Tetrahedral and Octahedral Symmetries in Nuclei

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29th September 2006

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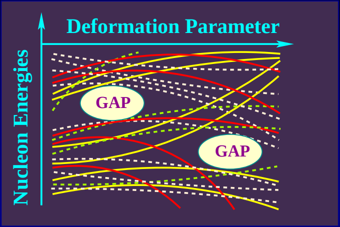
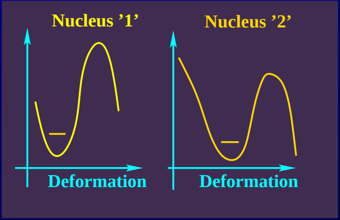
Part I

Introduction: Symmetry and Nuclear Stability

Nuclear Stability and Gaps in the Spectra [1]

- Here and in the following we use the nuclear mean-field approach
- The deformation-parameter axis represents often *many independent* deformations of the mean field
- The presence of the sufficiently strong gaps at Fermi levels leads to shape coexistence
- In what follows we focus on shell effects generated by high symmetry point-groups (see below)





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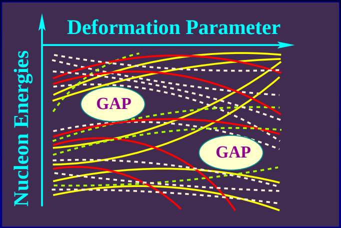

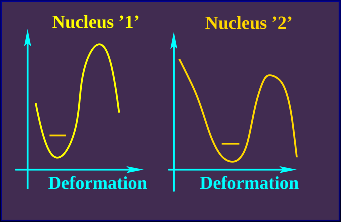

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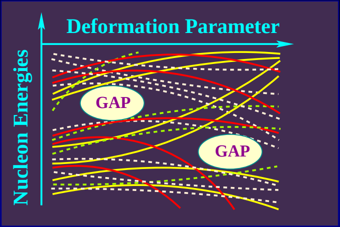

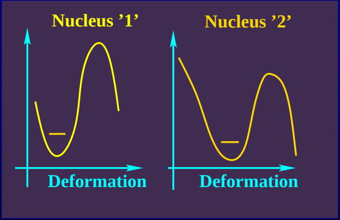

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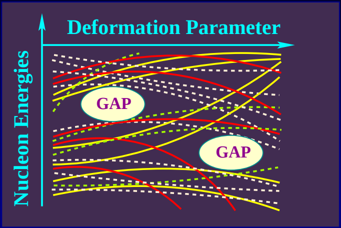

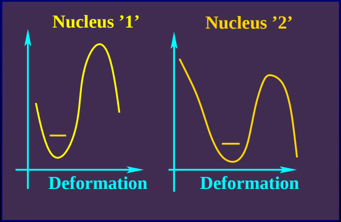

Single-Particle Gaps and Underlying Symmetries [1]

- One of the first mechanisms used in nuclear physics is related to the Harmonic Oscillator 'rule': $\omega_x : \omega_y : \omega_z = k : m : n$
- This mechanism manifests (is a special case) of SU_3 symmetry

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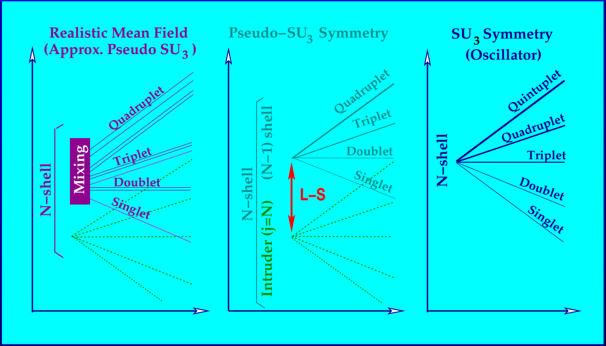

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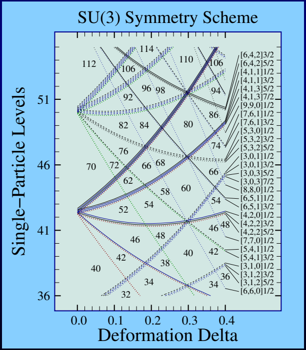

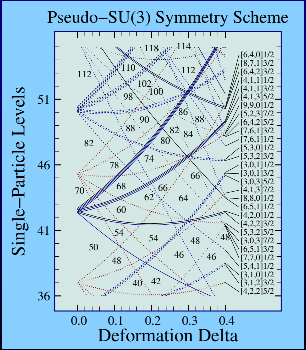

Symmetries and Implied Degeneracies of Levels [1]

- Both spherical gaps and deformed oscillator gaps arise because of the degeneracy of levels
- How to optimise the mathematical conditions so that the degeneracies are the strongest possible?
- The answer is:

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Symmetry Groups and Degeneracies of Levels

- Given Hamiltonian H and a group: $\mathcal{G} = \{\mathcal{O}_1, \mathcal{O}_2, \dots \mathcal{O}_f\}$
- Assume that $\mathcal{G}$ is a symmetry group of H i.e.

$$[H, \mathcal{O}_k] = 0 \quad \text{with} \quad k = 1, 2, \dots f$$

- Let irreducible representations of G be $\{\mathcal{R}_1, \mathcal{R}_2, \dots \mathcal{R}_r\}$
- Let their dimensions be $\{d_1, d_2, \dots d_r\}$, respectively
- Then the eigenvalues $\{\varepsilon_\nu\}$ of the problem

$$H\psi_\nu = \varepsilon_\nu \psi_\nu$$

appear in multiplets d_1-fold, d_2-fold ... degenerate

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What Are the Nuclear High-Level Symmetry Groups?

32 Point Groups: Subgroups

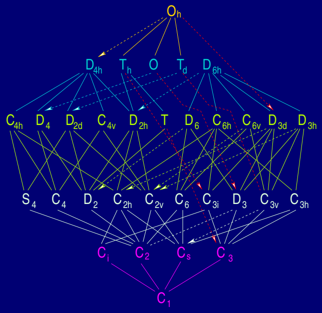


Figure: *Cubic group structure*

Dashed lines indicate that the sub-group marked is not invariant

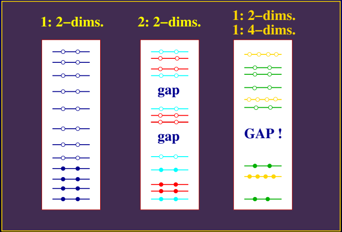
Trivial groups: $\mathrm{C}_1 \equiv \{\mathbb{I}\}$, $\mathrm{C}_s \equiv \{\mathbb{I}, \hat{\sigma}\}$ and $\mathrm{C}_i \equiv \{\mathbb{I}, \hat{\pi}\}$

Only the double groups O_h^D and T_d^D lead to four-fold degeneracies in the nucleonic spectra - all the others cause merely two-fold degeneracies. This is why the former are called *high-level* ...

Irreducible Representations and Gaps - Nuclear Context

- The nuclear potential depth is approximately constant - it depends only weakly on the particle numbers and on deformation
- The higher the dimensions of the irreps. $\rightarrow$ the higher the degeneracies of s.p. levels $\rightarrow$ the larger the gaps, on the average
- The highest dimensions of the irreducible representations correspond to the *Double* Tetrahedral & Octahedral Groups ($d = 4$)

Three 'repartitions' of single particle levels into various irreducible repres.:
Left: one two-dimensional irrep.
Middle: two two-dimensional irreps.
Right: one two-dimensional and 2 four-dimensional irreps.



Symmetries and Gaps in Nuclear Context: Summary

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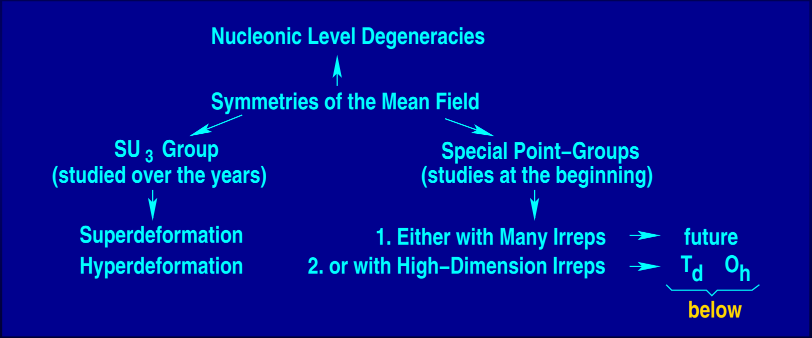

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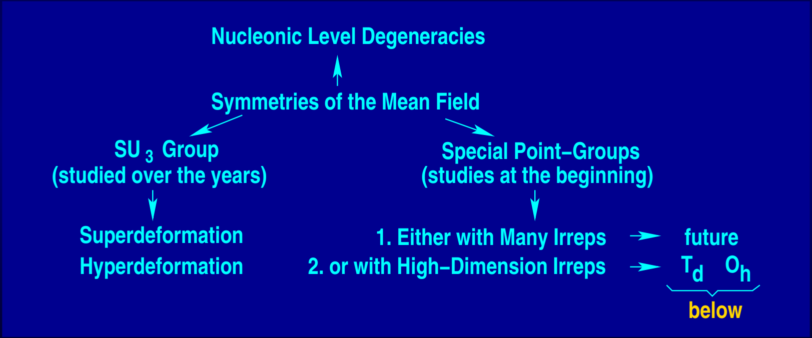

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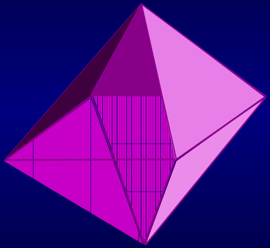
Part II

Octahedral and Tetrahedral Nuclei

Introducing Nuclear Octahedral Symmetry

Let us recall one of the magic forms introduced long time by Plato. The implied symmetry leads to the *octahedral group* denoted O_h

An octahedron has 8 equal walls. Its shape is invariant with respect to 48 symmetry elements that include inversion. However, the nuclear surface cannot be represented in the form of a diamond $\longrightarrow \longrightarrow \longrightarrow \longrightarrow \longrightarrow \longrightarrow \longrightarrow \longrightarrow$



... but rather in a form of a regular spherical harmonic expansion:

$$\mathcal{R}(\vartheta,\varphi) = R_0\, c(\{\alpha\})[1 + \sum_{\lambda}^{\lambda_{max}} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda,\mu}\; Y_{\lambda,\mu}(\vartheta,\varphi)]$$

A Basis for Octahedral Symmetry

Only special combinations of spherical harmonics may form a basis for surfaces with octahedral symmetry:

Three Lowest Orders: Rank $\leftrightarrow$ Multipolarity λ

$$\lambda = 4: \quad \alpha_{40} \equiv o_4; \quad \alpha_{4,\pm4} \equiv \pm\sqrt{\tfrac{5}{14}} \cdot o_4$$

$$\lambda = 6: \quad \alpha_{60} \equiv o_6; \quad \alpha_{6,\pm4} \equiv -\sqrt{\tfrac{7}{2}} \cdot o_6$$

$$\lambda = 8: \quad \alpha_{80} \equiv o_8; \quad \alpha_{8,\pm4} \equiv \sqrt{\tfrac{28}{198}} \cdot o_8; \quad \alpha_{8,\pm8} \equiv \sqrt{\tfrac{65}{198}} \cdot o_8$$

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Nuclear Octahedral Shapes - 3D Examples

Illustrations below show the octahedral-symmetric surfaces at three increasing values of rank $\lambda = 4$ deformations o_4: 0.1, 0.2 and 0.3:

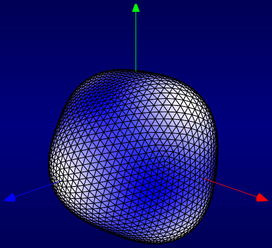


Figure: $o_4 = 0.1$

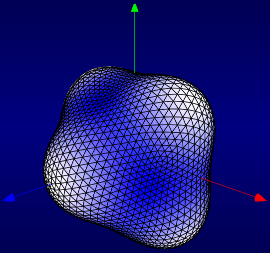


Figure: $o_4 = 0.2$

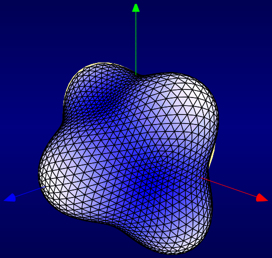
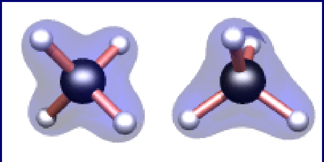


Figure: $o_4 = 0.3$

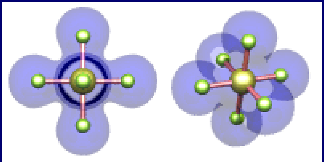
Recall: $\alpha_{40} \equiv o_4; \quad \alpha_{4,\pm 4} \equiv \pm\sqrt{\frac{5}{14}} \cdot o_4$

Among the Highest Symmetries in Molecular Physics

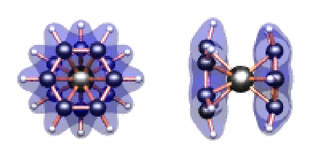
Group T_d - Molecule: $[CH_4]$



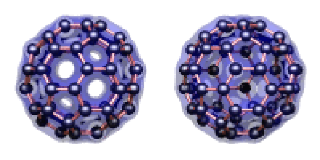
Group O_h - Molecule: $[SF_6]$



Group D_{6d} - Mol.: $[Cr(C_6H_6)_2]$



Group I_h - Molecule: $[C_{60}]$



From J. Goss, University of Newcastle

Nuclear Octahedral Shapes - Neutron Spectra

Double group O_h^D has four 2-dimensional and two 4-dimensional irreducible representations $\longrightarrow$ six distinct families of levels

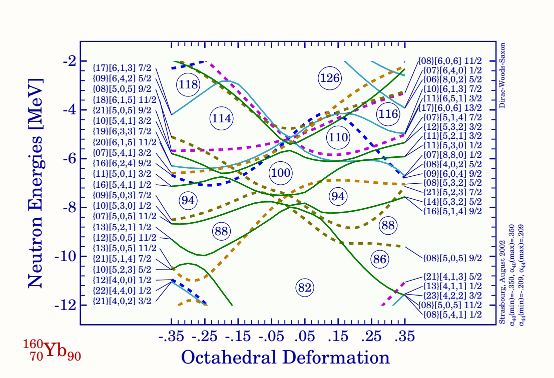


Figure: Full lines correspond to 4-dimensional irreducible representations - they are marked with double Nilsson labels. Observe huge gap at N=114.

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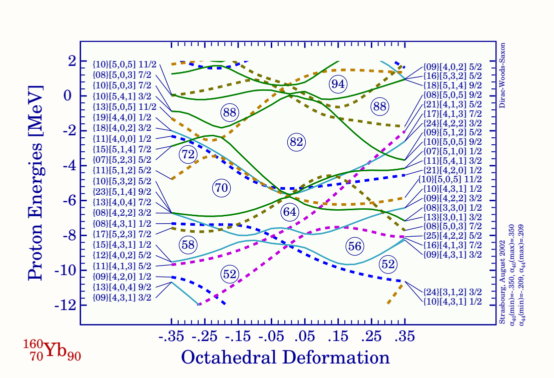
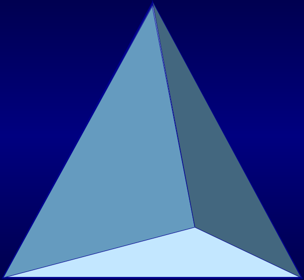


Figure: Full lines correspond to 4-dimensional irreducible representations - they are marked with double Nilsson labels. Observe huge gap at Z=70.

Discrete Symmetries in Nuclei

Let us recall one of the magic forms introduced long time by Plato. The implied symmetry leads to the *tetrahedral group* denoted T_d

A tetrahedron has four equal walls. Its shape is invariant with respect to 24 symmetry elements. Tetrahedron is *not* invariant with respect to the inversion. Of course nuclei cannot be represented by a sharp-edge pyramid



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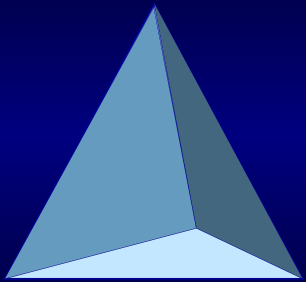

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Tetrahedral Symmetry in Heavy Zr Nuclei

The Table below shows the HFB energies relative to the energy of the tetrahedral minimum. Calculations with SLy4 parametrisation. Energy in MeV.

Nucleus	^{104}Zr	^{106}Zr	^{108}Zr	^{110}Zr	^{112}Zr
Tetrahedral	+0.00	+0.00	+0.00	+0.00	+0.00
Spherical	+0.22	+0.29	+0.39	+0.43	+0.03
Oblate	-1.57	-1.52	-1.10	+0.07	+0.30
Prolate	-2.07	-1.76	-0.68	+0.27	+1.01

Conclusion: In some exotic nuclei the ground-state energies may correspond to the tetrahedral minima

Part III

Tetrahedral Rare Earths - A Test-Ground

Abundance of Tetrahedral Nuclei along Periodic Table

- The tetrahedral/octahedral symmetric nuclei are predicted around magic closures $\{Z_t, N_t\} = \{32, 40, 56, 64, 70, 90, 132-136\}$

- ... and more precisely around the following nuclei:

$$^{64}_{32}\text{Ge}_{32},\ ^{72}_{32}\text{Ge}_{40},\ ^{88}_{32}\text{Ge}_{56},\ ^{80}_{40}\text{Zr}_{40},\ ^{110}_{40}\text{Zr}_{70},\ ^{112}_{56}\text{Ba}_{56},$$

$$^{126}_{56}\text{Ba}_{70},\ ^{146}_{56}\text{Ba}_{90},\ ^{134}_{64}\text{Gd}_{70},\ ^{154}_{64}\text{Gd}_{90},\ ^{160}_{70}\text{Yb}_{90},\ ^{222}_{90}\text{Th}_{132}$$

- The majority of these are either proton-rich or neutron-rich

- An important exception is the God's gift: Rare Earth Region around Gd and Yb mass A∼150-160 nuclei

- Therefore after having examined this 'easier' range the majority of the physics in question corresponds to the realm of Spiral 2

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- The majority of these are either proton-rich or neutron-rich
- An important exception is the God's gift: Rare Earth Region around Gd and Yb mass A∼150-160 nuclei
- Therefore after having examined this 'easier' range the majority of the physics in question corresponds to the realm of Spiral 2

Abundance of Tetrahedral Nuclei along Periodic Table

- The tetrahedral/octahedral symmetric nuclei are predicted around magic closures $\{Z_t, N_t\} = \{32, 40, 56, 64, 70, 90, 132-136\}$
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Tetrahedral vs. Ground-State Configurations

Tetrahedral minima compete with the prolate ground-state minima

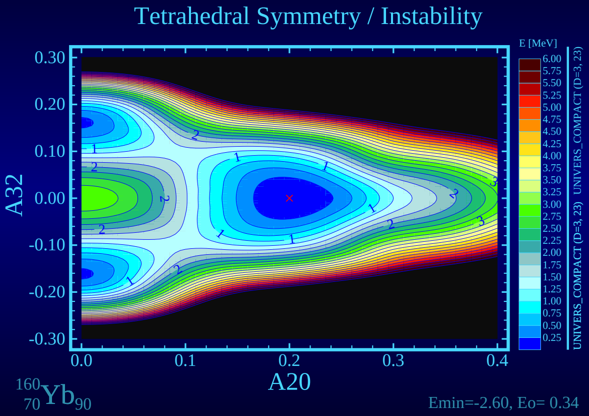

Figure: Barriers between the tetrahedral and quadrupole-deformed minima.

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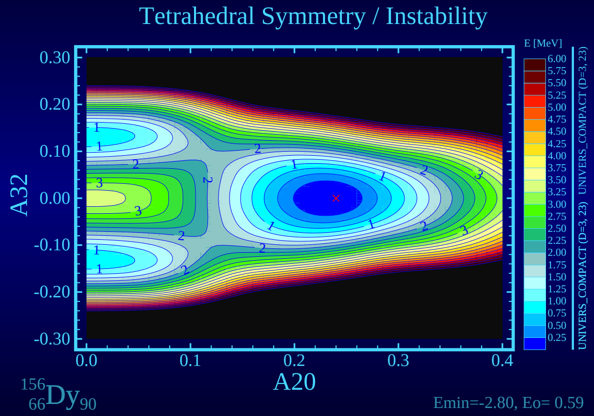

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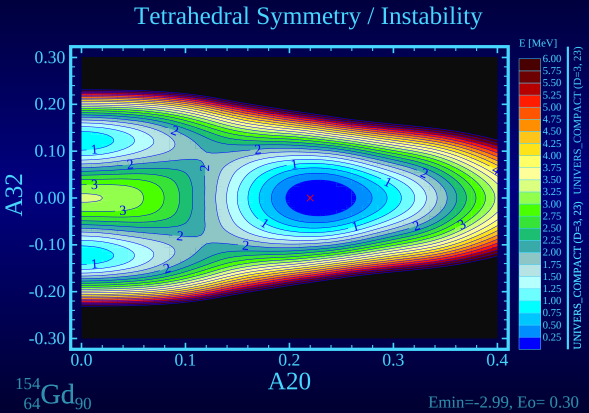

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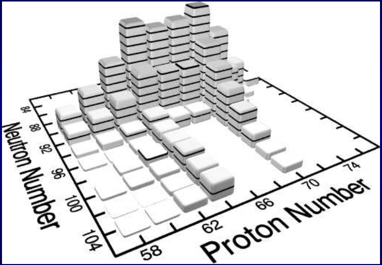
Tetrahedral Shapes in Rare Earth Nuclei - Stability

The chances to observe the new symmetries in experiment increase with the increasing heights of the barriers surrounding these minima



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- Conclusion: *The highest barriers correspond to the Gadolinum and Ytterbium nuclei with N∼90.*

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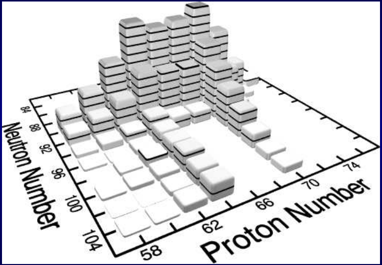

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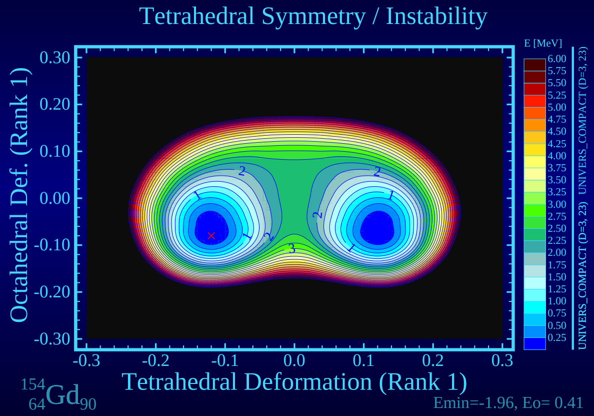

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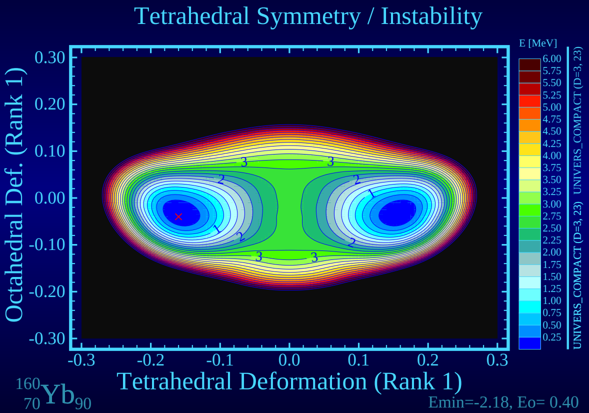

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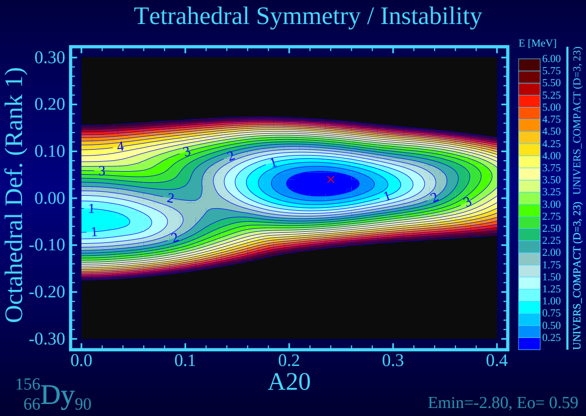

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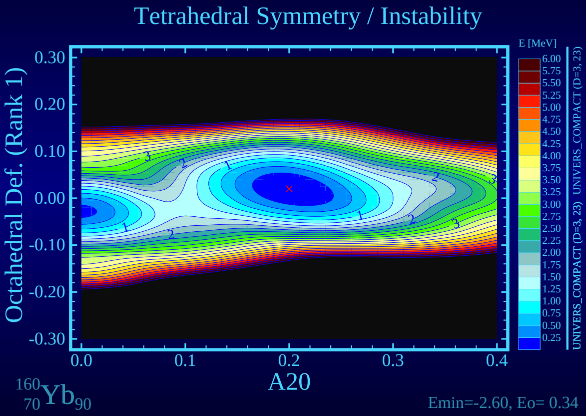

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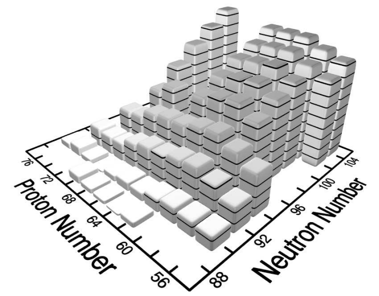
Tetrahedral vs. Normal-Deformed Energy Differences

The chances to observe the new symmetries in experiment increase with the decreasing energy difference: $(E_t - E_{nd})$



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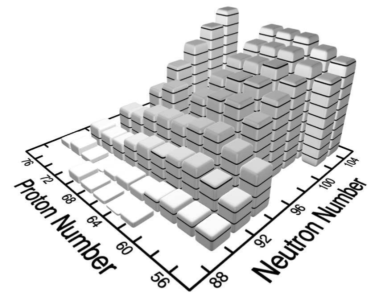

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About Enormous Instability of Spherical Shapes

Let us examine the energy gain in spherical nuclei by allowing them to get tetrahedral and/or octahedral deformed

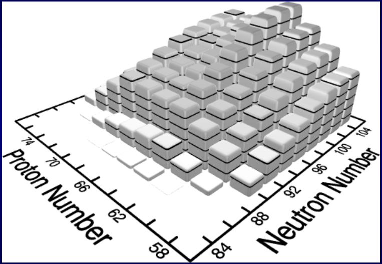

Figure: Energy difference $\Delta E \equiv (E_{sph} - E_t)$ between the spherical and tetrahedral minima. Brick-size 500 keV.

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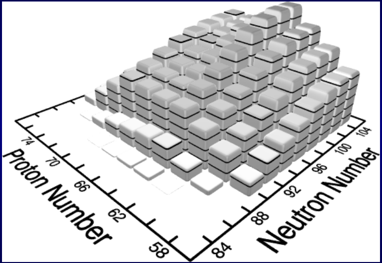

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Tetrahedral/Octahedral Shapes Have No Q_2-Moments

At the exact tetrahedral symmetry the quadrupole moments vanish.

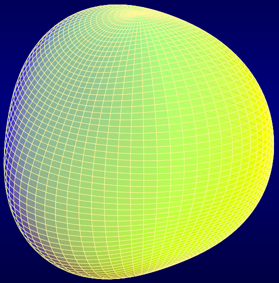
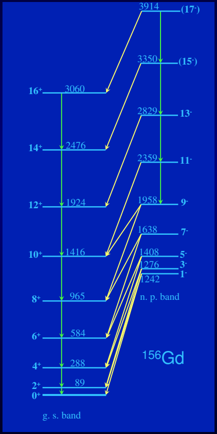


Figure: Equilibrium shape $t_1 = 0.15$.



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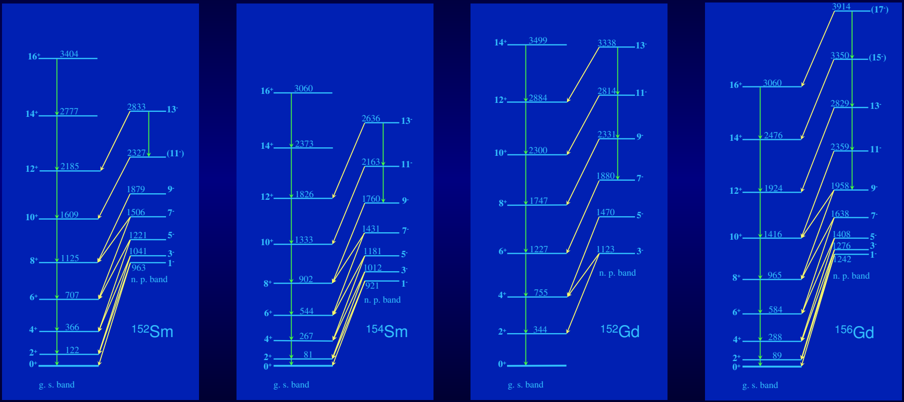

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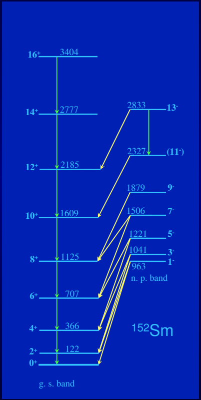

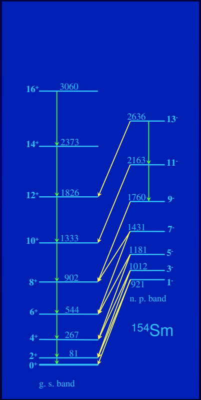

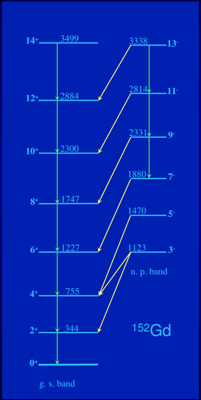

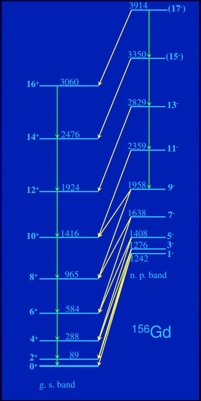

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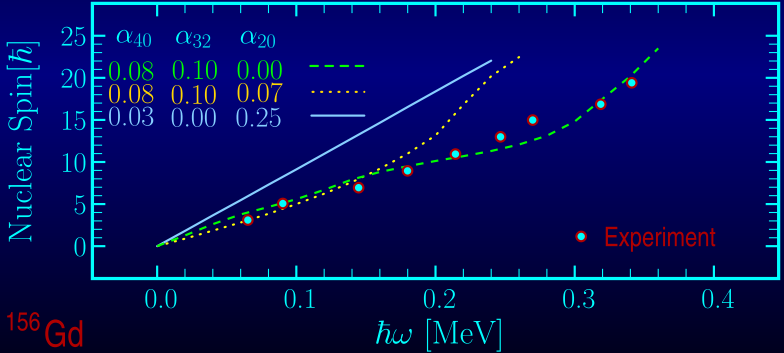
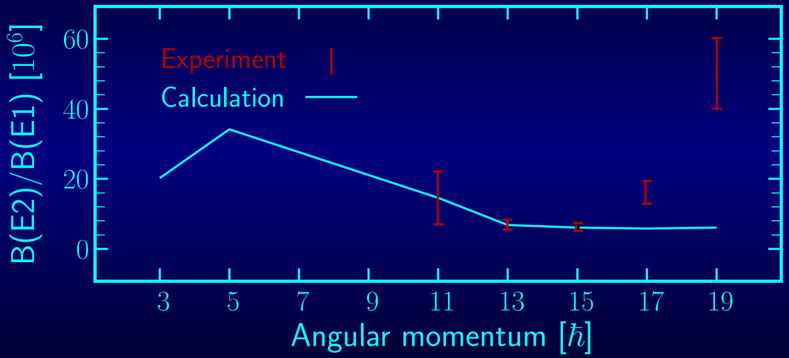
Comparison Theory - Experiment: Alignments

Three hypotheses: 1. Tetrahedral and Octahedral $\leftrightarrow$ *(microscopic);*
2. Tetrahedral and Octahedral + Zero-Point Motion ($\alpha_{20}^{pol.} = 0.07$)*;*
3. Prolate ('standard')



Comparison Theory - Experiment: B(E2)/B(E1)

Comparison between the mean-field predictions and experiment



Possible Further Signs of Tetrahedral Symmetry

Table: Experimental ratios $B(E2)_{in}/B(E1)_{out}$ $\times 10^6$

Spin	^{152}Gd	^{156}Gd	^{154}Dy	^{160}Er	^{164}Er	^{162}Yb	^{164}Yb
19^-	-	50	-	-	-	-	-
17^-	-	16	-	-	-	-	-
15^-	-	6	-	60	24	-	-
13^-	14	7	15	18	23	-	17
11^-	4	15	5	9	0	10	11
9^-	4	0	-	0	-	11	10
7^-	0	0	0	-	-	0	0

Above: Branching ratios related to the negative parity bands interpreted as tetrahedral, inter-band transitions to g.s.band

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Conclusion: The suspected bands in Rare Earth nuclei behave very differently as compared e.g. to 'classical' octupole ^{222}Th band!

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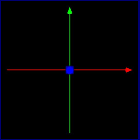
Part IV

Spatial Representation of Nuclear Shells

Spatial Structure of Orbitals (Spherical ^{132}Sn) ($|\psi(\vec{r})|^2$)

Limit 80%



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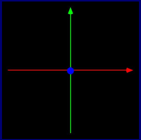

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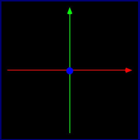

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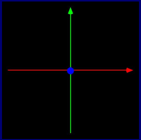

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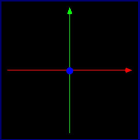

Density distribution $|\psi_\pi(\vec{r})|^2 \geq$ Limit, for $\pi = [2,0,2]1/2$ orbital

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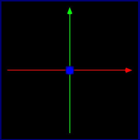
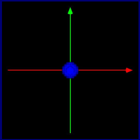


Limit 50%



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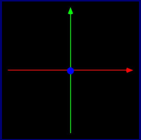

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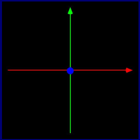

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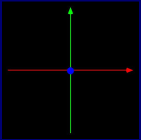

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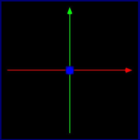

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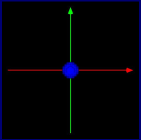

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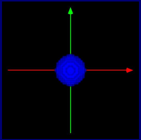

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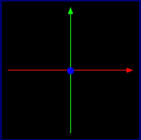

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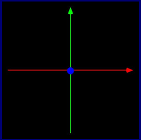

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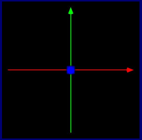

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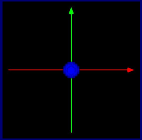

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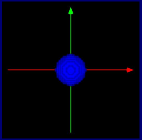
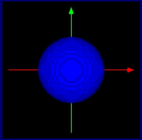
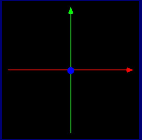


Limit 3%



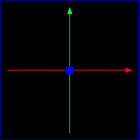
Limit ??%



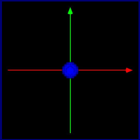
Density distribution $|\psi_\pi(\vec{r})|^2 \geq$ Limit, for $\pi = [2,0,2]1/2$ orbital

Spatial Structure of Orbitals (Spherical ^{132}Sn) ($|\psi(\vec{r})|^2$)

Limit 80%



Limit 50%



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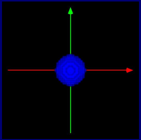
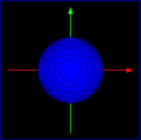
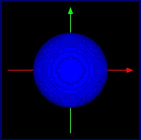


Limit 3%



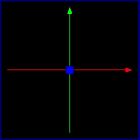
Limit 1%



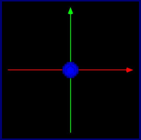
Density distribution $|\psi_\pi(\vec{r})|^2 \geq$ Limit, for $\pi = [2, 0, 2]1/2$ orbital

Spatial Structure of Orbitals (Spherical ^{132}Sn) ($|\psi(\vec{r})|^2$)

Limit 80%



Limit 50%



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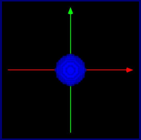

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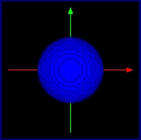

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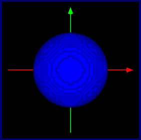

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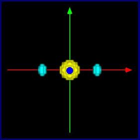

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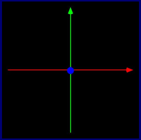

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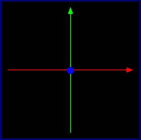

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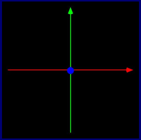
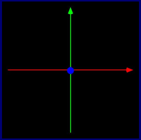


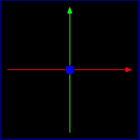
Limit ??%



Bottom: N=3 shell b-[303]7/2, w-[312]5/2, y-[321]3/2, p-[310]1/2

Spatial Structure of Orbitals (Spherical ^{132}Sn) ($|\psi(\vec{r})|^2$)

Limit 80%



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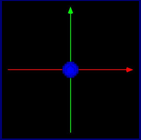

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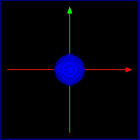

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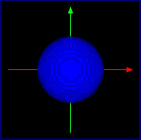

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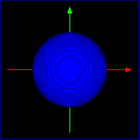

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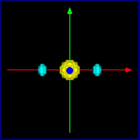
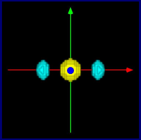


Limit 15%



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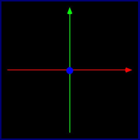

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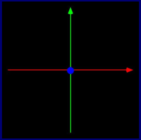

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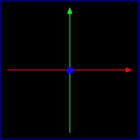
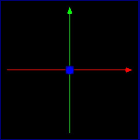


Bottom: N=3 shell b-[303]7/2, w-[312]5/2, y-[321]3/2, p-[310]1/2

Spatial Structure of Orbitals (Spherical ^{132}Sn) ($|\psi(\vec{r})|^2$)

Limit 80%



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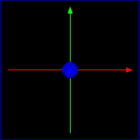

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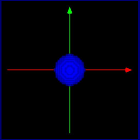

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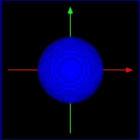

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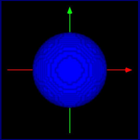

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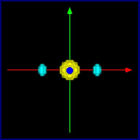

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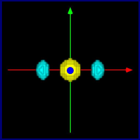

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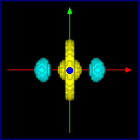

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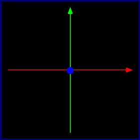

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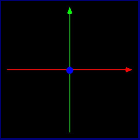
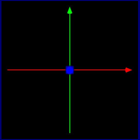


Bottom: N=3 shell b-[303]7/2, w-[312]5/2, y-[321]3/2, p-[310]1/2

Spatial Structure of Orbitals (Spherical ^{132}Sn) ($|\psi(\vec{r})|^2$)

Limit 80%



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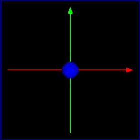

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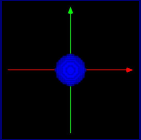

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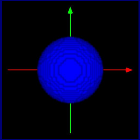

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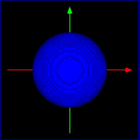

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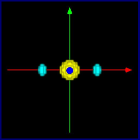

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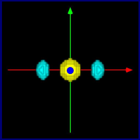

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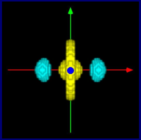

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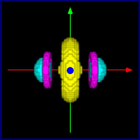

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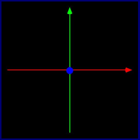
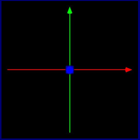


Bottom: N=3 shell b-[303]7/2, w-[312]5/2, y-[321]3/2, p-[310]1/2

Spatial Structure of Orbitals (Spherical ^{132}Sn) ($|\psi(\vec{r})|^2$)

Limit 80%



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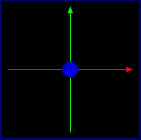

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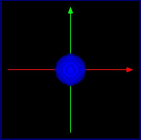

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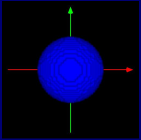

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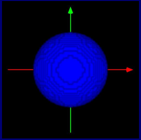

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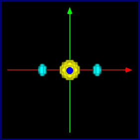

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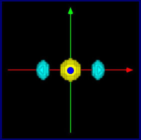

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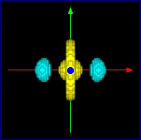

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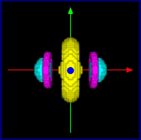

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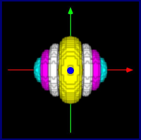
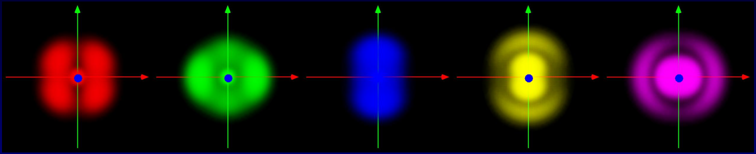
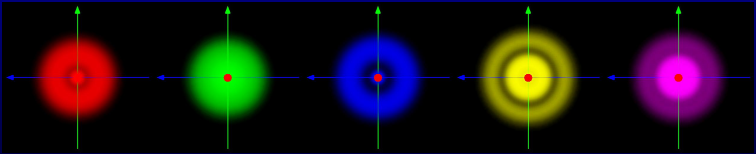


Bottom: N=3 shell b-[303]7/2, w-[312]5/2, y-[321]3/2, p-[310]1/2

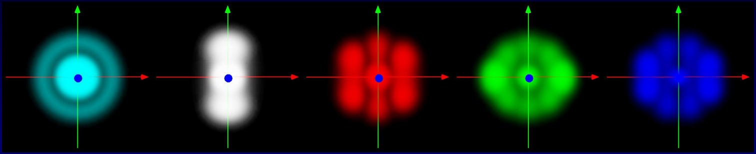
Spatial Structure of N=3 Spherical Shell ($|\psi_\nu(\vec{r})|^2$)

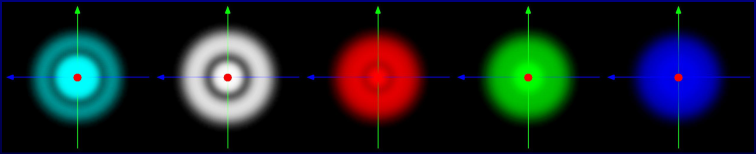




^{132}Sn: Distributions $|\psi_\nu(\vec{r})|^2$ for single proton orbitals. Top $\mathcal{O}_{xz}$, bottom $\mathcal{O}_{yz}$. Proton $e_\nu \leftrightarrow [\nu{=}30, 32, \ldots 38]$ for spherical shell

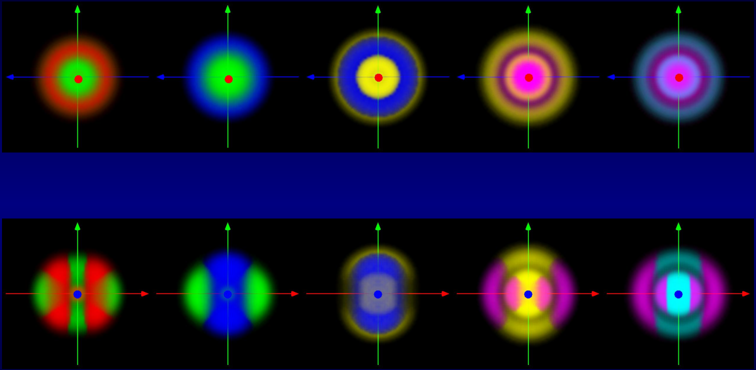
Spatial Structure of N=3 Spherical Shell ($|\psi_\nu(\vec{r})|^2$)





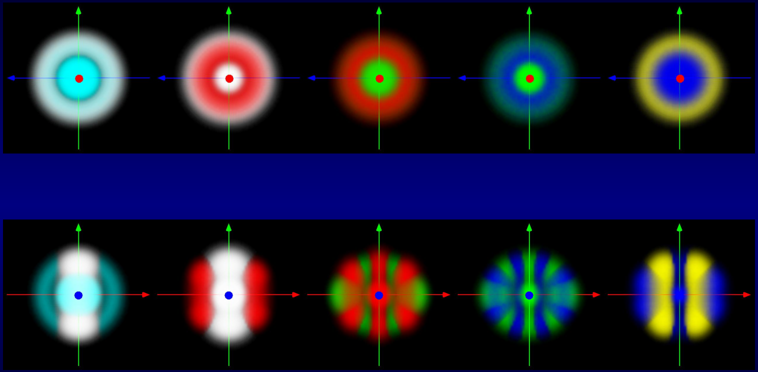
^{132}Sn: Distributions $|\psi_\nu(\vec{r})|^2$ for single proton orbitals. Top $\mathcal{O}_{xz}$, bottom $\mathcal{O}_{yz}$. Proton $e_\nu \leftrightarrow [\nu{=}40, 42, \dots 48]$ for spherical shell

Spatial Structure of N=3 Spherical Shell ($|\psi_\nu(\vec{r})|^2$)



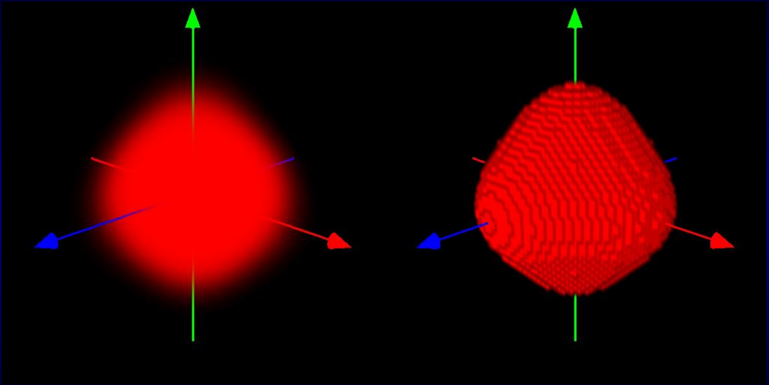
^{132}Sn: distributions $|\psi_\nu(\vec{r})|^2$ for consecutive pairs of orbitals. Top $\mathcal{O}_{xz}$, bottom $\mathcal{O}_{yz}$. Proton $e_\nu \leftrightarrow$ [n=30:32, ... 38:40], spherical shell

Spatial Structure of N=3 Spherical Shell ($|\psi_\nu(\vec{r})|^2$)



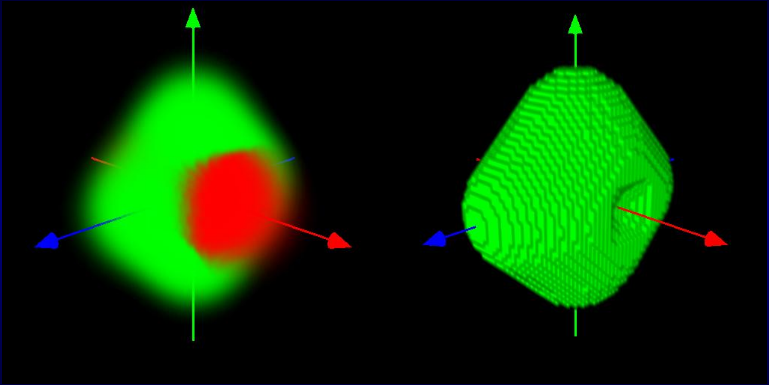
^{132}Sn: distributions $|\psi_\nu(\vec{r})|^2$ for consecutive pairs of orbitals. Top $\mathcal{O}_{xz}$, bottom $\mathcal{O}_{yz}$. Proton $e_\nu \leftrightarrow [n{=}40{:}42, \dots\ 48{:}50]$, spherical shell

The First Octahedral Shell (20 Nucleons)) ($|\psi(\vec{r})|^2$)



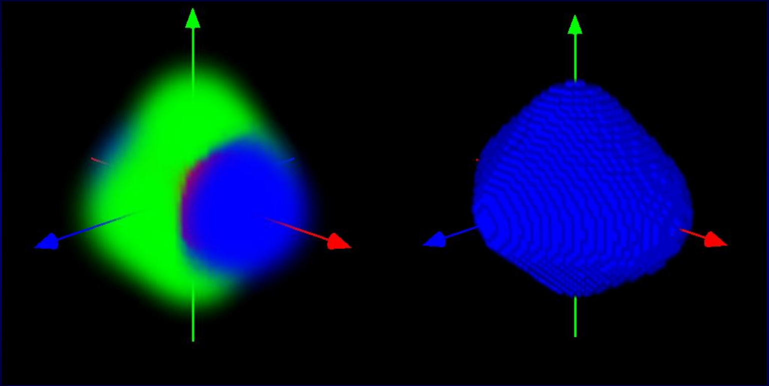
Left: accumulating image of all orbitals; Right: Single Orbital (No.1)

The First Octahedral Shell (20 Nucleons)) ($|\psi(\vec{r})|^2$)



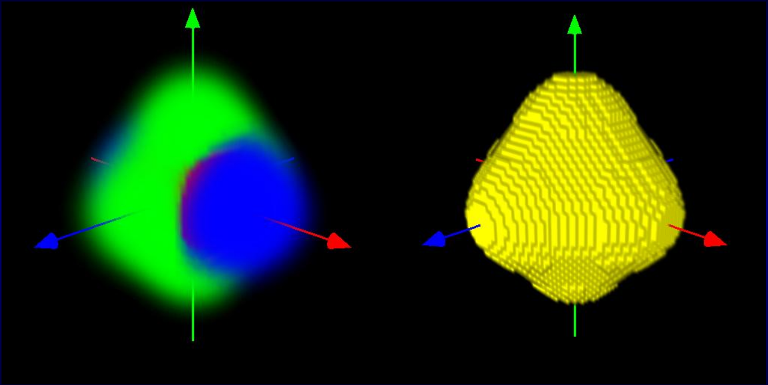
Left: accumulating image of all orbitals; Right: Single Orbital (No.2)

The First Octahedral Shell (20 Nucleons)) ($|\psi(\vec{r})|^2$)



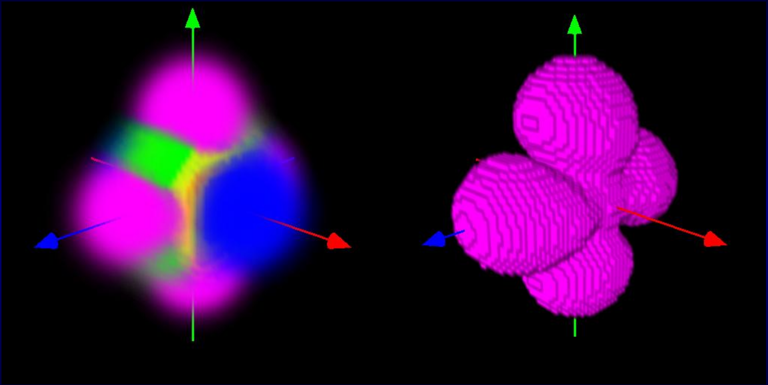
Left: accumulating image of all orbitals; Right: Single Orbital (No.3)

The First Octahedral Shell (20 Nucleons)) ($|\psi(\vec{r})|^2$)



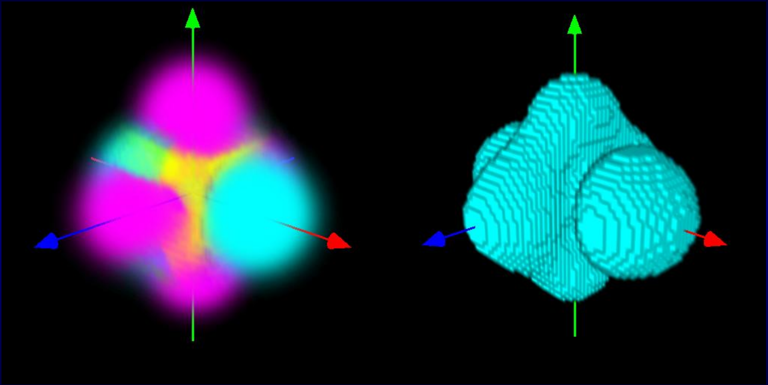
Left: accumulating image of all orbitals; Right: Single Orbital (No.4)

The First Octahedral Shell (20 Nucleons)) ($|\psi(\vec{r})|^2$)



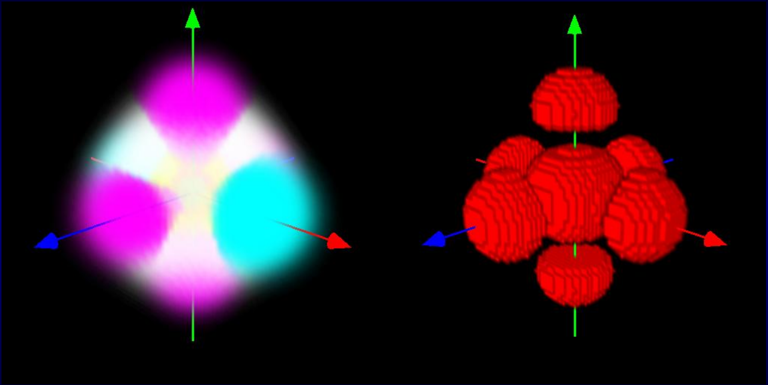
Left: accumulating image of all orbitals; Right: Single Orbital (No.5)

The First Octahedral Shell (20 Nucleons)) ($|\psi(\vec{r})|^2$)



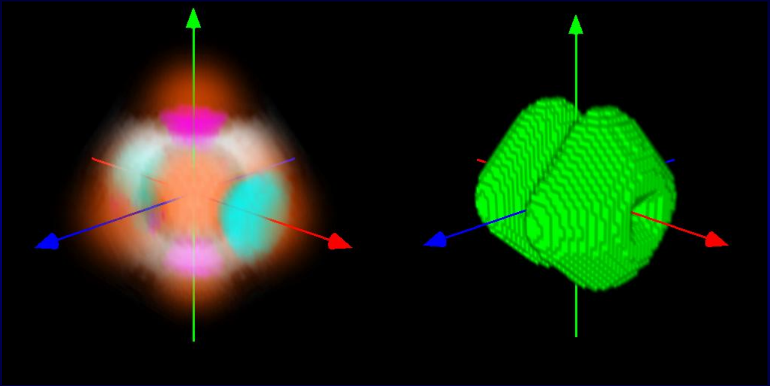
Left: accumulating image of all orbitals; Right: Single Orbital (No.6)

The First Octahedral Shell (20 Nucleons)) ($|\psi(\vec{r})|^2$)



Left: accumulating image of all orbitals; Right: Single Orbital (No.7)

The First Octahedral Shell (20 Nucleons)) ($|\psi(\vec{r})|^2$)



Left: accumulating image of all orbitals; Right: Single Orbital (No.8)

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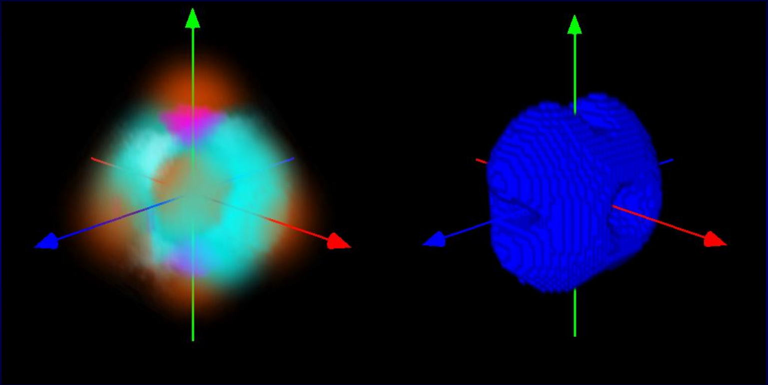

Left: accumulating image of all orbitals; Right: Single Orbital (No.9)

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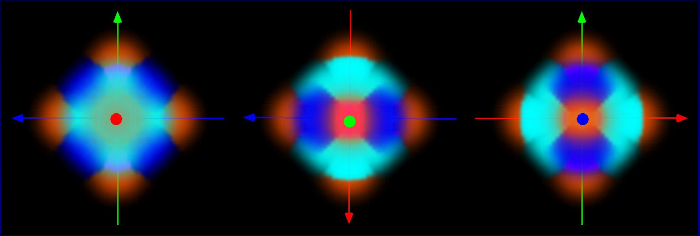

Three space perspectives of the full octahedral shell (n=20 nucleons)

Part V

Nuclear Quantum Rotor

Quantum Systems and Rotation: Preliminaries [1]

Microscopic 'True' H –> All Solutions

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Alternative:

Mean Field H –> Individual Nucleons

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Alternative:

Collective Rotor H –> Rotational Bands

Quantum Systems and Rotation: Preliminaries [2]

Collective Rotor Hamiltonian

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Collective Rotor Hamiltonian

↓

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Quantum Systems and Rotation: Preliminaries [2]

Collective Rotor Hamiltonian

$$H_{quant} = H(\{ T_{\lambda\mu}(I_+, I_-, I_0) \})$$

$$H_{class} = 1/2 \sum_j \sum_k B_{jk}(\{x,p\})\, \dot{\alpha}_j\, \dot{\alpha}_k + V(\{\alpha;x,p\})$$

Mean Field and Implied Rotor Symmetries

- Suppose a system manifests a spontaneous symmetry breaking
- We find it convenient to describe it with the help of $H_{mf}(\{6A\})$
- We wish to stress rotational degrees of freedom by introducing

$$\boxed{H_{eff}(\{6A\}) = H_{rot} + H_{mf}(\{6A\})}$$

- Thus the symmetries of $H_{eff}(\{6A\})$ and of $H_{mf}(\{6A\})$ coincide
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What Must - and What Must Not be Attempted

The only direct observables are energy and angular momentum

Refusing:

- No use of the classical concepts such as 'inertia' or 'rigid body'
- No use of derived concepts as VMI or Harris parametrisations

Accepting:

- Nuclear modelling focuses on the mechanism under interest

  - To study individual-nucleonic features → Basis of $\hat{x}_k$ & $\hat{p}_k$
  - For collective rotation → Basis of $\{\hat{I}_+, \hat{I}_-, \hat{I}_0\}$ & $\{\alpha, \beta, \gamma\}$

$$\text{Basis } \{\hat{I}_+, \hat{I}_-, \hat{I}_0\} : \hat{T}_{\lambda\mu}(n) \stackrel{df.}{=} \underbrace{[(\hat{I} \otimes \hat{I}) \otimes \ \ldots \otimes \hat{I}]_{\lambda\mu}}_{n \ \ factors}$$

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Nuclear Rotor: Collective vs. Intrinsic Hamiltonians

- One of the most successful models of nuclear structure is the rotor model based on the Hamiltonian

$$\hat{\mathrm{H}}^{[nucl.]} = \hat{\mathrm{H}}^{[rot.]} + \hat{\mathrm{H}}^{[intr.]} + \hat{\mathrm{H}}^{[inter.]}$$

- Neglecting the interaction term as an approximation leads to

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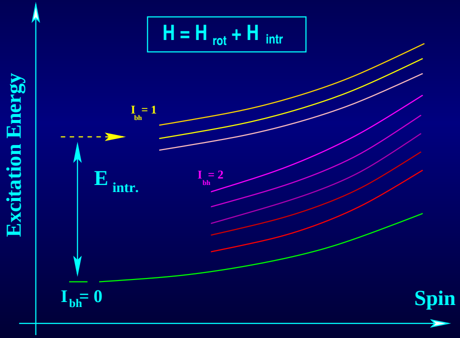
- *To compare the rotor spectra with experiment, we must shift every band upwards, introducing the band-head energies*



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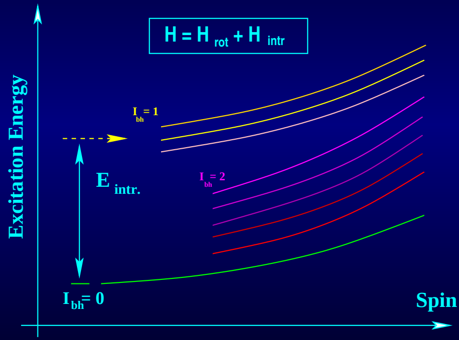

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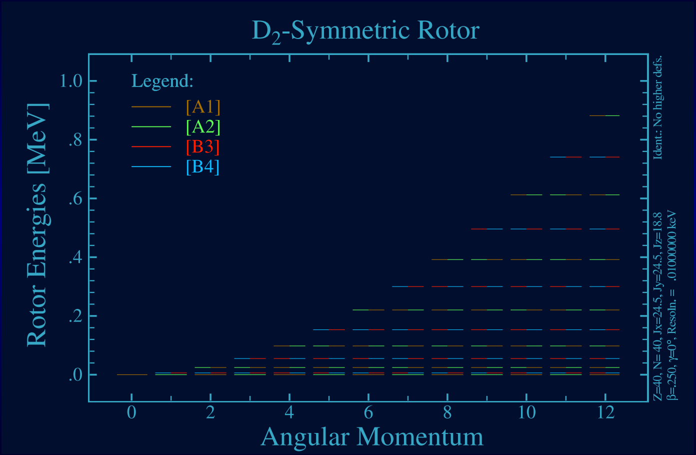
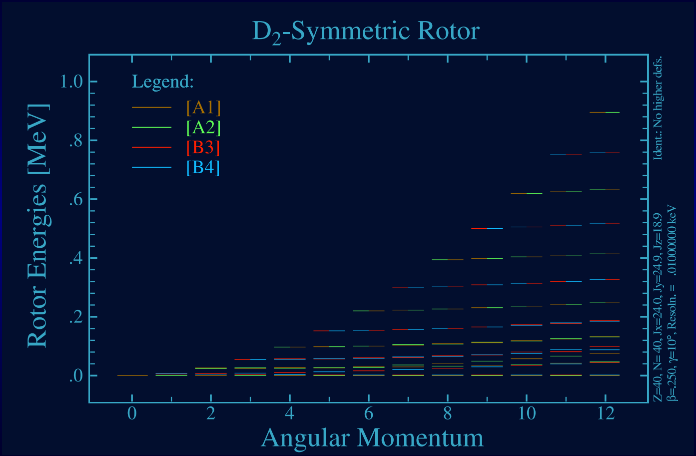
Ellipsoidal Rotor: Energy Spectrum at $\gamma = 0^o$



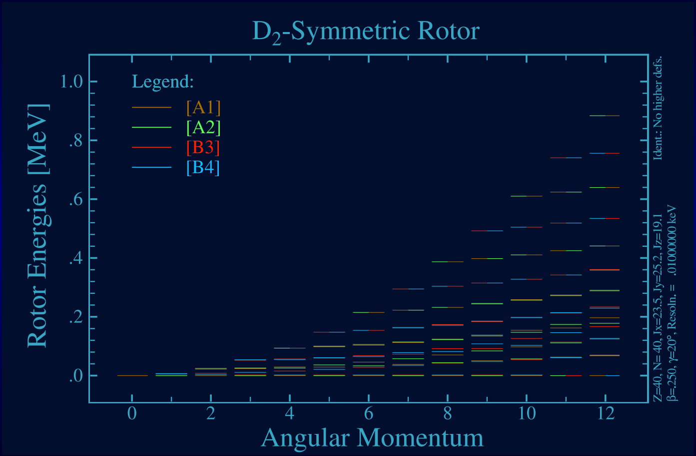
To facilitate reading, the spectrum is normalised to the yrast line

Ellipsoidal Rotor: Energy Spectrum at $\gamma = 10^o$



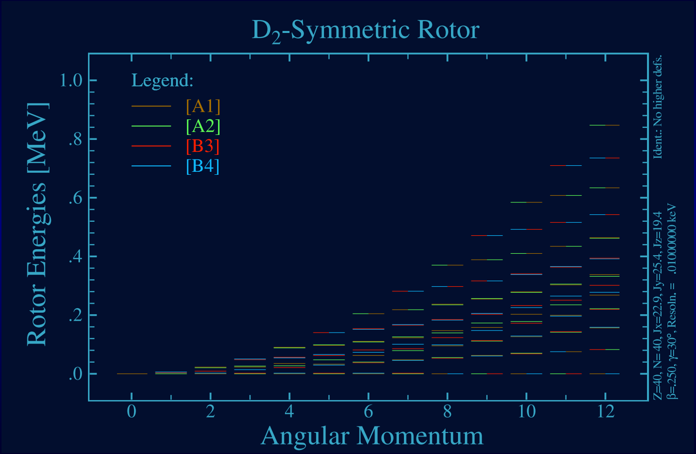
To facilitate reading, the spectrum is normalised to the yrast line

Ellipsoidal Rotor: Energy Spectrum at $\gamma = 20^o$



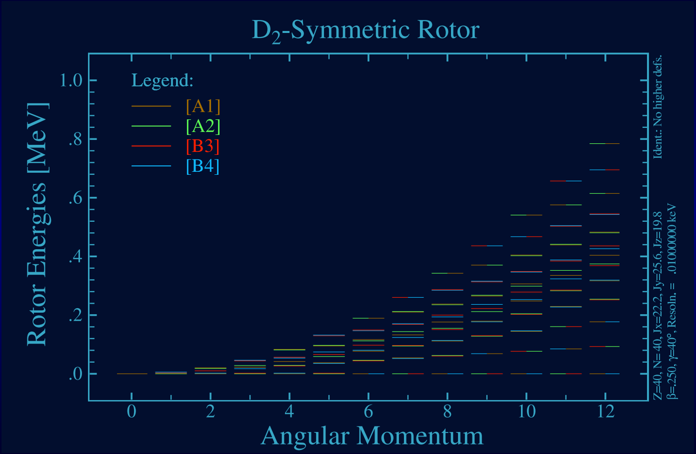
To facilitate reading, the spectrum is normalised to the yrast line

Ellipsoidal Rotor: Energy Spectrum at $\gamma = 30^o$



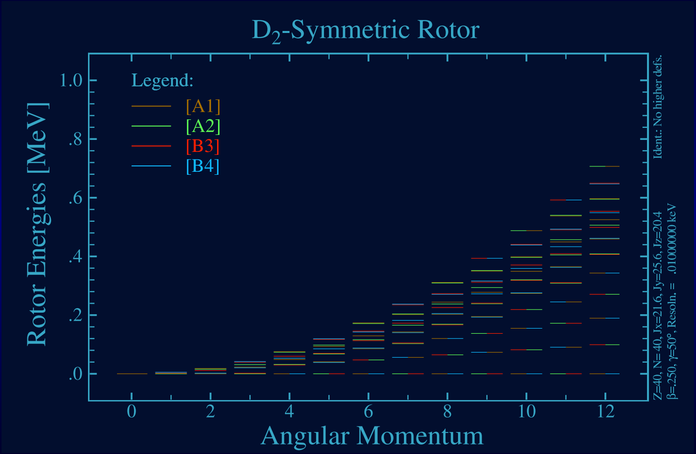
To facilitate reading, the spectrum is normalised to the yrast line

Ellipsoidal Rotor: Energy Spectrum at $\gamma = 40^o$



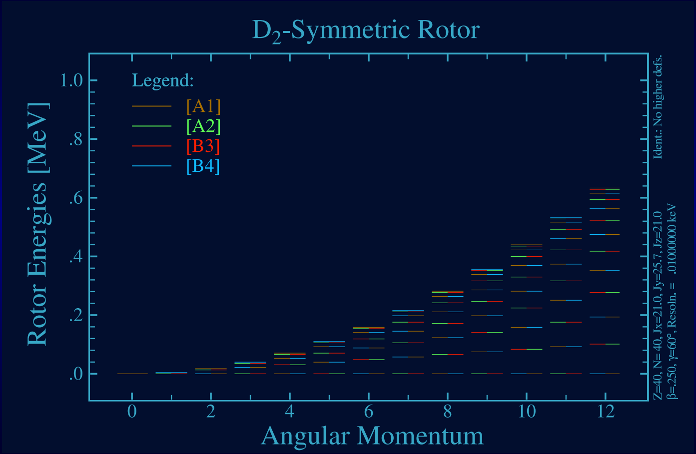
To facilitate reading, the spectrum is normalised to the yrast line

Ellipsoidal Rotor: Energy Spectrum at $\gamma = 50^o$



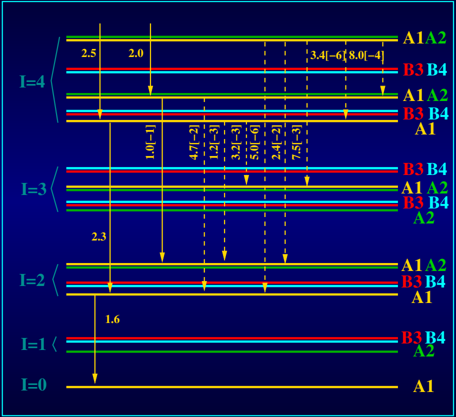
To facilitate reading, the spectrum is normalised to the yrast line

Ellipsoidal Rotor: Energy Spectrum at $\gamma = 60^o$



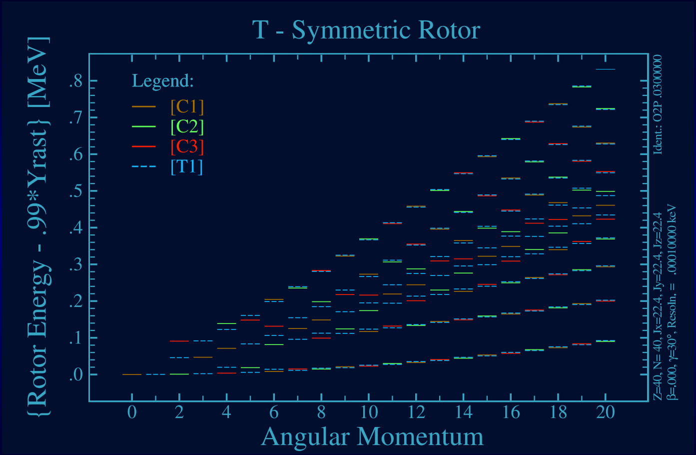
To facilitate reading, the spectrum is normalised to the yrast line

Stretched Qudrupole Transitions in the D_2 Rotor



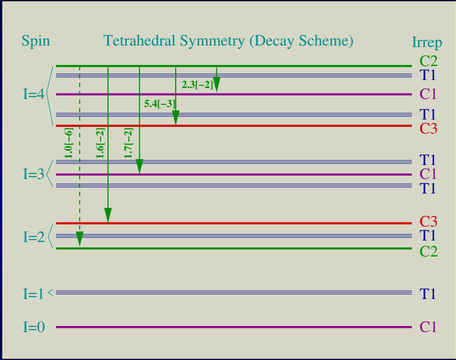
Observe the domination of $\Delta I = 2$ stretched E2→g.s. transitions

Reduced Spectrum of the Tetrahedral-Symmetric Rotor



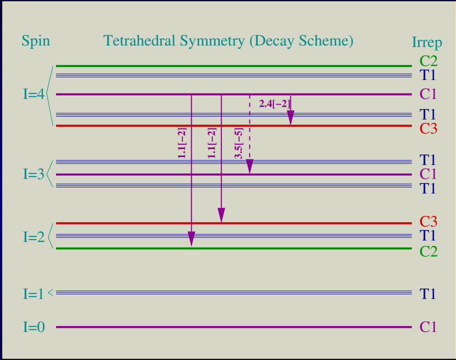
Spectrum is normalised to 99 % of the yrast line

Electro-Magnetic E2 Transitions from $I_{[9]}[C2]=4$ State



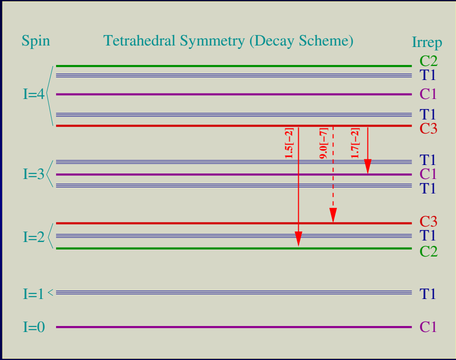
Stretched and non-stretched E2 transitions: observe retardation of the C2→C2 transitions

Electro-Magnetic E2 Transitions from $I_{[5]}$[C2] State



Stretched and non-stretched E2 transitions: observed retarded C1→C1 transitions

Electro-Magnetic E2 Transitions from $I_{[1]}$[C2] State



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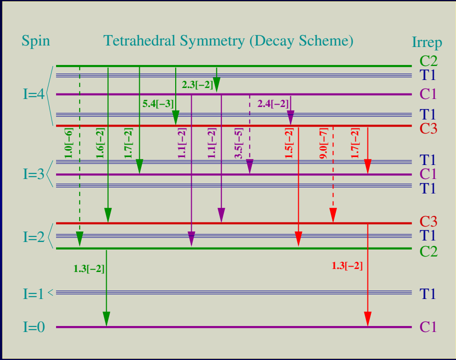
Electro-Magnetic E2 Transitions - Comparison



Observe the *__domination__* *of Ci⟶C(k≠i) transitions and* *__retardation__* *of the Ci⟶Ci type transitions*

Summarising the T-Group Selection Rules

- We expect a competition between the minima of tetrahedral and quadrupole-deformed (prolate and/or oblate) shapes
- The decay patterns of the quadrupole bands are very neat; stretched E2-transitions dominate $\leftrightarrow$ a distinct feature
- The tetrahedral minima are expected to be 'contaminated' with quadrupole deformation. Contamination with $\beta \sim 0.03$ implies already a significant E1-E2 competition
- There is a degeneracy pattern different in the D_2 and T-symmetry cases: tetrahedral symmetry implies three-fold degeneracies

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E2 Selection - Detailed Predictions [2]

- E2-transitions connect C1, C2 and C3 symmetry states among themselves - and T1-symmetry states among themselves!
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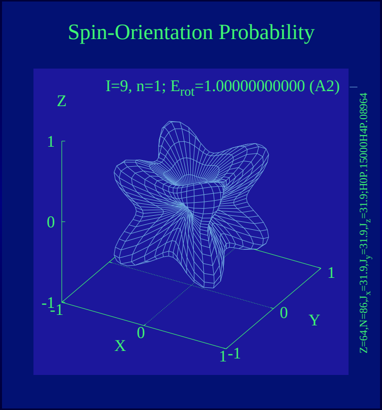
Statistical Wobbling in Space: O_h-Case



Spin-Orientation Probability I=9, n=1, Representation A2

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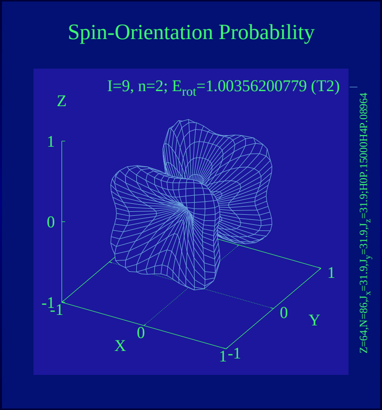
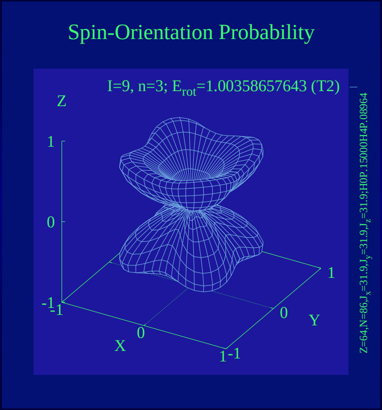


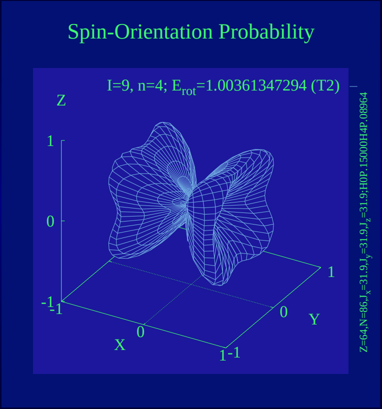
Spin-Orientation Probability I=9, n=2, T2 (3-fold degenerate)

Statistical Wobbling in Space: O_h-Case



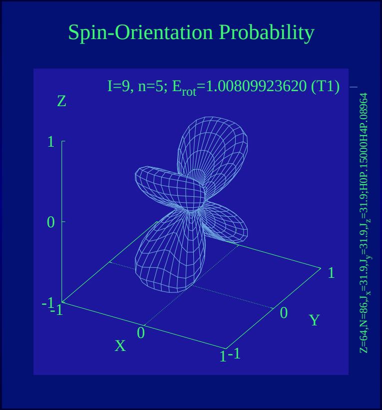
Spin-Orientation Probability I=9, n=3, T2 (3-fold degenerate)

Statistical Wobbling in Space: O_h-Case



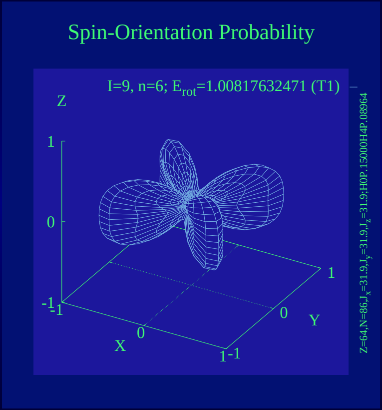
Spin-Orientation Probability I=9, n=4, T2 (3-fold degenerate)

Statistical Wobbling in Space: O_h-Case



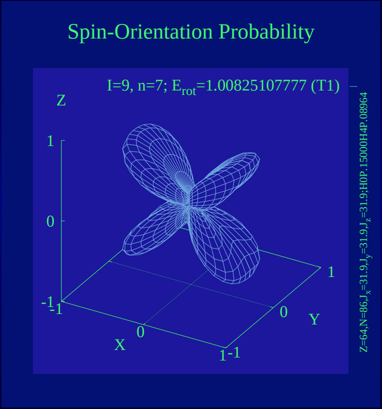
Spin-Orientation Probability I=9, n=5, T1 (3-fold degenerate)

Statistical Wobbling in Space: O_h-Case



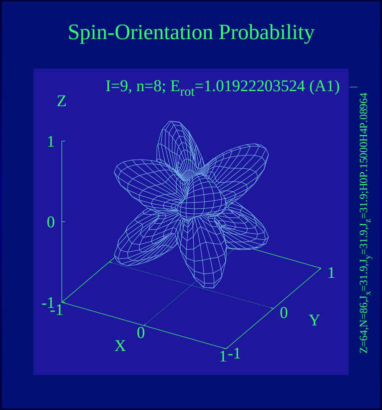
Spin-Orientation Probability I=9, n=6, T1 (3-fold degenerate)

Statistical Wobbling in Space: O_h-Case



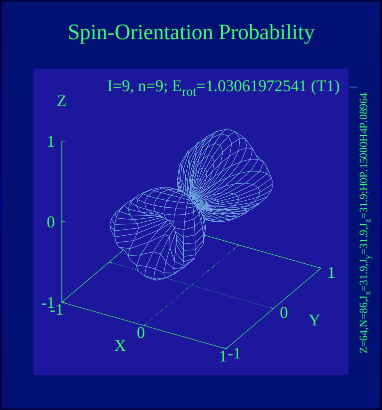
Spin-Orientation Probability I=9, n=7, T1 (3-fold degenerate)

Statistical Wobbling in Space: O_h-Case



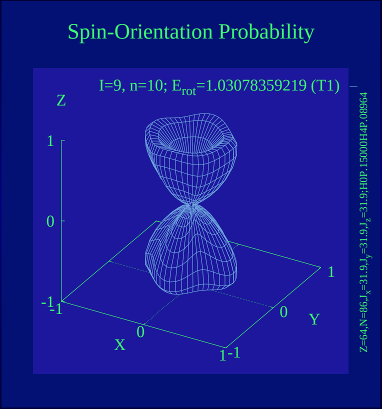
Spin-Orientation Probability I=9, n=8, A1 (1-fold degenerate)

Statistical Wobbling in Space: O_h-Case



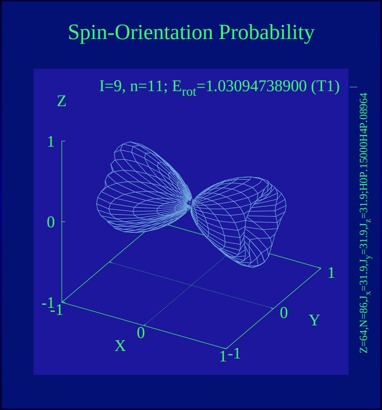
Spin-Orientation Probability I=9, n=9, T1 (3-fold degenerate)

Statistical Wobbling in Space: O_h-Case



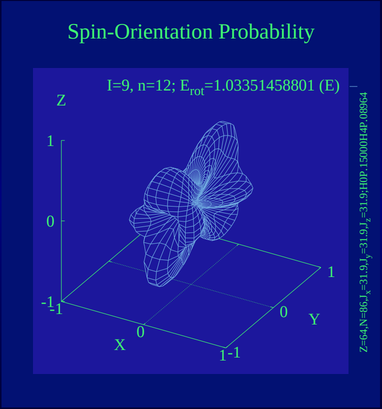
Spin-Orientation Probability I=9, n=10, T1 (3-fold degenerate)

Statistical Wobbling in Space: O_h-Case



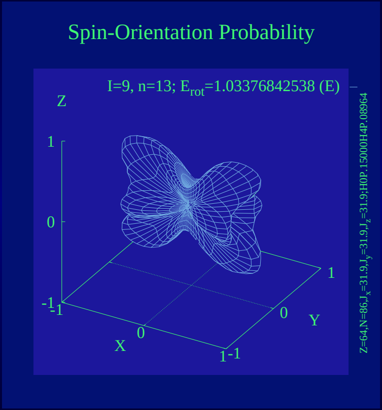
Spin-Orientation Probability I=9, n=11, T1 (3-fold degenerate)

Statistical Wobbling in Space: O_h-Case



Spin-Orientation Probability I=9, n=12, E (2-fold degenerate)

Statistical Wobbling in Space: O_h-Case



Spin-Orientation Probability I=9, n=13, E (2-fold degenerate)

Part VI

Summary & Perspectives

Summary

Search for the high-rank symmetries in nuclei presents a genuinely new physics challenge in nuclear structure domain for large facilities

1. Tetrahedral symmetry in nuclei is predicted as an abundant phenomenon in numerous islands throughout the Periodic Table
2. Most of the nuclei in question are either neutron rich or proton rich and thus of interest for exotic-nuclei projects
3. Paradoxally, the high-level symmetries have most likely been seen already with 'old' non-existing facilities - not realising it!
4. The tetrahedral Rare-Earth nuclei are about the only non-exotic nuclei that can be studied with relatively modest facilities
5. The latter can be seen as a cadeau du ciel allowing to learn the new physics "inexpensively"

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A few points about the TETRANUC collaboration:

Goal: Demonstrate the existence and study the structure of tetrahedral nuclei at Ganil with Spiral 1 and 2

Laboratories today: Strasbourg, Ganil, Orsay, GSI-Darmstad, Cracow, Legnaro, Madrid, Surrey and Warsaw
New members are very welcome!

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Most Urgent: First Proposals to Study Branching Ratios

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The LOGO (!) of the new collaboration suggests our principal interest:

Find for the first time the experimental evidence of the tetrahedral symmetry in subatomic physics.

First proposals: To study the suspect tetrahedral band in ^{156}Gd. Using known transition energies aim at as precise as possible B(E2)/B(E1) ratios, also Q_2.

Figure: Logo by N. Dubray

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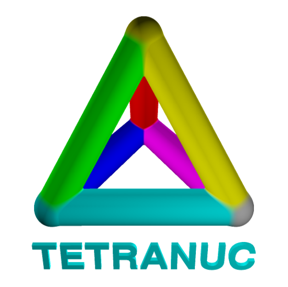

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1. *From the basic physics point of view the tetrahedral symmetry in sub-atomic physics seems comparably or more fundamental and/or interesting than e.g. super-deformation*

2. *Measurements will involve:*
   a. Life-times (to establish weak quadrupole moments);
   b. Electromagnetic transition ratios e.g. B(E2)/B(E1) ...

3. *... and nuclei placed around the tetrahedral magicity centres:*

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