

A many-particles plus rotor approach to magnetic rotation

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Structure of this talk

Overview of the model

Hamiltonian, Basis states etc.

Application 1, Magnetic rotation

Tilted axis cranking model description:

proton and neutron spin vectors align around static tilt angle.

Is the same description obtained when spins are coupled properly ?

Application 2, Wobbling motion

Wobbling bands only observed in odd-even Lu isotopes ?

What can be expected in neighboring nuclei ?

Model I

System divided into
macroscopic rotor + microscopic valence particles

New features

- large particle space
- parameters from mean field models

Model II

Particle-rotor Hamiltonian without pairing:

$$H^{PR} = \sum_i \frac{R_i^2}{2\mathcal{J}_i} + h_{s.p.} = \sum_i \frac{(I_i - j_i)^2}{2\mathcal{J}_i} + h_{s.p.}$$

Nilsson potential:

$$h_{s.p.}(\varepsilon_2, \gamma, \varepsilon_4) = \sum_i e_i a_i^\dagger a_i$$

Strong coupling wave functions chosen as basis states

$$\begin{aligned} |\Psi_{MK\alpha}^I\rangle &= N_{\alpha K} \frac{1}{\sqrt{2}} (1 + e^{-ij2\pi} e^{iI2\pi}) \sqrt{\frac{2I+1}{8\pi^2}} |IMK\rangle |\alpha\rangle \\ &= N_{\alpha K} \sqrt{\frac{2I+1}{16\pi^2}} (|IMK\rangle |\alpha\rangle + (-1)^{I-K} |IM - K\rangle |\tilde{\alpha}\rangle) \end{aligned}$$

A basis of Slater determinants is used for the valence particles

$$|\alpha\rangle = \left(\prod_{i=1}^{Z_1} a_{\beta_i}^\dagger \right) \left(\prod_{i=1}^{N_1} a_{\gamma_i}^\dagger \right) |0\rangle$$

Model III

PAC calculation for the rotor part

$$H^{PAC} = h_{s.p.}(\varepsilon_2, \gamma, \varepsilon_4) - \omega j_x$$

gives:

energy $E(I_x)$, quadrupole moments Q_{20}, Q_{22} and effective gyromagnetic moment g_R

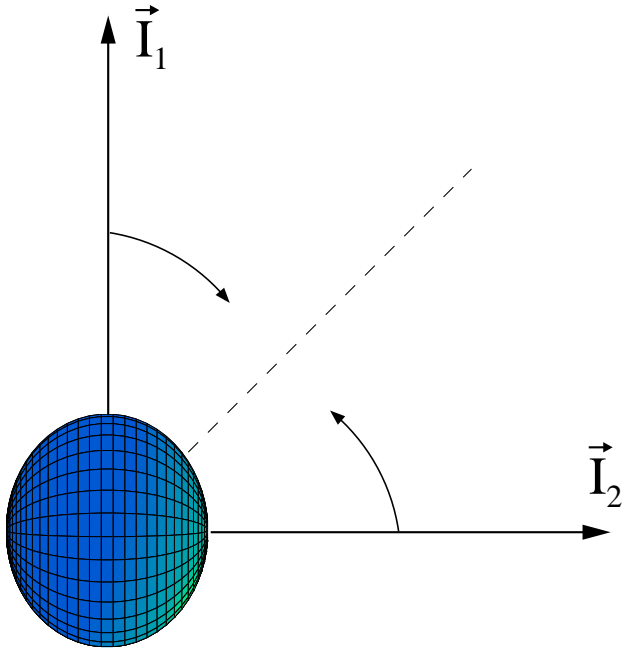
$$g_R(I_x) = [g_{l,\pi} \langle j_{\pi,x} \rangle + (g_{s,\pi} - g_{l,\pi}) \langle S_{\pi,x} \rangle + g_{s,v} \langle S_{v,x} \rangle] / I_x$$

$$Q_{20}(I_x) = \sum_i^Z q_{20,i}$$

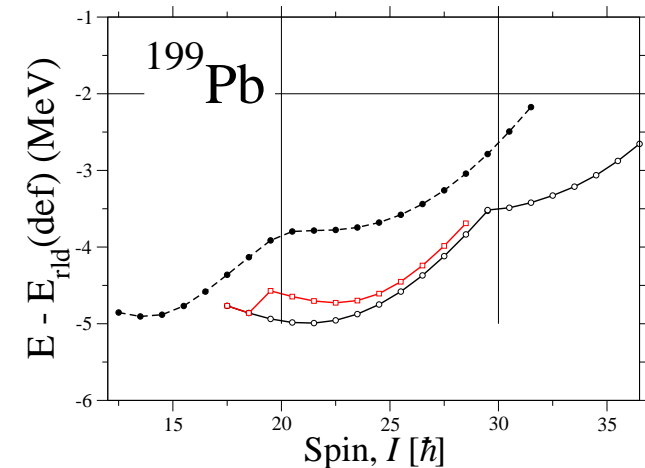
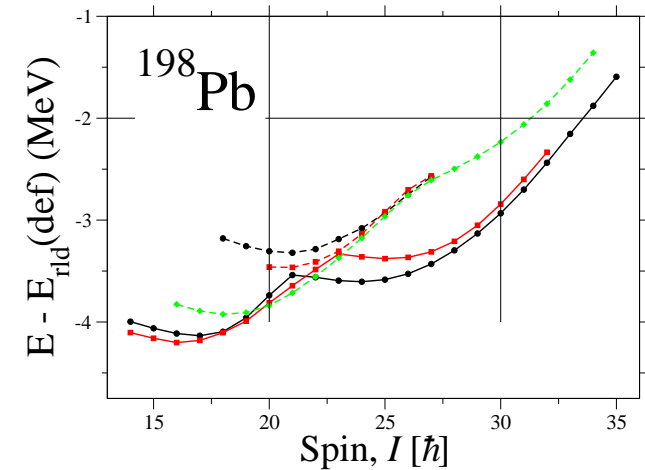
Well-behaved core,

$$g_R(I_x) \simeq g_R, Q_{20}(I_x) \simeq Q_{20} \text{ and } E(I_x) \simeq E_0 + \frac{I_x^2}{2\mathcal{J}_x}$$

Magnetic rotation or the Shears mode



- Oblate shapes ($\varepsilon_2 \sim -0.14, \gamma \sim 0$)
- Alignment around non-principal axis
- Large $B(M1)$ values
- Several bands found in lead isotopes
(see H. Hübel *et.al.*, Prog. Part. Phys. 54 (2005))



Example for ^{198}Pb

Case A

4 neutron holes in a $i_{13/2}$ shell

1 proton in a $i_{13/2}$ shell

1 proton in a $h_{9/2}$ shell.

=> 84084 basis states of each signature

$$I_{max}^{core} = 8\hbar$$

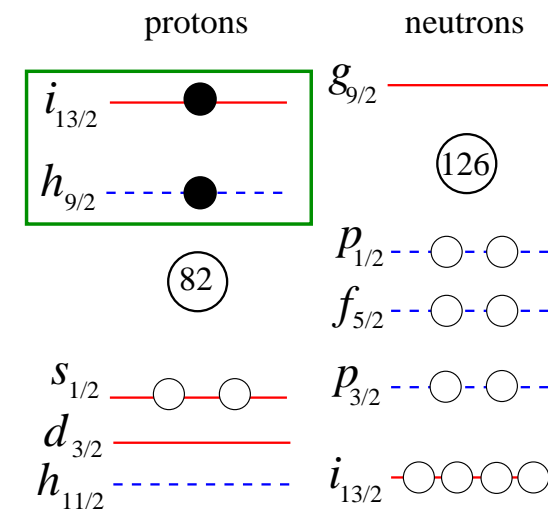
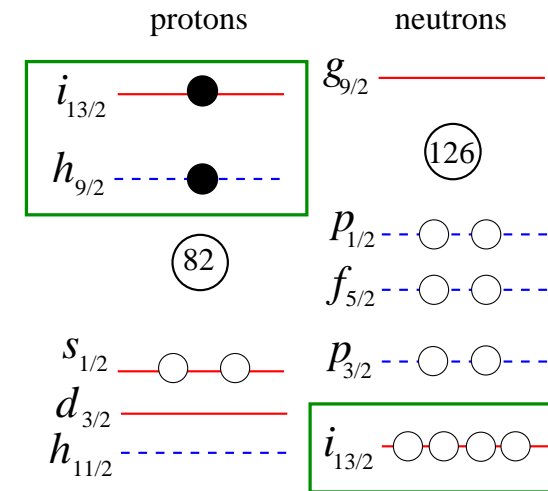
Case B

Neutrons holes treated as core

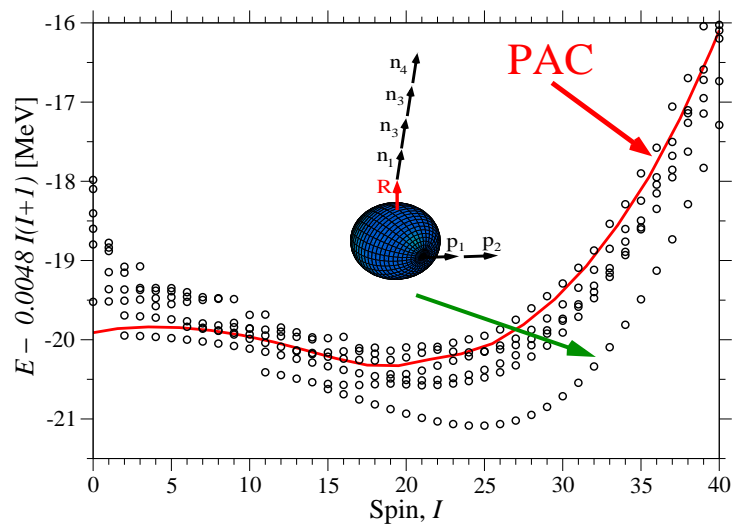
=> 84 basis states of each signature

$$I_{max}^{core} = 8 + 20 = 28\hbar$$

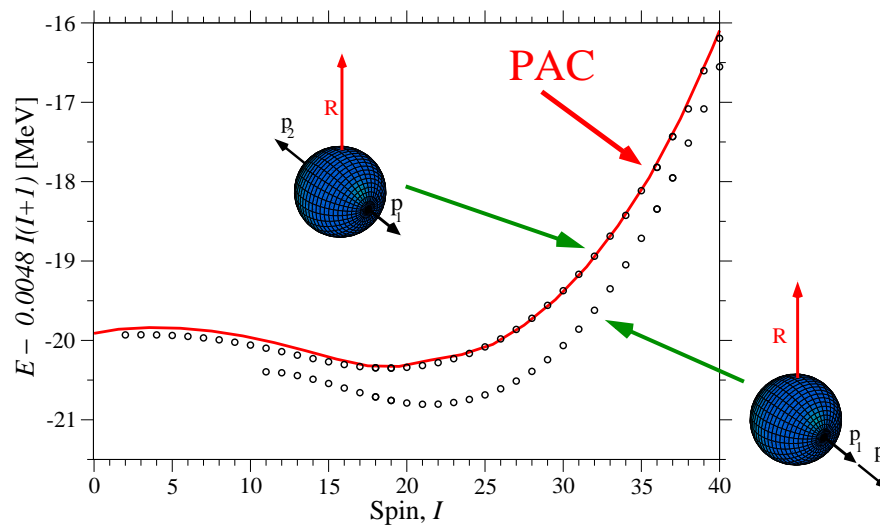
Configuration rel. ^{208}Pb



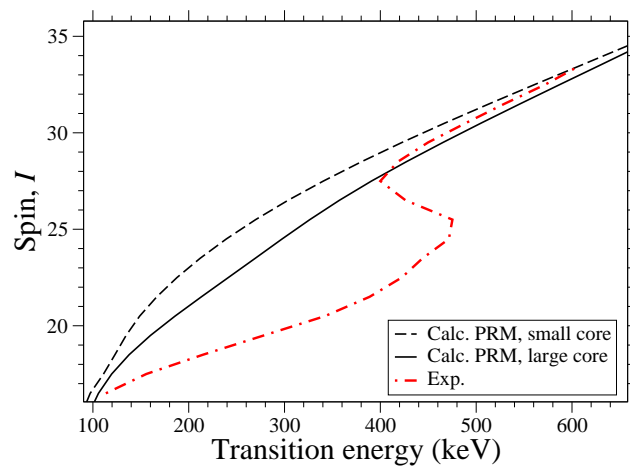
198Pb: Numerical Computation, energies



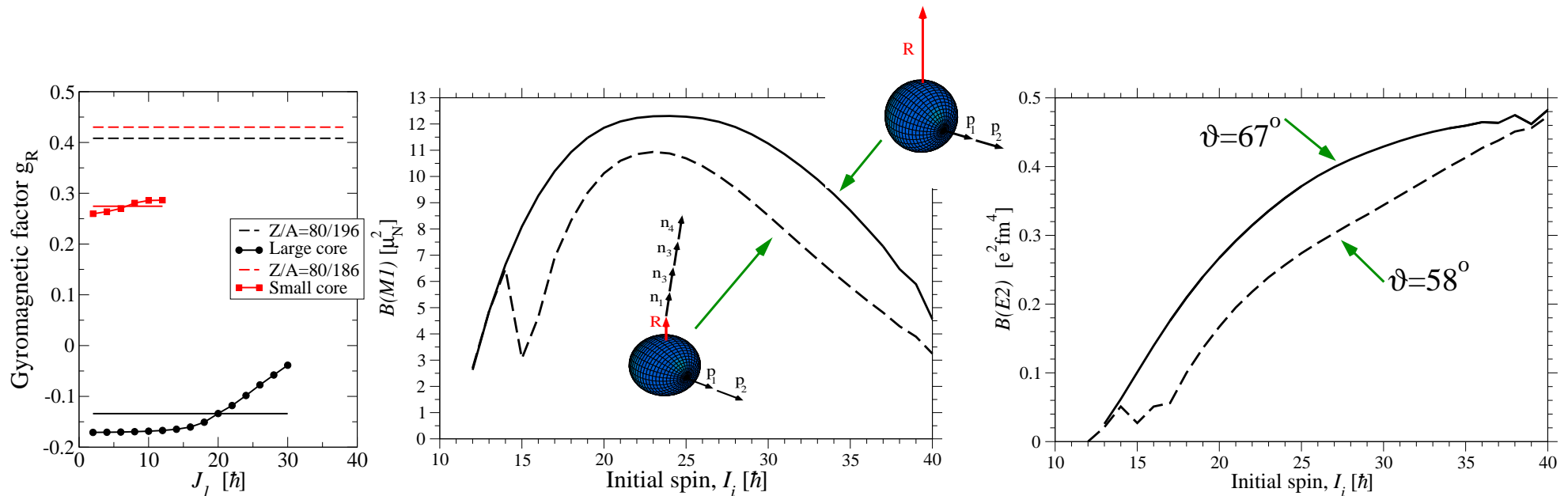
Case A



Case B



198 Pb: Numerical Computation, transitions



Dashed line = Case A

Full line = Case B

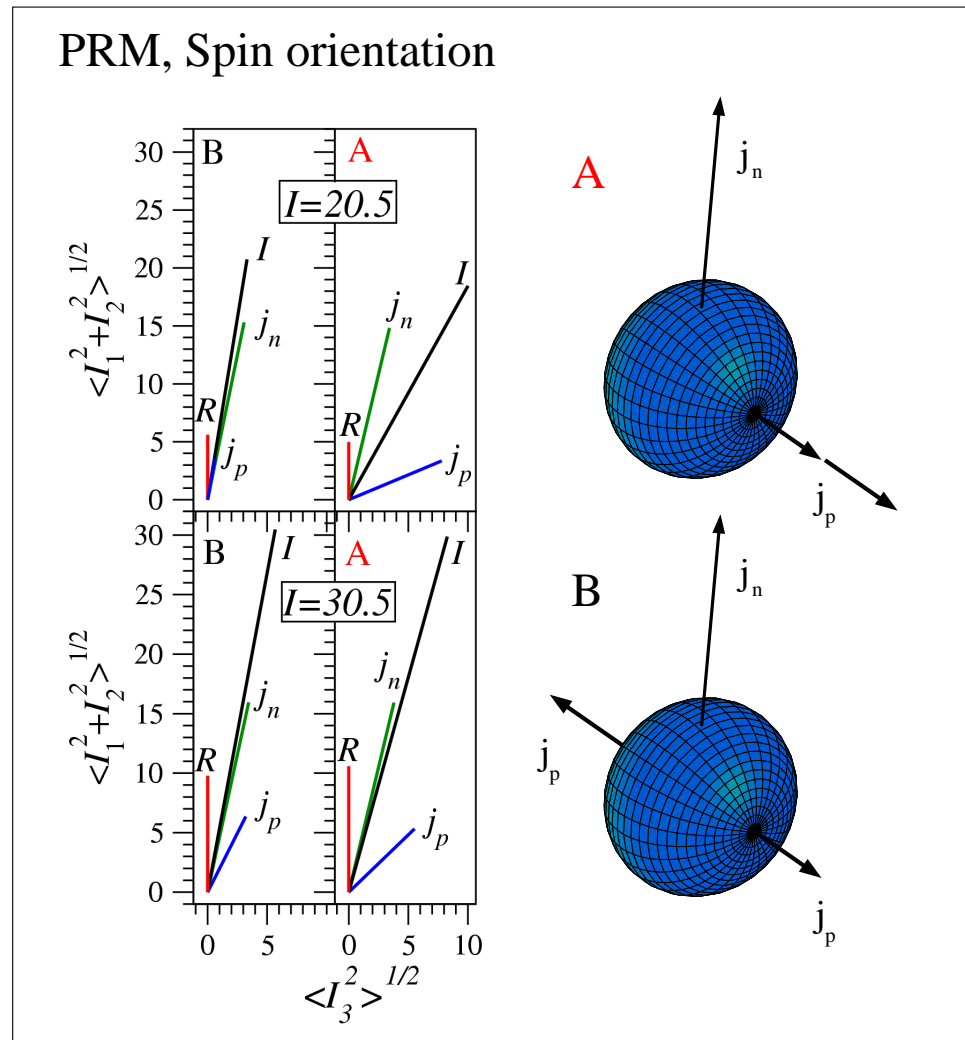
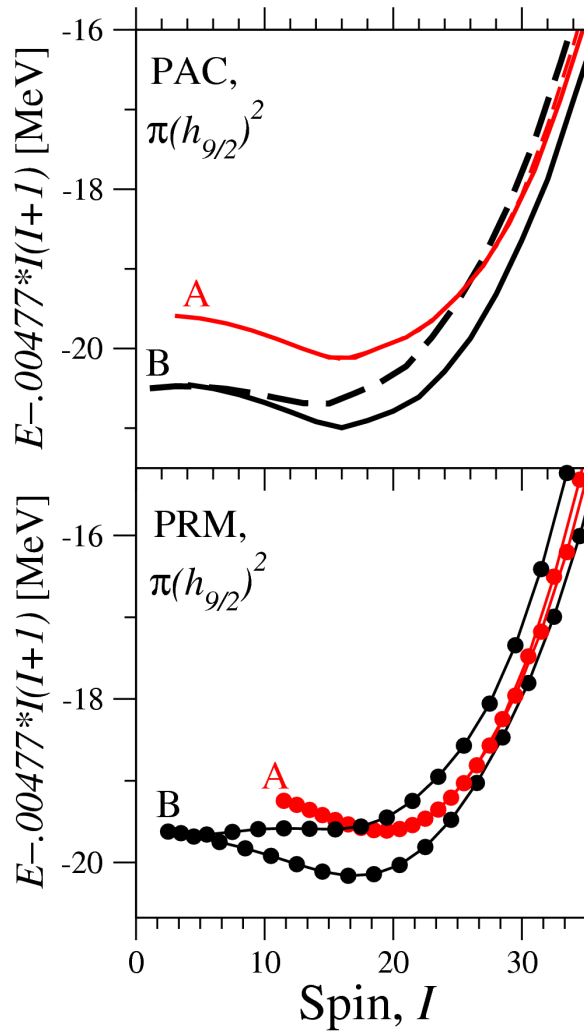
$$\pi(i_{13/2})^1 (h_{9/2})^1 \otimes v(i_{13/2})^{-4}$$

$$\pi(i_{13/2})^1 (h_{9/2})^1$$

$$v = \arccos \left(\frac{\sqrt{\langle I_z^2 \rangle}}{\sqrt{I(I+1)}} \right)$$

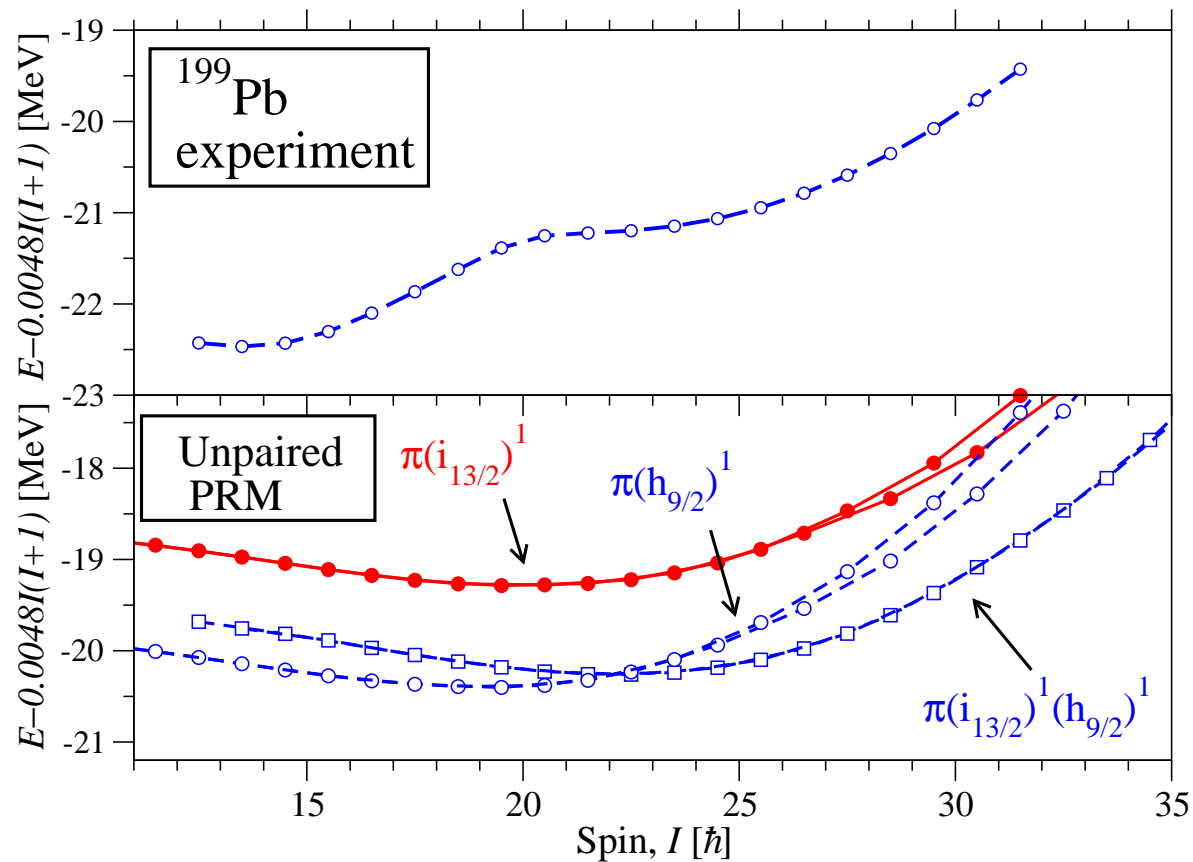
$$B(E2) = \frac{15}{128\pi} (eQ_{20})^2 \sin^4 v$$

$^{199}\text{Pb } \pi(h_{9/2})^2 \otimes \nu(i_{13/2})^{-3}$

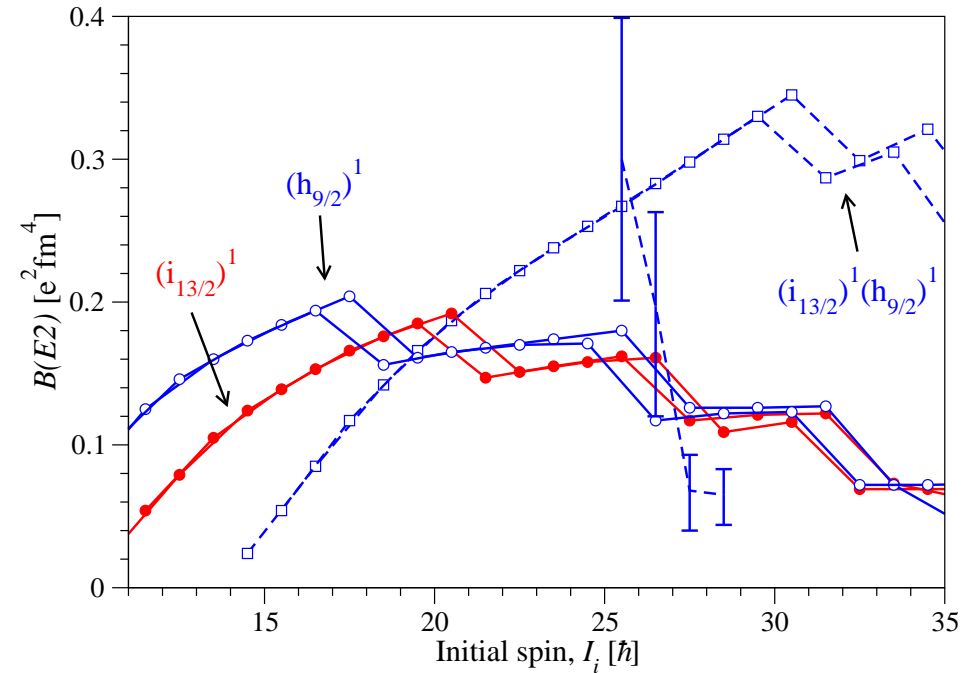
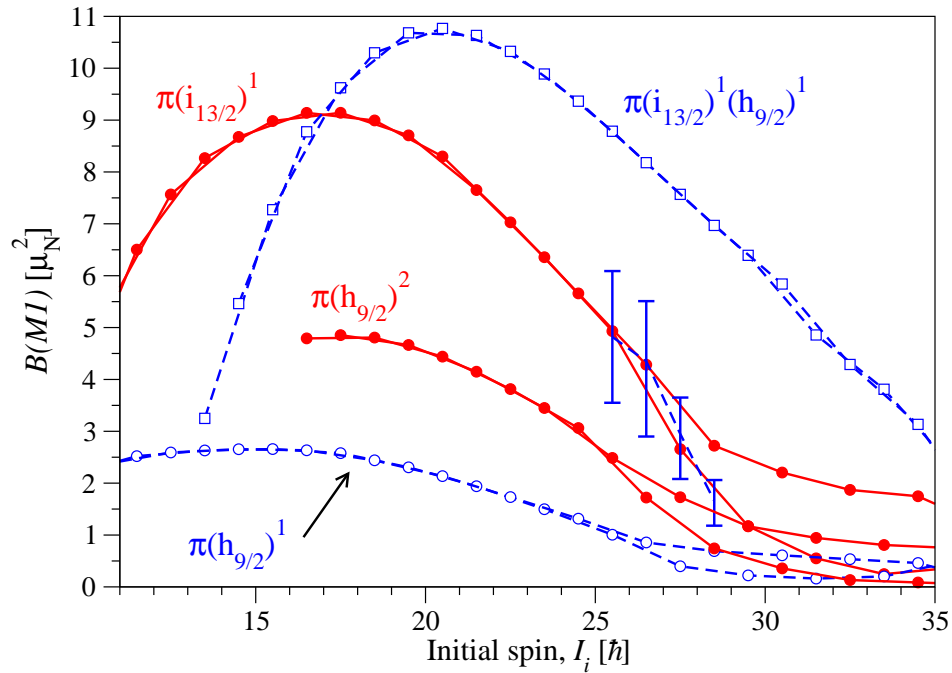


Comparison with experiments, ^{199}Pb

$v(i_{13/2})^{-3}$ coupled with different proton configurations.
Energy minimized with respect to ε_2



Comparison with experiments, ^{199}Pb



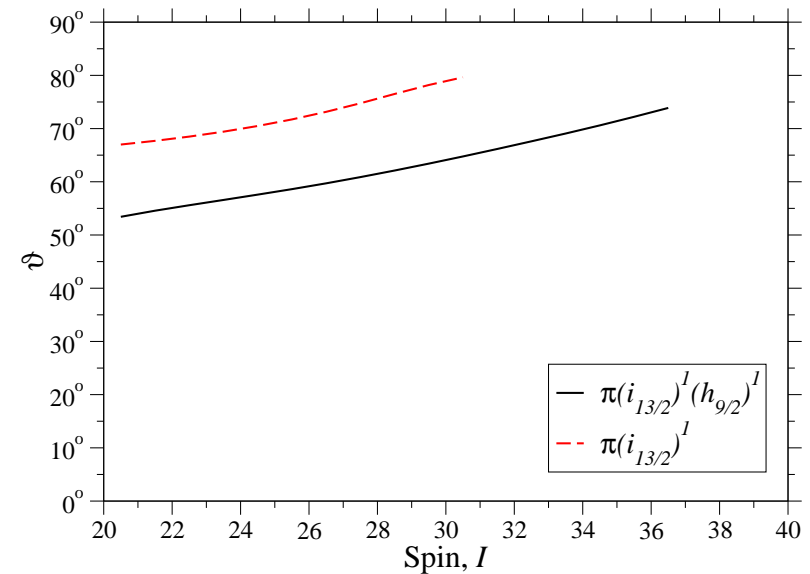
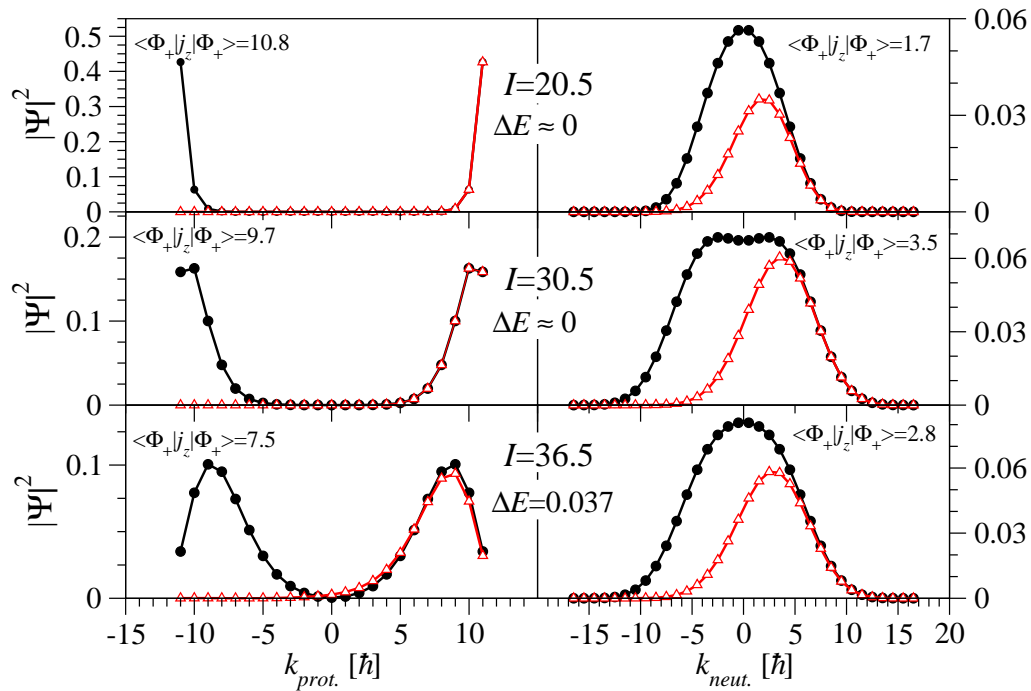
$B(E2)$ values indicate low collectivity already around $I = 30\hbar$

Spin orientation ^{199}Pb

Spin projection for

$$\pi(i_{13/2})^1 (h_{9/2})^1 \otimes \nu(i_{13/2})^{-3} \text{ cfg.}$$

Spin angle



Wavefunction separates into two degenerate parts:

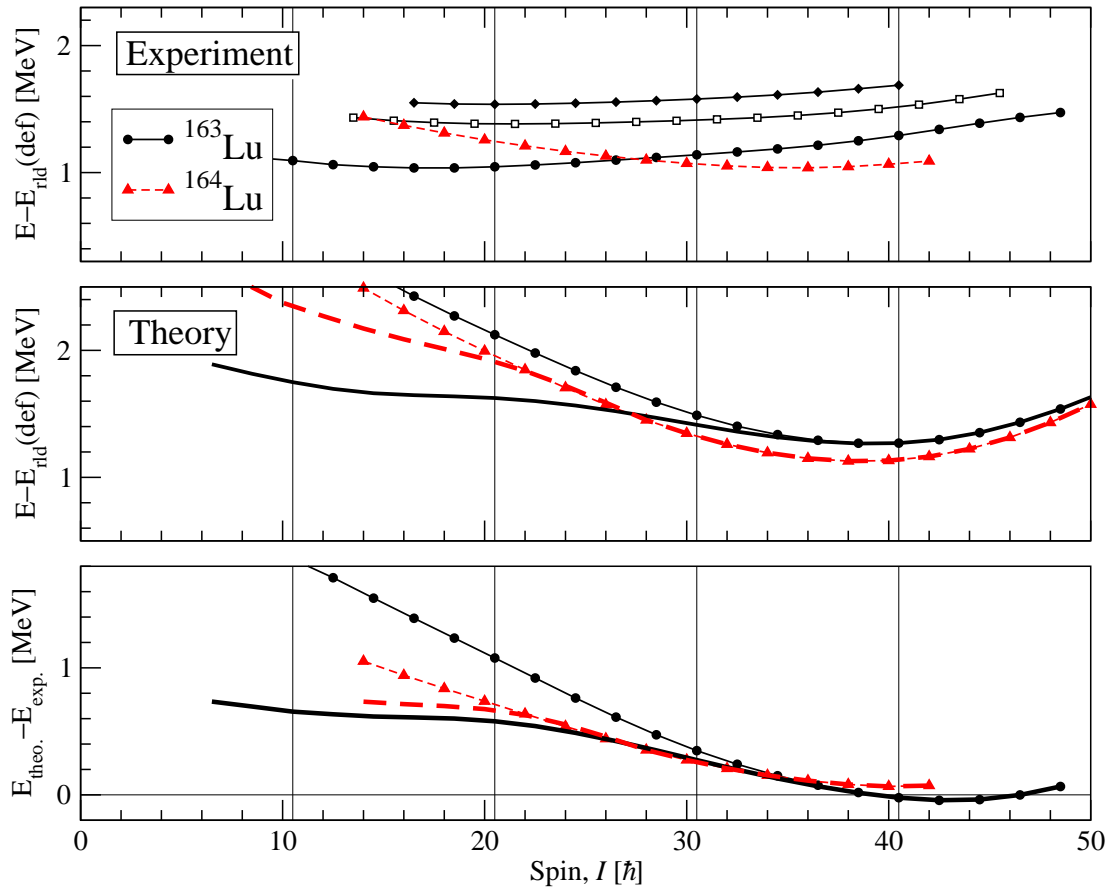
$$|\Psi^I\rangle = \frac{1}{\sqrt{2}} (|\Phi_{k_p > 0}\rangle + |\Phi_{k_p < 0}\rangle)$$

Summary

- In several cases the many particles + rotor calculations with parameters from cranking give similar results as the PAC calculations
- Calculations for ^{199}Pb revealed different types of bands:
 - ▷ with 2 high- j protons, shears effect + collective spin
 - ▷ with 1 high- j proton, less shears effect, some signature splitting at high spins
 - ▷ 2 protons in same shell => antiparallel coupling is favored
- Core spin constrained to point in one direction, effect on result ?

(to be published G.B Carlsson and Ingemar Ragnarsson, Phys. Rev. C)

Triaxial superdeformed bands (TSD)



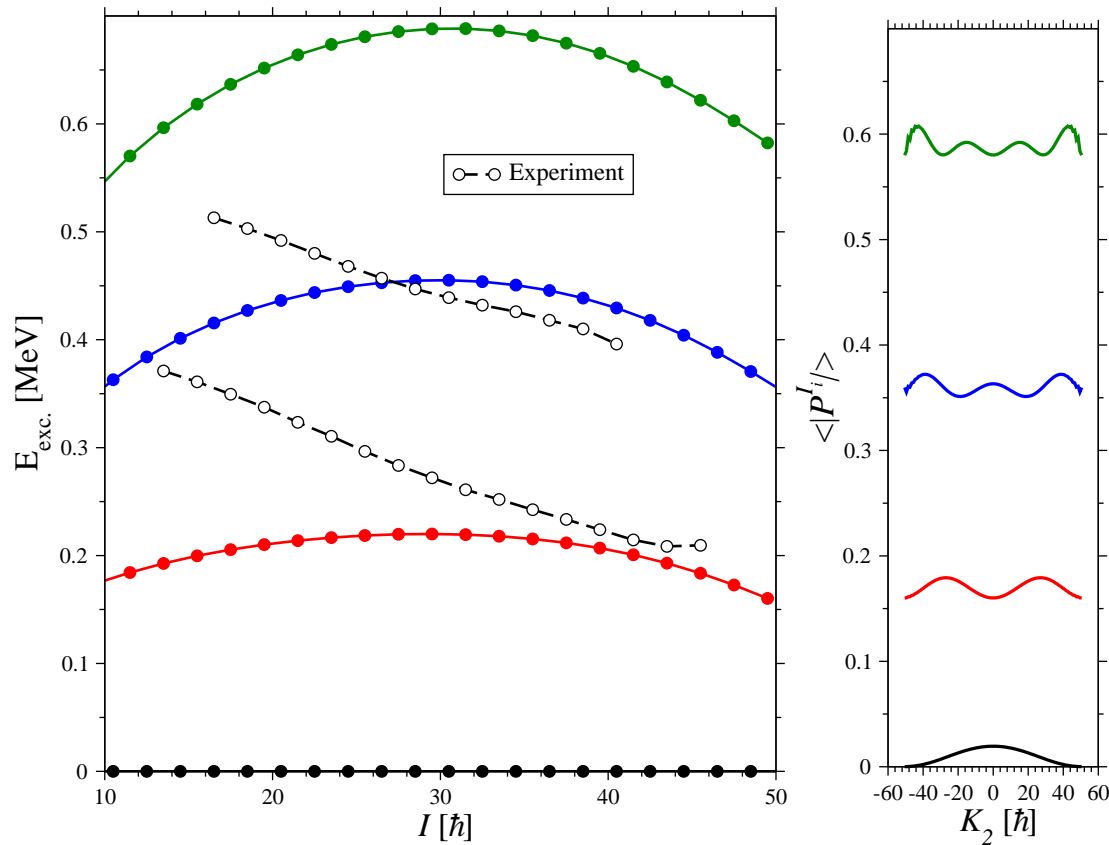
TSD bands in
 $^{163,164}\text{Lu}$

($\varepsilon_2 \sim 0.20, \gamma \sim 40^\circ$)

Two wobbling bands
in ^{163}Lu

6 unconnected bands
in ^{164}Lu

Wobbling excitation energy in ^{163}Lu



Configuration with respect to the rotor core

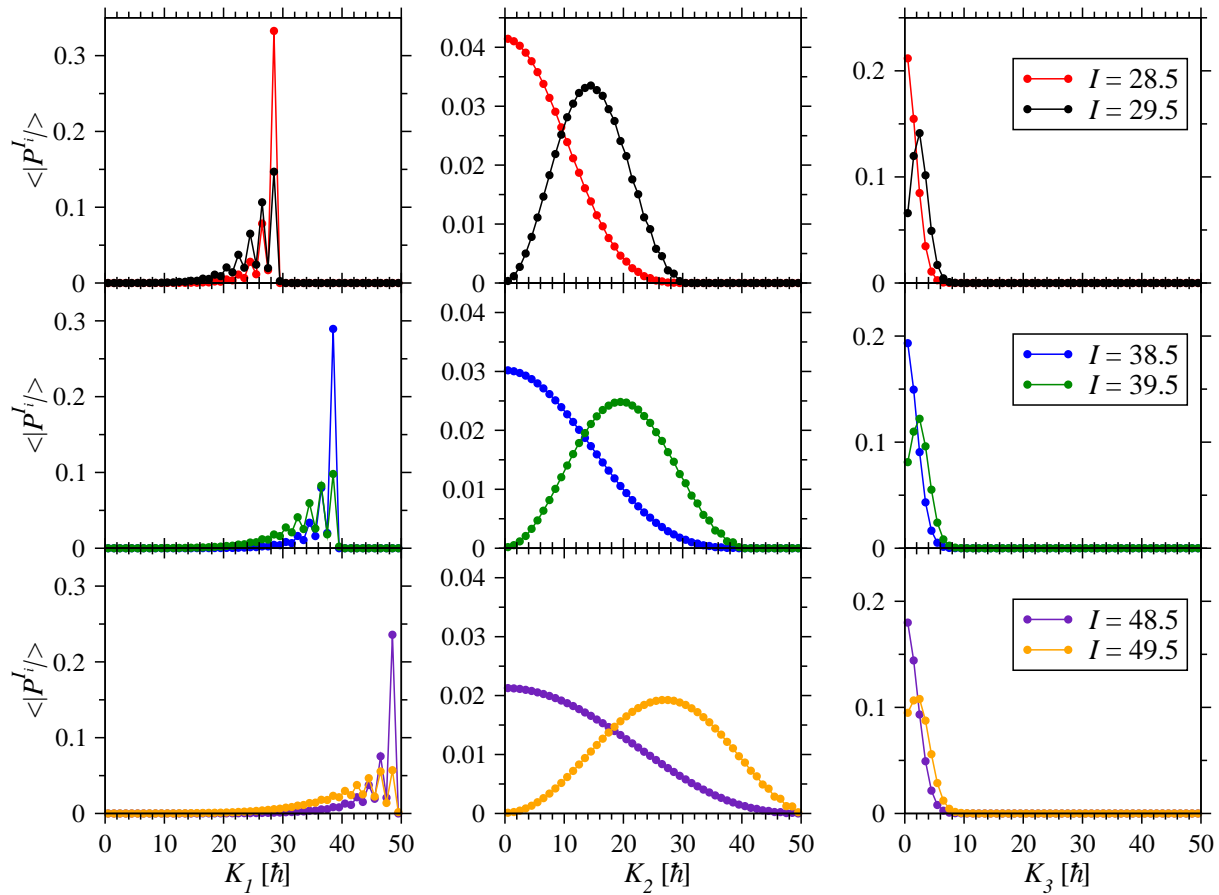
$$\pi(i_{13/2})^1 v(i_{13/2})^6$$

rotor moments of inertia obtained by cranking in 3 directions

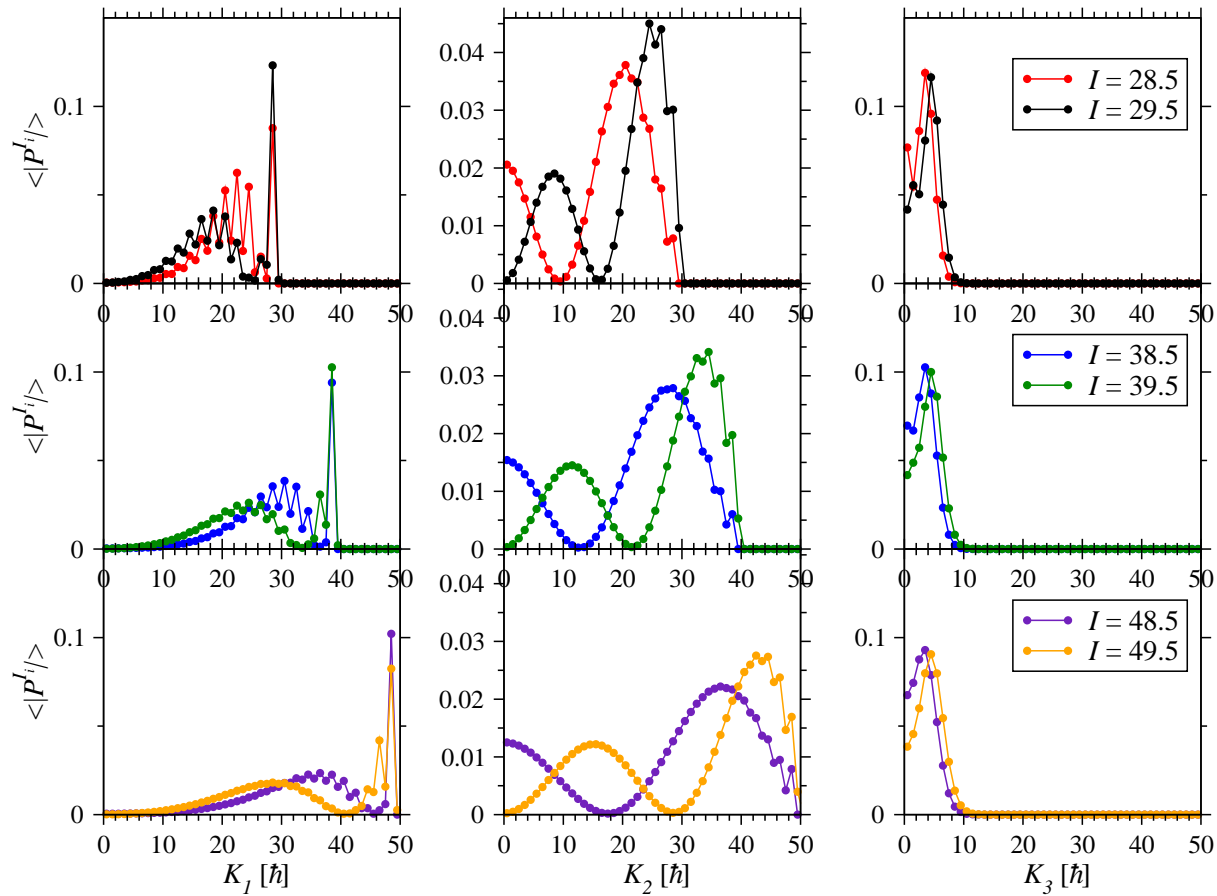
Summary

- Wobbling excitation energies in high-spin limit reasonable
- More work needed to obtain transition probabilities and to go to neighboring nuclei

Spin projection

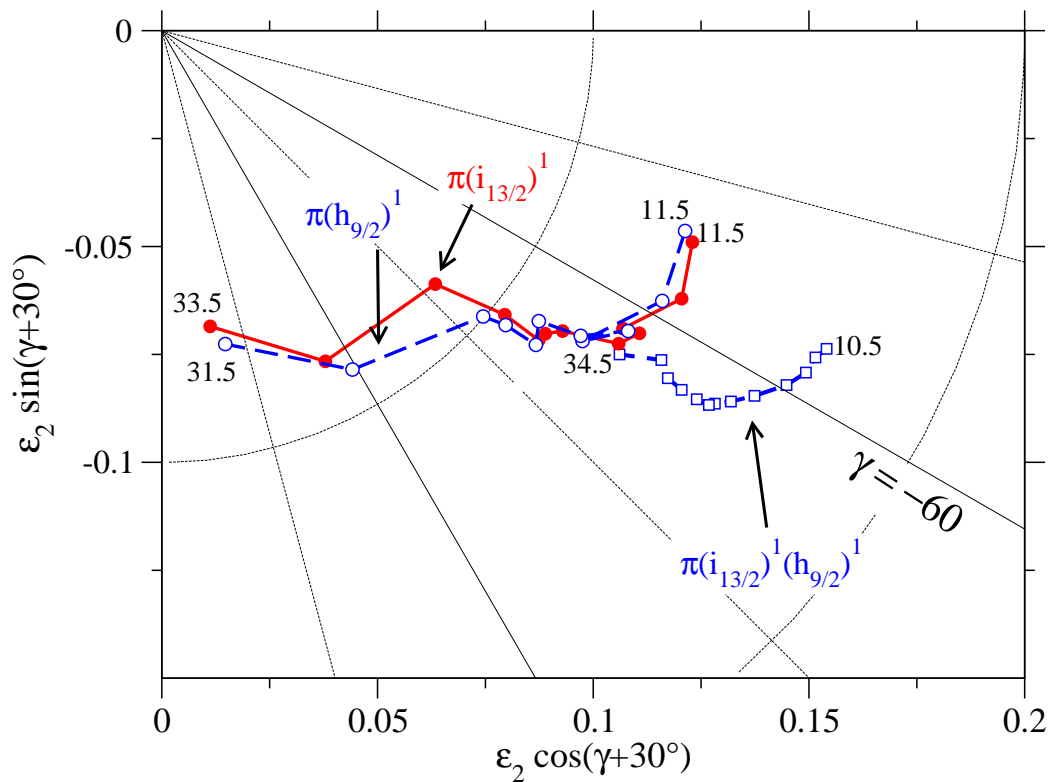


Spin projection



Comparison with experiments, ^{199}Pb

Deformations calculated using principal axis cranking



protons		neutrons	
$i_{13/2}$	●	$g_{9/2}$	—
$h_{9/2}$	●	(126)	○
(82)	○	$p_{1/2}$	○
$s_{1/2}$	○	$f_{5/2}$	○
$d_{3/2}$	○	$p_{3/2}$	○
$h_{11/2}$	○	$i_{13/2}$	○

So far we have assumed axial symmetry in the rotor calculation