states to nuclear level density in the range near neutron separation energies.

Vibrational enhancement calculations

Response function method

$K = \rho / \rho_0$ with ρ within saddle-point approximation

$$\rho(U, A) = \left(4\pi^2 D\right)^{-1/2} \exp S(\alpha_0, \beta_0),$$

$$S(\alpha_0, \beta_0) = -\alpha_0 A + \beta_0 E + \ln Z(\alpha_0, \beta_0)$$

- nuclear entropy;

$$Z(\alpha, \beta) = \mathrm{Tr}\left[\exp(-\beta \tilde{H})\right]$$

- partition function;

$$\Omega(\alpha, \beta) = -\ln Z(\alpha, \beta) / \beta$$

- thermodynamic potential; $\tilde{H} \equiv \hat{H} - \mu \hat{A}$; $\mu = \alpha / \beta$;

Equations of thermodynamic state:

$$A = \partial \ln Z / \partial \alpha \,|_{\alpha_0, \beta_0},$$

$$E = -\partial \ln Z / \partial \beta \,|_{\alpha_0, \beta_0}$$

$1/\beta_0 = T$ - the temperature; $\mu = \alpha_0 / \beta_0$ - chemical potential

Coherent separable interactions

$$V_{k_L,res}(i,j) = k_L \sum_{L,\mu} q^*_{L\mu}(\vec{r}_i) q_{L\mu}(\vec{r}_j), \quad q_{L\mu}(\vec{r}) = r^L Y_{L\mu}(\hat{r})$$

The total Hamiltonian: $\hat{H}_k = \hat{H}_0 + \hat{V}_{k,res}$,

Vibrational state addition to partition function

$$\Delta Z = Z / Z_0 = \exp(-\beta\Delta\Omega), \; Z_0 => \hat{H}_0$$

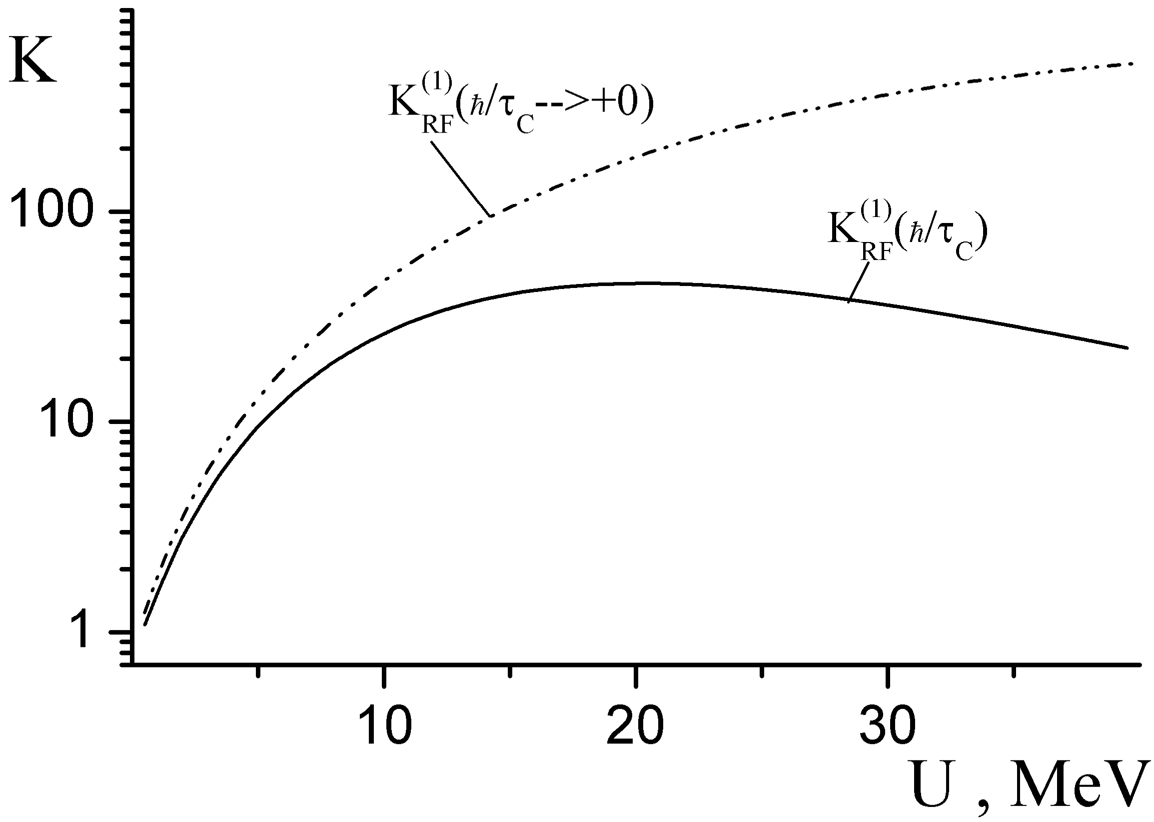
Green's function method result for thermodynamic potential

$$\Delta\Omega = \sum_L \Delta\Omega_L \;;$$

$$\Delta\Omega_L = -\frac{2L+1}{2\pi} \int_0^{k_L} dk' \int_{-\infty}^{+\infty} \frac{\hbar}{1-e^{-\beta\hbar\omega}} \times$$

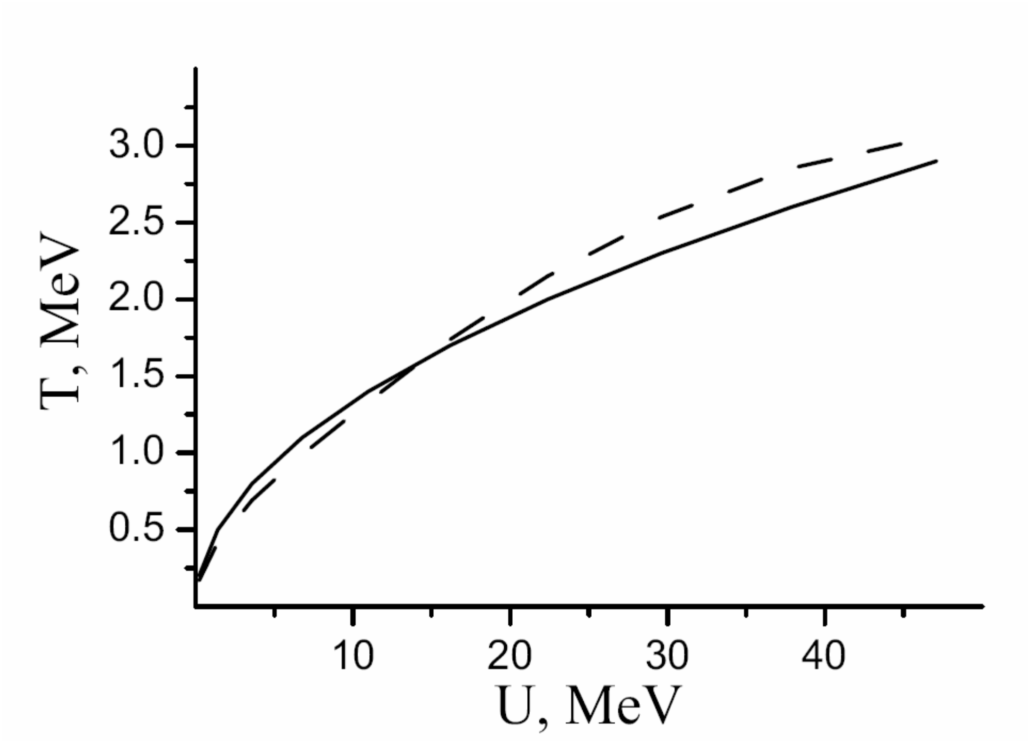
$$\mathrm{Im}(\chi_L^{k'}(\omega) - \chi_L^0(\omega))d\omega \;;$$

$\chi_L^{\kappa'}(\omega) = \mathrm{Tr}_{\{1\}}\left(q_{L0}(\vec{r}) \cdot \delta\rho(\vec{r};\omega)\right)_{k'}$ - RF within Vlasov-Landau kinetic equation

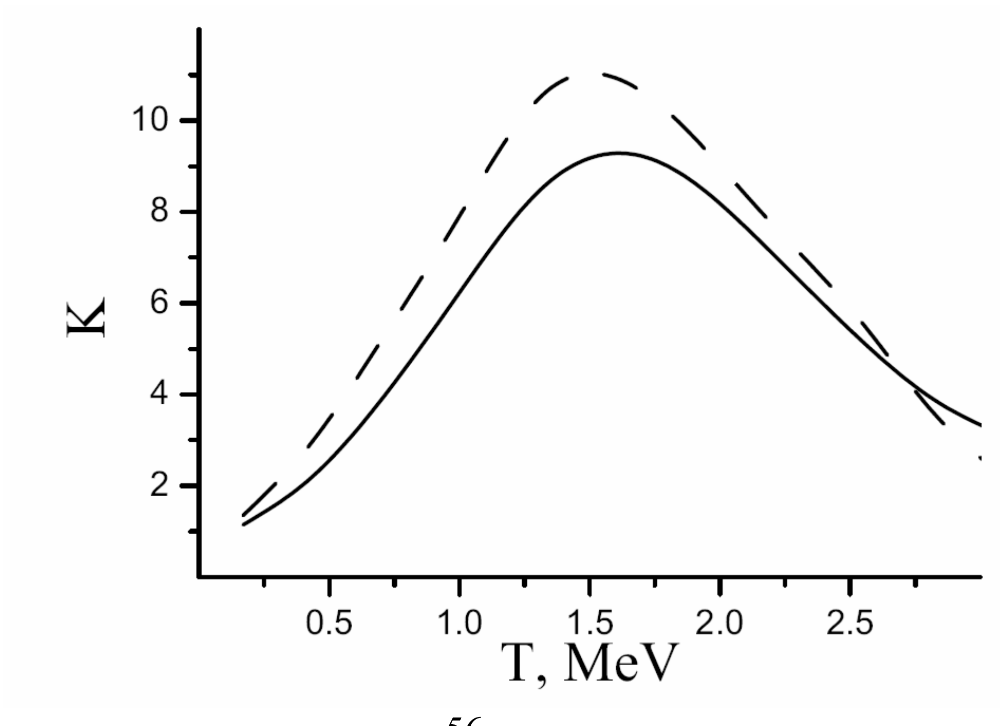


Effect of relaxation on K in ^{56}Fe:

$K_{RF}^{(1)}$ - RF method in one-resonance approach



The temperature in ^{56}Fe (2^{+}): with allowance for vibrational state (– – –) and without (——)



Enhancement factor in ^{56}Fe: – – – - saddle-point method, —— - adiabatic approach

Phenomenological expressions for vibrational enhancement factor

Boson partition function with damped occupation numbers

[Ignatyuk A.V., Weil J.L., Raman S., Kahane S. PRC 47 (1993) 1504]

$$K = \exp\left(\bar{S} - \bar{U}/T\right) \equiv K_{DN},$$

$$\bar{S} = \sum_L (2L+1)\left[(1+\bar{n}_L)\ln(1+\bar{n}_L) - \bar{n}_L \ln \bar{n}_L\right],$$

$$\bar{U} = \sum_L (2L+1)\hbar\omega_L \bar{n}_L, \qquad \bar{n}_L = \frac{\exp\left[-\Gamma_L/(2\hbar\omega_L)\right]}{\exp(\hbar\omega_L/T) - 1},$$

$$\Gamma = C \cdot \left[(\hbar\omega)^2 + 4\pi^2 T^2\right], \qquad C = 0.0075 A^{1/3} \;\; MeV^{-1}$$

Boson partition function with complex energies

[Blokhin A.I, Ignatyuk A.V., Shubin Yu.N. Yad. Fiz. 48 (1988) 371]

$$K = \prod_L \left| \frac{\delta Z_{B,L}(\hbar\omega_L + i\gamma_L;T)}{\delta Z_{B,L}(\hbar\tilde{\omega}_L + i\tilde{\gamma}_L;T)} \right| =$$

$$= \prod_L \left| \frac{1 - \exp\left[-(\hbar\omega_L + i\gamma_L)/T\right]}{1 - \exp\left[-(\hbar\tilde{\omega}_L + i\tilde{\gamma}_L)/T\right]} \right|^{-(2L+1)} \equiv K_{CE}$$

$$[\hbar\omega_L]^2 = [\hbar\tilde{\omega}_L]^2 - \xi(T)\{[\hbar\tilde{\omega}_L]^2 - [\hbar\omega_{L,\exp}]^2\},$$

$$\xi(T) = \exp\{-C_1 T^2/[\hbar\omega_{L,\exp}]\}$$

K using boson partition function with average occupation numbers

$$K = K_{BAN} \equiv K(\Delta Z = \Delta Z_{BAN})$$

$$\Delta Z = \prod_L (1 + \overline{n}_L(\omega_L))^{2L+1} \equiv \Delta Z_{BAN}$$

$$\overline{n}_L = \frac{1}{T_p} \int_0^{T_p} n_L \exp(-\Gamma_L t) dt =$$

$$= \left(\exp(\hbar\omega_L / T) - 1\right)^{-1} \left(1 - \exp(-2\pi\Gamma_L / \hbar\omega_L)\right) \hbar\omega_L / 2\pi\Gamma_L$$

Liquid drop partition function with reduction

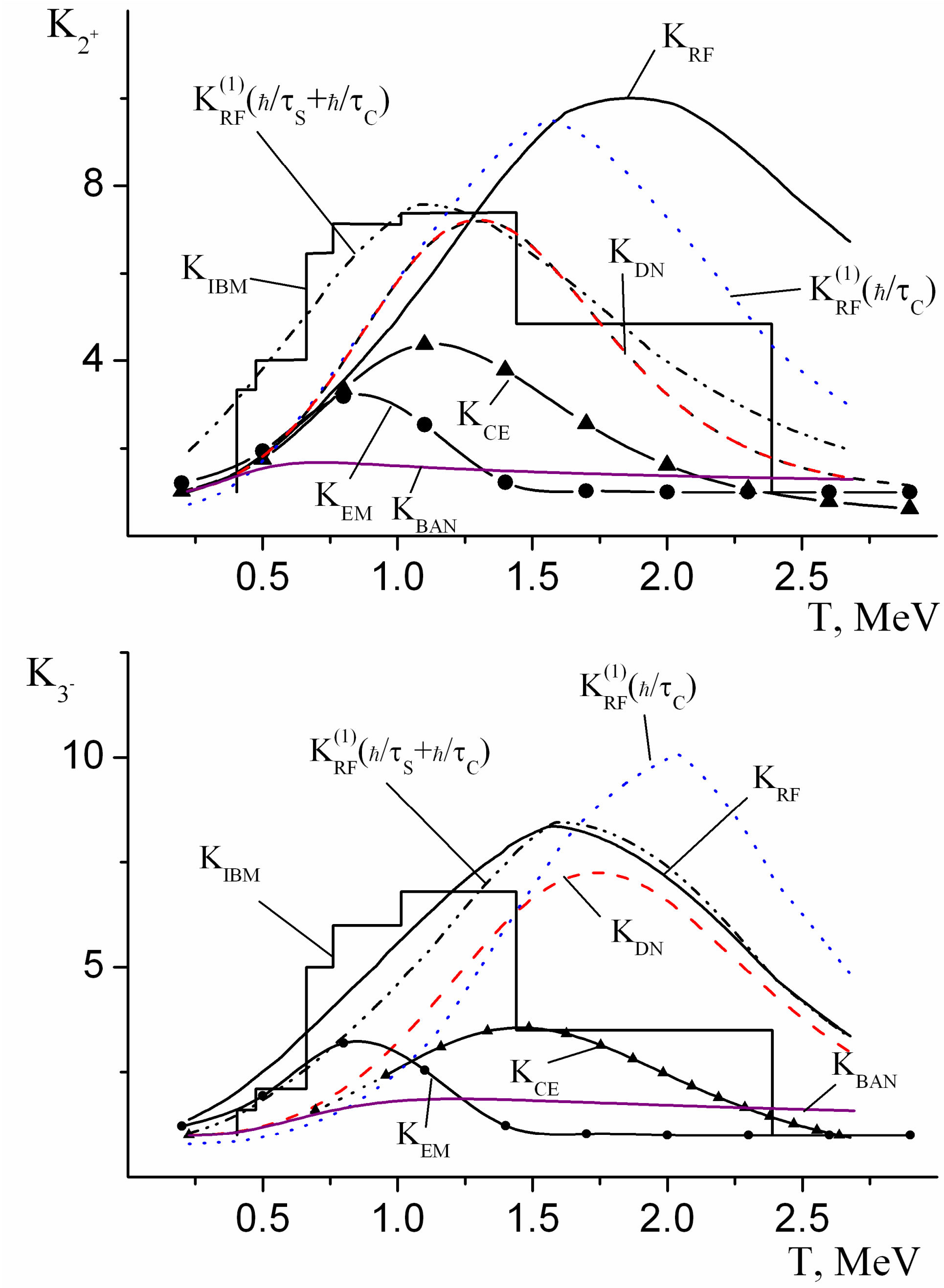
[Empire-II code by Herman M., Capote-Noy R., Oblozinsky P. et al. Journ. Nucl. Sc. Techn. Suppl.2 (2002) 116]

$$K = K_{LDM}(1 - Q_{damp}) + Q_{damp} \equiv K_{EM} ,$$

$$Q_{damp} = 1/[1 + \exp\{(T_{1/2} - T)/DT\}],$$

$$DT = 0.1 \;\; MeV, \qquad T_{1/2} = 1 \;\; MeV$$

$$K_{LDM} = \exp\left[C_{4/3} \left(\frac{\rho_0}{\hbar^2 \sigma} \right)^{2/3} R_0^2 T^{4/3} \right] = \exp\left[C_3 \, A^{2/3} \cdot T^{4/3} \right]$$



Comparison of enhancement factor in ^{146}Sm: K_{IBM} - finite temperature IBM [Mengoni A., et al. Journ. Nucl. Sc. Techn. Suppl.2 (2002) 766]; $K_{RF}^{(1)}$ - RF method in one-pole approach

Comparison with experimental data

Observable values:

- Neutron resonance spacing

$$D = 1/\begin{cases} \rho(S_n, I = 1/2)/2, & if\ I_0 = 0, \\ (\rho(S_n, I = I_0 - 1/2) + \rho(S_n, I = I_0 + 1/2))/2, & if\ I_0 \neq 0, \end{cases}$$

- Cumulative number of nuclear levels

$$\mathbb{N} = \int_0^{U_0} \rho(U)\,dU$$

NUCLEAR LEVEL DENSITY

$$\rho = K \cdot \rho_0$$

Generalized Superfluid Model for ρ_0

Ignatyuk A.V., et al, PRC 47 (1993) 1504;RIPL2

Collective enhancement factor

$$K = K_{vibr} \cdot K_{rot}$$

Rotational contribution

[Ignatyuk A.V.,et al, PRC 47 (1993) 1504; RIPL2]

$$K_{rot}(U) = \frac{(J_{\perp} T(1+\beta/3)-1)}{(1+\exp(U-U_{cr})/d_{cr})} + 1,$$

$$U_{cr} = 120 A^{1/3} \beta^2 \ MeV, \quad d_{cr} = 1400 A^{-2/3} \beta^2 \ MeV$$

$J_{\perp}$ - moment of inertia, β - quadrupole deformation

Effect of neutron-proton asymmetry in asymptotic value of level density parameter "a"

Tested shapes of level density parameter

$$\tilde{a} = \alpha_V A + \alpha_S A^{2/3} \quad MeV^{-1}$$

Handbook for calculations of nuclear reaction data. RIPL2
(http://161.5.7.5/RIPL-2)

$$\tilde{a} = \alpha_V A / \exp(\alpha_{AS} (N - Z)^2)$$

$$\tilde{a} = (\alpha_V A + \alpha_S A^{2/3}) / \exp(\alpha_{AS} (N - Z)^2)$$

[Grimes S.M. AIP Conference Proc. 769 (2005) 1253,
Al-Quraishi S.I., Grimes S.M. et al. PRC 67 (2003) 015803]

$$\tilde{a}(A, I = (N - Z) / A) = \alpha_V A(1 - \alpha_{VI} I^2) +$$
$$+ \alpha_S A^{2/3} (1 - \alpha_{SI} I^2) + \alpha_C Z^2 / A^{1/3}$$

[Nerlo-Pomorska B., Pomorski K., Bartel J., Dietrich K. PRC 66 (2002) 051302(R)]

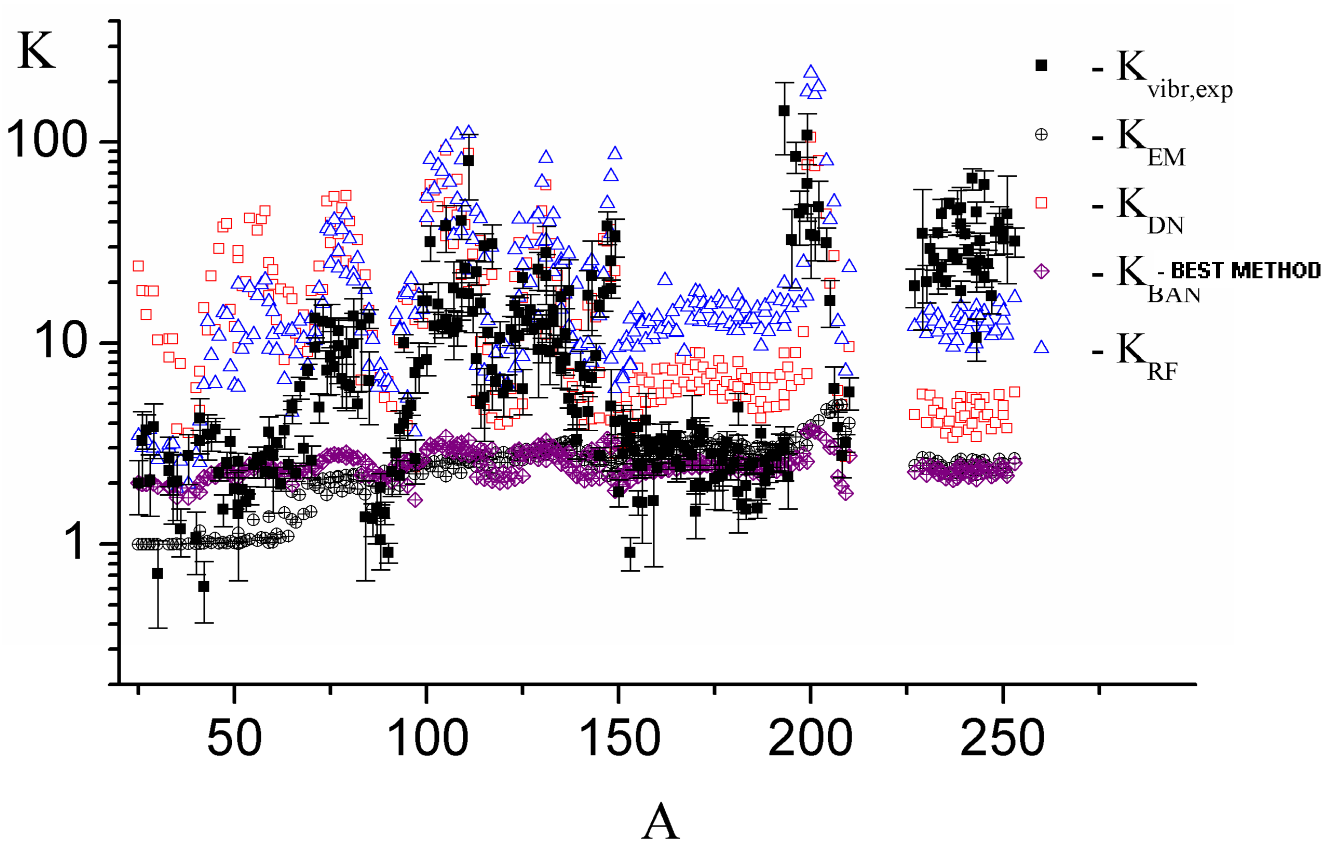
Test of approach acceptance → minimum $\left\{\chi^2,\ f\right\}$

$$\chi^2 = \sum_i^A \left[\left(\frac{\rho_i - \rho_{\exp,i}}{\Delta\rho_{\exp,i}} \right)^2 + \frac{\left(\mathbb{N}_i - \mathbb{N}_{\exp,i}\right)^2}{\mathbb{N}_{\exp,i}} \right],$$

$$f = \exp\left[\frac{1}{A} \sum_i^A \ln^2 \frac{\rho_{\exp,i}}{\rho_i} \right]^{1/2},$$

$\rho_{\exp,i}$ - level density at S_n, $\mathbb{N}_{\exp,i} = \mathbb{N}(A_i)$.

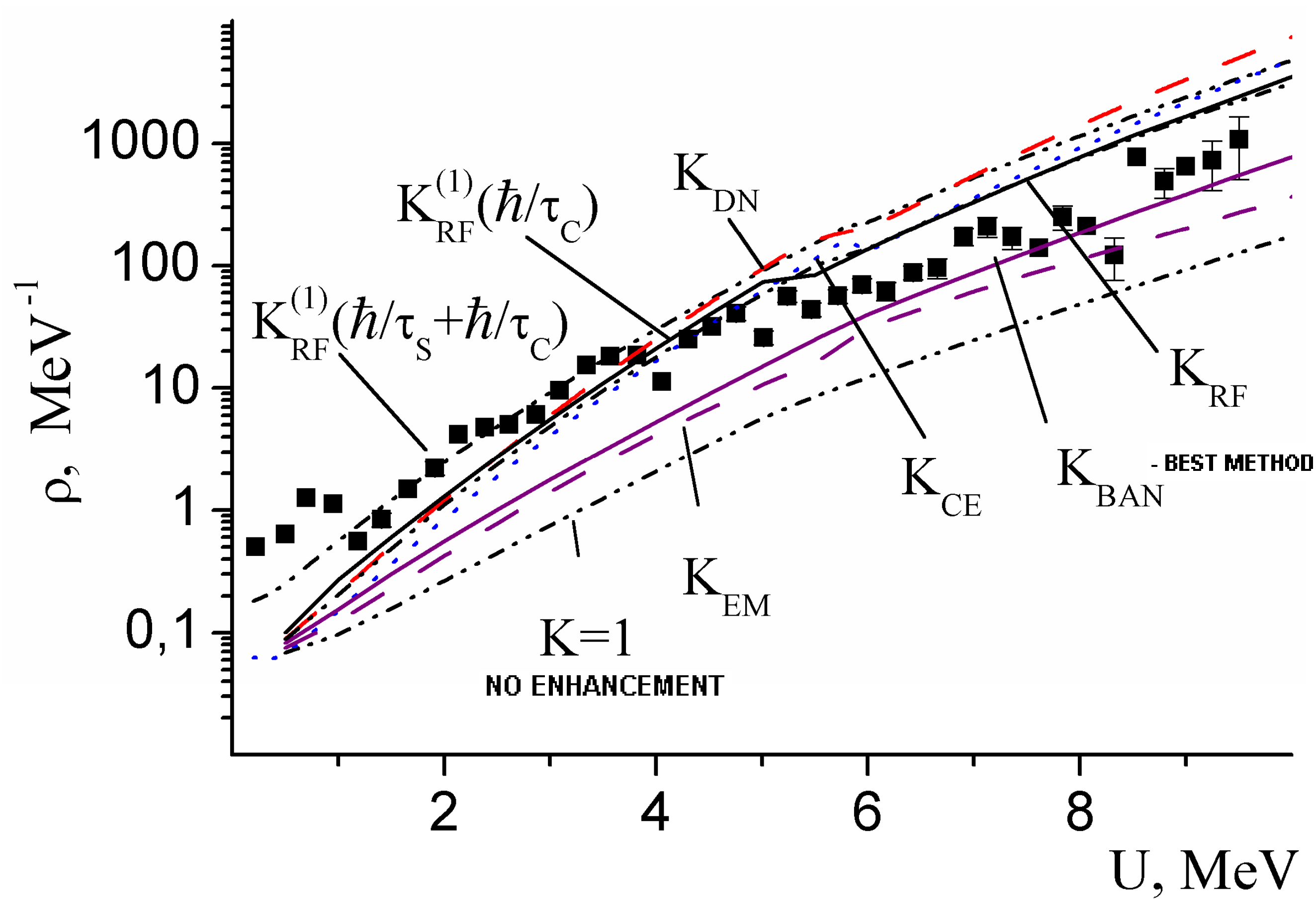
K_{vibr}, $K_{rot}=1$	χ^2			
	$\tilde{a} = \alpha_V A + \alpha_S A^{2/3}$	$\tilde{a} = \frac{\alpha_V A}{\exp(\alpha_{AS}(N-Z)^2)}$	$\tilde{a} = \frac{(\alpha_V A + \alpha_S A^{2/3})}{\exp(\alpha_{AS}(N-Z)^2)}$	$\tilde{a}(A,I) = \alpha_V A(1-\alpha_{VI} I^2) + \alpha_S A^{2/3}(1-\alpha_{SI} I^2) + \alpha_C Z^2 / A^{1/3}$
$K=1$	36.1	29.9	29.0	28.4
K_{DN}	29.7	29.9	28.4	29.8
K_{BAN}	32.3	26.8	26.1	25.1



The comparison of the different vibrational enhancement factors $(2^+ + 3^-)$ with experimental data: $U = S_n$.

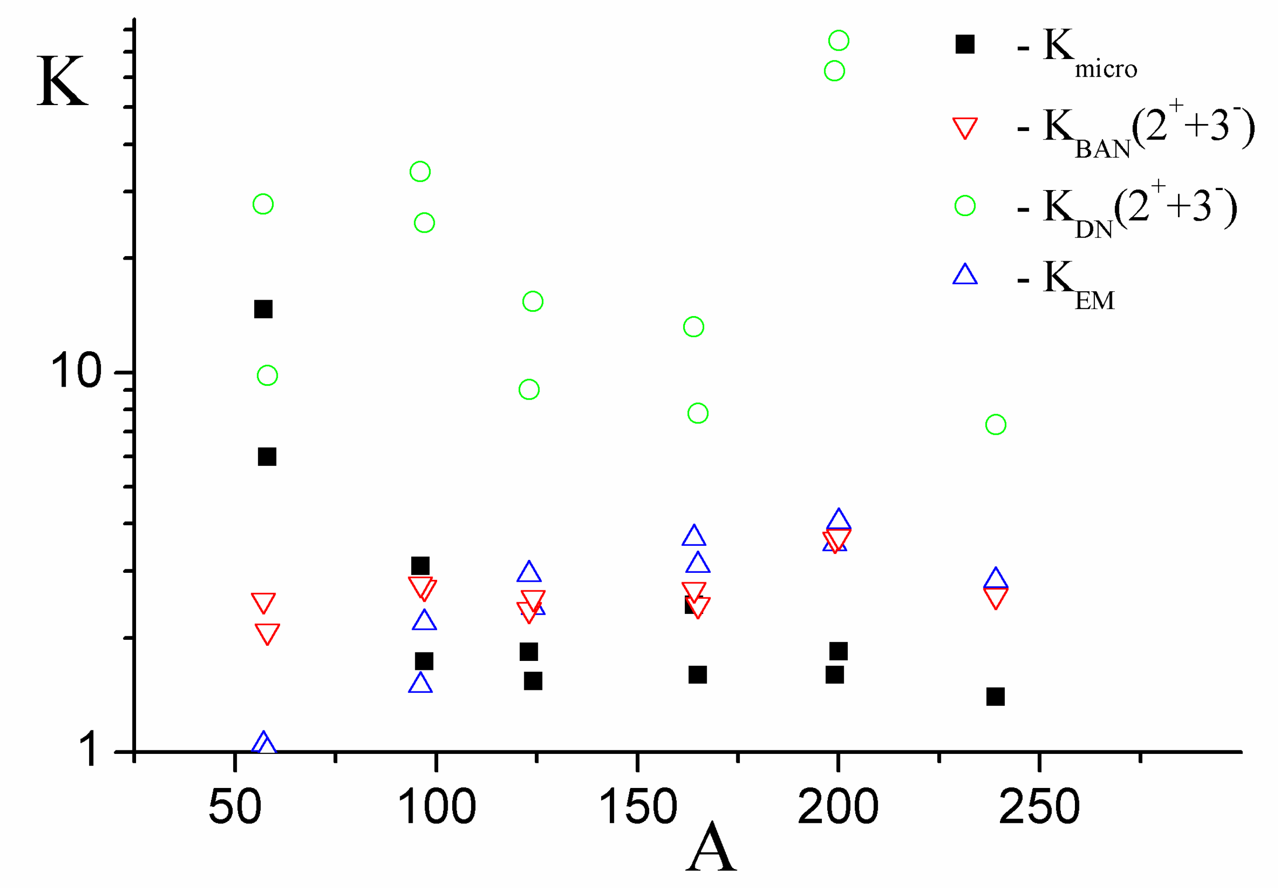
$$\tilde{a} = (0.1286A - 0.04872A^{2/3}) / \exp(0.0002986(N-Z)^2)$$

Smallest χ^2 value corresponds to method of boson partition function with average occupation numbers (BAN approach).



Dependence of total level density on excitation energy in *Fe* : ■ - experimental data

$$\tilde{a} = (0.1286A - 0.04872A^{2/3}) / \exp(0.0002986(N-Z)^2)$$



Comparision of different parametrizations for enhancement factor with microscopical calculation K_{micro}

[Vdovin A.I., Voronov V.V., Malov L.A. Soloviev V.G., Stoyanov Ch. Fiz.El.Chas.At.Yad. 7 (1976) 952]

SUMMARY

•Response function (RF) method was considered and tested to describe the effect of the vibrational states on nuclear level density. The method allows to take into account the damping of vibrational states in a standard semiclassical way.

•The calculation of the vibrational $(2^+ + 3^-)$ enhancement factor within the RF method and different phenomenological approaches with allowance for damping were compared.

•The calculations demonstrate rather strong dependence of the level density on the relaxation time shape.

• K using boson partition function with average occupation numbers (BAN) is the best method of the vibrational enhancement factor calculation when N/Z dependence of asymptotic level density parameter is taken into account in the form

$$\tilde{a} = (0.1286A - 0.04872A^{2/3}) / \exp(0.0002986(N-Z)^2)$$

$$K_{vibr}(S_n) \sim 2 \div 5 \quad (A \sim 100)$$

• Method of boson partition function with damping occupation numbers (DN) is the best one without allowance for N/Z dependence of asymptotic level density parameter

$$\tilde{a} = 0.103A - 0.105A^{2/3} \quad (RIPL2)$$

$$K_{vibr}(S_n) \sim 15 \div 30 \quad (A \sim 100)$$

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