Collective quadrupole excitations within a selfconsistent approach

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Microscopic models

phenomenological potentials (eg Nilsson),
mean field theory (Skyrme int., Gogny, RMF)
pairing interactions: constant G, δ force.

$\downarrow$

ATDHFB (cranking), GCM

$\downarrow$

Hamiltonian in the collective space

quadrupole variables: β, γ + Euler angles
"pairing" variables: $\Delta_{\mathrm{p.n}}, \phi_{\mathrm{p,n}}$

Example: ^{128}Xe

1. ATDHFB

$$R = \begin{pmatrix} \rho & \kappa \\ -\kappa^* & 1-\rho^* \end{pmatrix}, \quad \rho_{\mu\nu} = u_\mu^2 \delta_{\mu\nu}, \quad \kappa_{\mu\nu} = s_{\bar{\mu}} u_\mu v_\mu \delta_{\bar{\mu}\nu}$$

$R(q_k(t))$, collective variables q_k;

Collective (classical) energy

$$E_c = E_{\rm kin} + V = \frac{1}{2} \sum_{kj} \dot{q}_j \dot{q}_k B_{kj} + V(q_k)$$

Quantum hamiltonian

$$H = -\frac{\hbar^2}{2} \frac{1}{\sqrt{\det B}} \sum_{ij} \frac{\partial}{\partial q_i} \sqrt{\det B}\,(B^{-1})_{ij} \frac{\partial}{\partial q_j} + V(q_k)$$

Mass parameters

$$B_{kj} = \frac{\hbar^2}{2} \sum_{\mu\nu} \frac{f_{j,\mu\nu} f^*_{k,\mu\nu} + f^*_{j,\mu\nu} f_{k,\mu\nu}}{(E_\mu + E_\nu)}$$

$$\begin{pmatrix} 0 & f_k \\ -f^*_k & 0 \end{pmatrix} = \left(\frac{\partial R_0}{\partial q_k} \right)_{qp}$$

2. Collective variables

Quadrupole part: $q_\mu = \langle Q_{2\mu} \rangle \longrightarrow q_0,\ q_2\ ,\ \Omega$ — Euler angles

$$q_0 = \langle \sum_{n=1}^{A} (3z_n^2 - r_n^2) \rangle$$

$$q_2 = \langle \sum_{n=1}^{A} (x_n^2 - y_n^2) \rangle$$

$$\beta \cos\gamma = q_0 \sqrt{\pi/5} / A \langle r^2 \rangle$$

$$\beta \sin\gamma = q_2 \sqrt{3\pi/5} / A \langle r^2 \rangle$$

$$\min \langle | \mathcal{H} - \sum_i \alpha_i Q_i | \rangle \quad \text{with} \quad \langle | Q_i | \rangle = q_i \longrightarrow R(q_i)$$

Pairing part: Δ, ϕ

$v_i \longrightarrow v_i(\Delta, \lambda)$ as in BCS, λ from $\langle N \rangle = N_0$

$$R(\Delta, \phi) = \begin{pmatrix} \rho & e^{-2i\phi}\kappa \\ -e^{2i\phi}\kappa^* & \rho \end{pmatrix}$$

Other possibility

$$\tilde{\Delta} \sim \langle \sum_{k>0} e^{-2i\phi} s_k c^+_{\bar{k}} c^+_k + e^{2i\phi} s^*_k c_k c_{\bar{k}} \rangle \sim \sum_{k>0} u_k v_k$$

3. Density matrix derivatives

$$f_{\alpha,\mu\nu} = \frac{s_\nu}{E_\mu + E_\nu} Q_{\mu\bar{\nu}}(u_\mu v_\nu + v_\mu u_\nu)$$

$$f_{\Delta,\mu\nu} = s_\nu \delta_{\mu\bar{\nu}} \frac{1}{2E_\mu^2}((\epsilon_\mu - \lambda) + \Delta\partial_\Delta\lambda)$$

$$f_{\phi,\mu\nu} = i s_\nu \delta_{\mu\bar{\nu}} \frac{\Delta}{E_\mu}$$

$$\frac{\partial q}{\partial \alpha} = \sum_{\gamma\mu} \frac{|Q_{\bar{\mu}\gamma}|^2 (u_\gamma v_\mu + v_\gamma u_\mu)^2}{E_\mu + E_\gamma}$$

$$\frac{\partial q}{\partial \Delta} = \Delta \sum_{\mu>0} \frac{Q_{\mu\mu}((\epsilon_\mu - \lambda) + \Delta\partial_\Delta\lambda)}{E_\mu^3}$$

Matrix of mass parameters

$$\begin{pmatrix} (B_{\mathrm{rot}}) & 0 & 0 \\ 0 & (B_{\mathrm{vib}}) & 0 \\ 0 & 0 & (B_{\mathrm{rotpair}}) \end{pmatrix}$$

$$B_{\mathrm{vib}} = \begin{pmatrix} B_{\beta\beta} & B_{\beta\gamma} & B_{\beta\Delta_p} & B_{\beta\Delta_n} \\ B_{\gamma\beta} & B_{\gamma\gamma} & B_{\gamma\Delta_p} & B_{\gamma\Delta_n} \\ B_{\Delta_p\beta} & B_{\Delta_p\gamma} & B_{\Delta_p\Delta_p} & B_{\Delta_p\Delta_n} \\ B_{\Delta_n\beta} & B_{\Delta_n\gamma} & B_{\Delta_n\Delta_p} & B_{\Delta_n\Delta_n} \end{pmatrix}$$

$$B_{\mathrm{rotpair}} = \begin{pmatrix} B_{\phi_p\phi_p} & 0 \\ 0 & B_{\phi_n\phi_n} \end{pmatrix}$$

4. Basis functions

$$F_j(\beta,\gamma,\Omega) f_k^{(p)}(\Delta_p) f_m^{(n)}(\Delta_n)$$

Eigenfunctions of the hamiltonian without deformation-pairing coupling

$$\begin{pmatrix} (B_{\rm def}(q,\Delta)) & 0 \\ 0 & (B_{\rm pair}(q,\Delta)) \end{pmatrix}$$

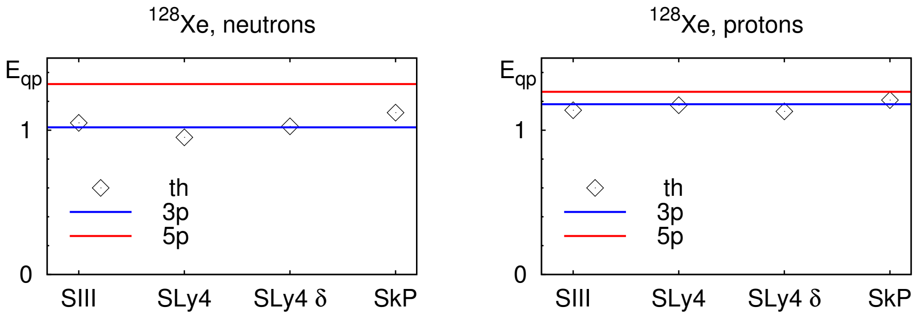
Matrix elements of some operators (eg $1/J_k$)

$$\langle\Psi_j|\,P\,|\Psi_k\rangle = \int dq_{\rm def} F_j F_k \int P(q_{\rm def},\Delta_n,\Delta_p)|f_0^p|^2|f_0^n|^2 g_{\rm pair}^p g_{\rm pair}^n d\Delta_p d\Delta_n =$$
$$= \int dq_{\rm def} F_j F_k P_{\rm av}(q_{\rm def})$$

Example: ^{128}Xe

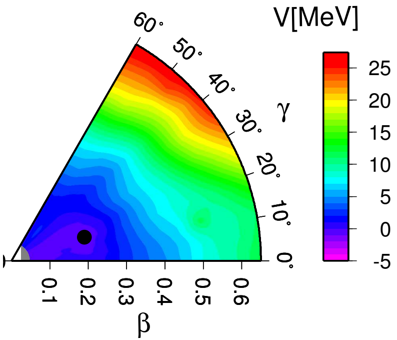
Skyrme interactions: SIII, SLy4, SkP

Pairing strength

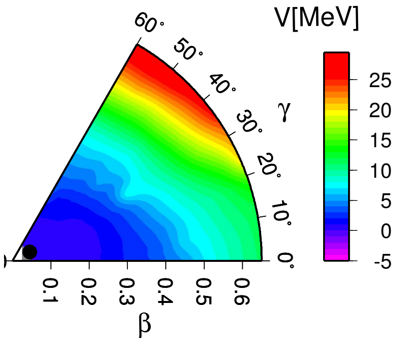


$$E_{\text{qp}} = \langle E_{\text{qp,min}} \rangle$$

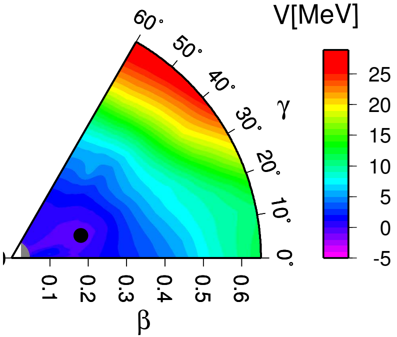
^{128}Xe, SIII



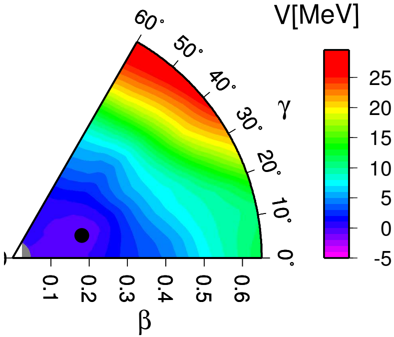
^{128}Xe, SkP

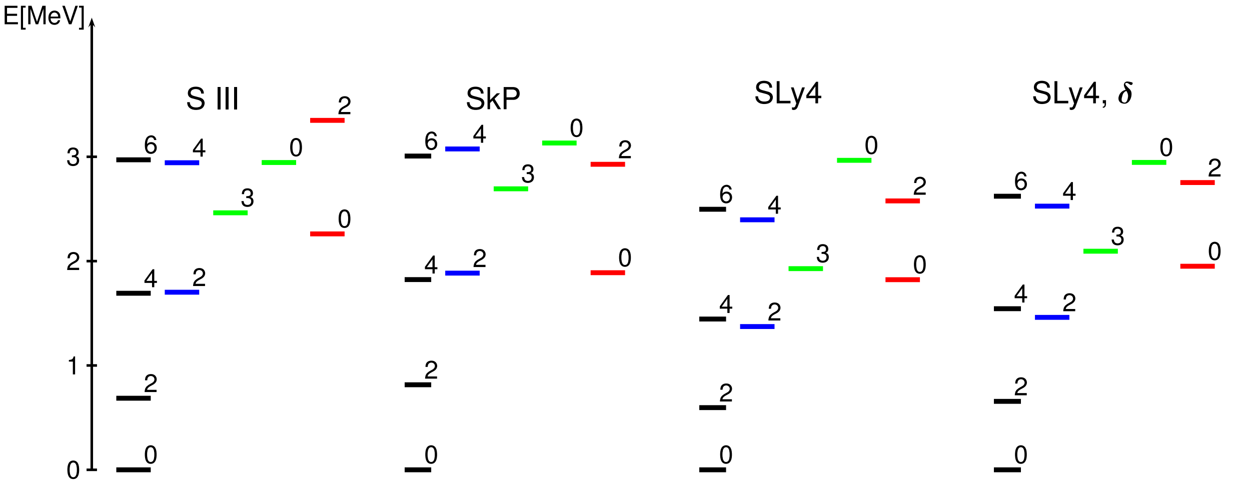


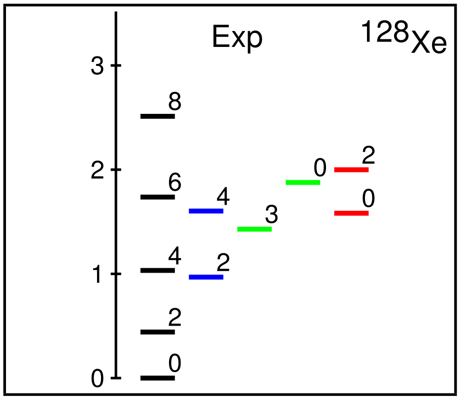
^{128}Xe, SLy4



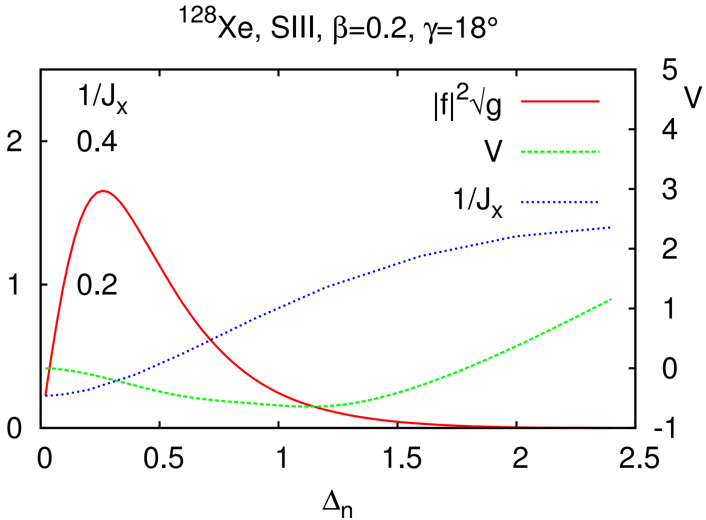
^{128}Xe, SLy4, δ force




E[MeV]
S III
SkP
SLy4
SLy4, δ
0
1
2
3
6
4
2
0
3

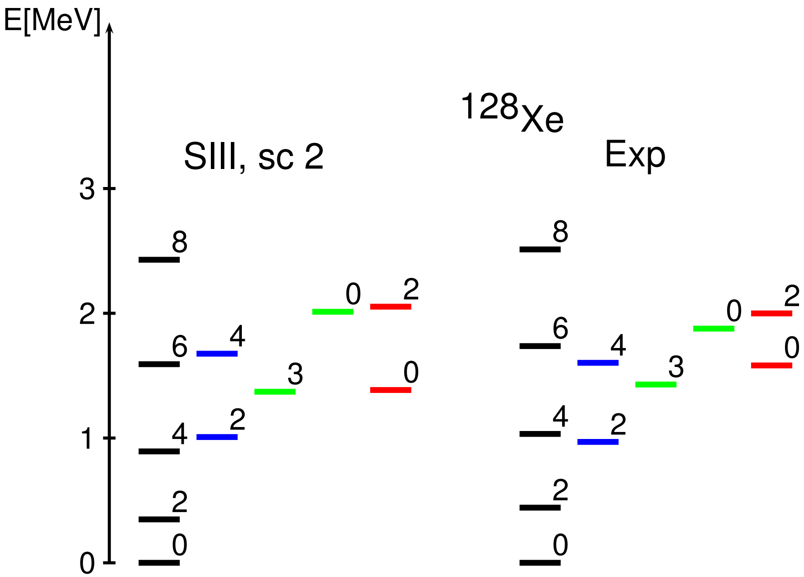

Exp
128Xe
0
1
2
3
8
6
4
2
0
3

Ground state functions of the collective pairing hamiltonian



Average scaling factor for $1/J_k$, $B^{-1}_{\mu\nu} \sim 2$

Estimates for ^{178}Hf, ^{238}U: ~ 1.15


E[MeV]
^{128}Xe
SIII, sc 2
Exp
0
1
2
3
8
6
4
2
0
4
2
3
0
2
0
8
6
4
2
0
4
2
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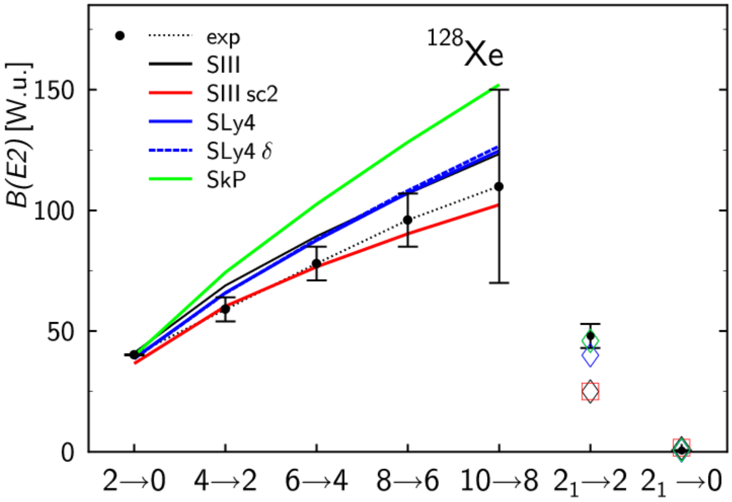
0
2
0

E2 transitions



5. Conclusions

Possible large effect of pairing dynamical degrees of freedom on mass parameters

To do
Larger scale calculations, question of adibaticity of 'pairing' variables, δ interaction, ...