Fission dynamics in the four-dimensional deformation space

Johann Bartel, IReS-CNRS and ULP Strasbourg, France
Krzysztof Pomorski, University MCS, Lublin, Poland[a]

- Introduction
- Spherical harmonics expansion of the shape of fissioning nuclei
- Generalized Trentalage-Koonin-Sierk shape definition
- Funny-Hills deformation parameters
- Modified Funny-Hills parameterization
- Non-axial deformation of fissioning nuclei
- Werner-Wheeler approximation in case of nonaxiality
- Summary

[a] Seminar presented at 12th Nuclear Physics Workshop in Kazimierz, September 2005

Introduction

We describe the fission dynamics by the Langevin equation

$$\frac{dq_i}{dt} = \sum_j \mathcal{M}_{ij}^{-1}\, p_j$$

$$\frac{dp_i}{dt} = -\frac{dV(\vec{q})}{dq_i} - \frac{1}{2}\sum_{j,k} \frac{d\mathcal{M}_{jk}^{-1}}{dq_i}\, p_j\, p_k - \sum_{j,k} \gamma_{ij}\, \mathcal{M}_{jk}^{-1}\, p_k + \mathcal{F}_i(t) \ ,$$

where $V(\vec{q})$ is the collective potential and $\mathcal{M}(\vec{q})$ and $\gamma(\vec{q})$ are the mass and friction tensors, respectively.

The random Langevin force $\mathcal{F}_i(t)$ is given by:

$$\mathcal{F}_i(t) = \sum_j \mathcal{D}_{ij}^{1/2}(\vec{q})\, \vec{\mathcal{G}}_j(t) \ ,$$

where $\vec{\mathcal{G}}(t)$ is a stochastic function. The diffusion $\mathcal{D}$ tensor is related to friction through the Einstein relation

$$\mathcal{D}(\vec{q}) = \gamma(\vec{q})\, T \ .$$

Here T corresponds to the nuclear temperature.

Rayleigh expansion

Lord Rayleigh theoretical studies of the stability of electrified liquid drops [a] where based on the spherical harmonics expansion of the surface-radius of deformed body:

$$R(\theta, \varphi) = R_0([\alpha_{\lambda\mu}]) \left[1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta, \varphi) \right] .$$

Nowadays, this classical Ansatz of Rayleigh is commonly applied to the shape of deformed nuclei.

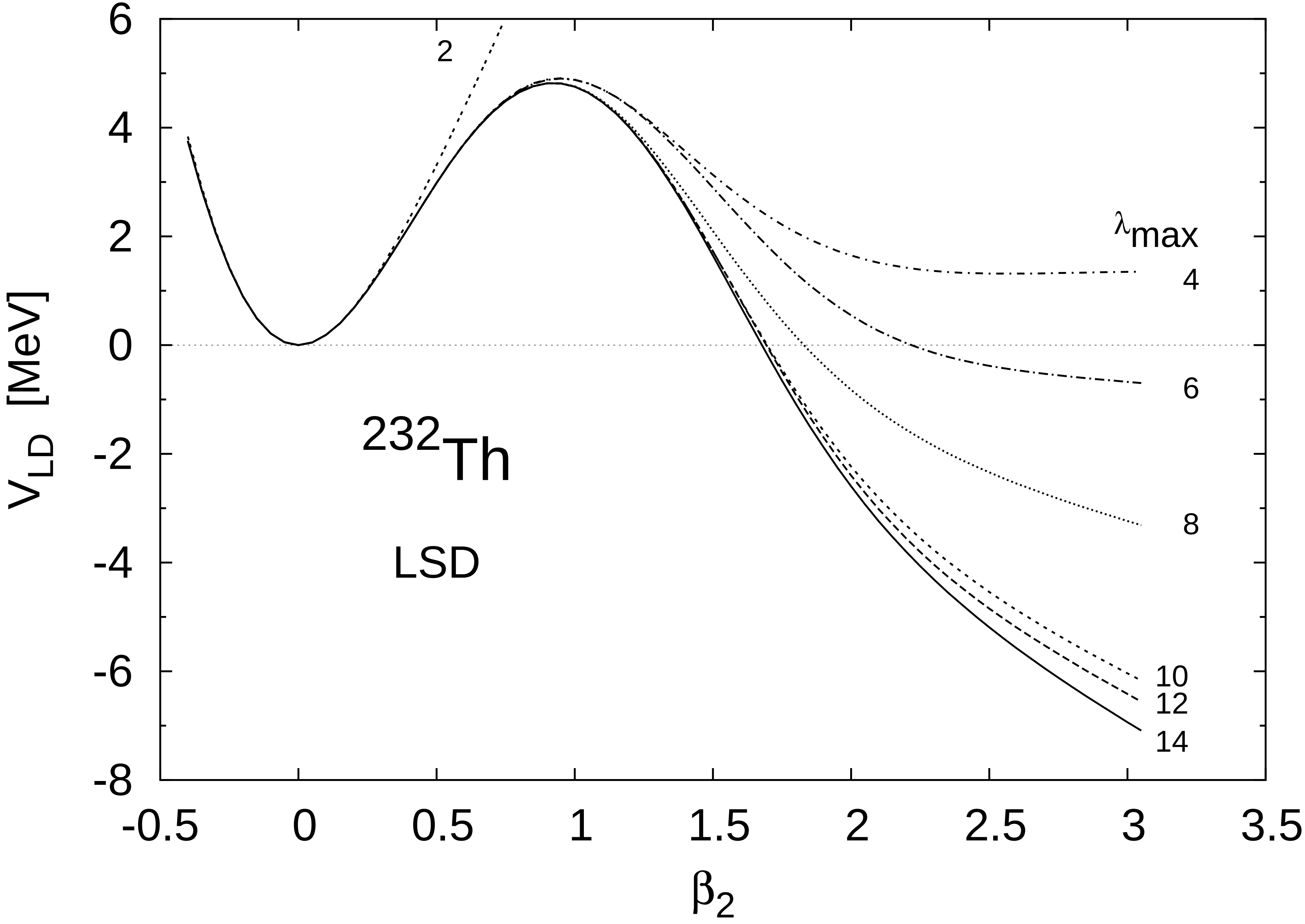
Frequently is also used its simplified version for axially symmetric shapes:

$$R(\theta) = R_0([\beta_\lambda]) \left[1 + \sum_{\lambda} \beta_\lambda P_\lambda(\cos\theta) \right] .$$

---

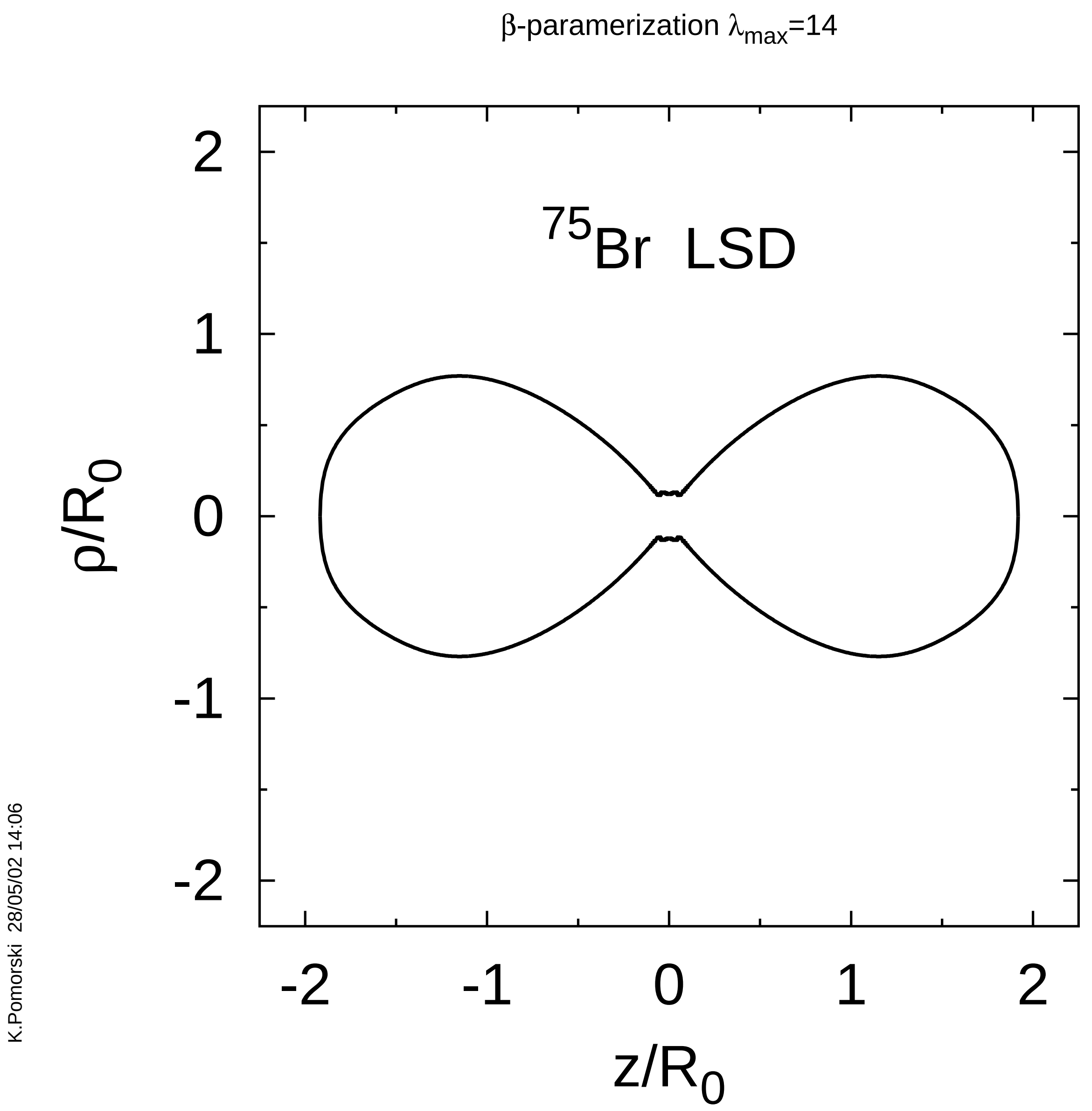
[a] Lord Rayleigh, *The Theory of Sound*, vol. II, MacMillan, New-York, 1896

How good is the Rayleigh's expansion?

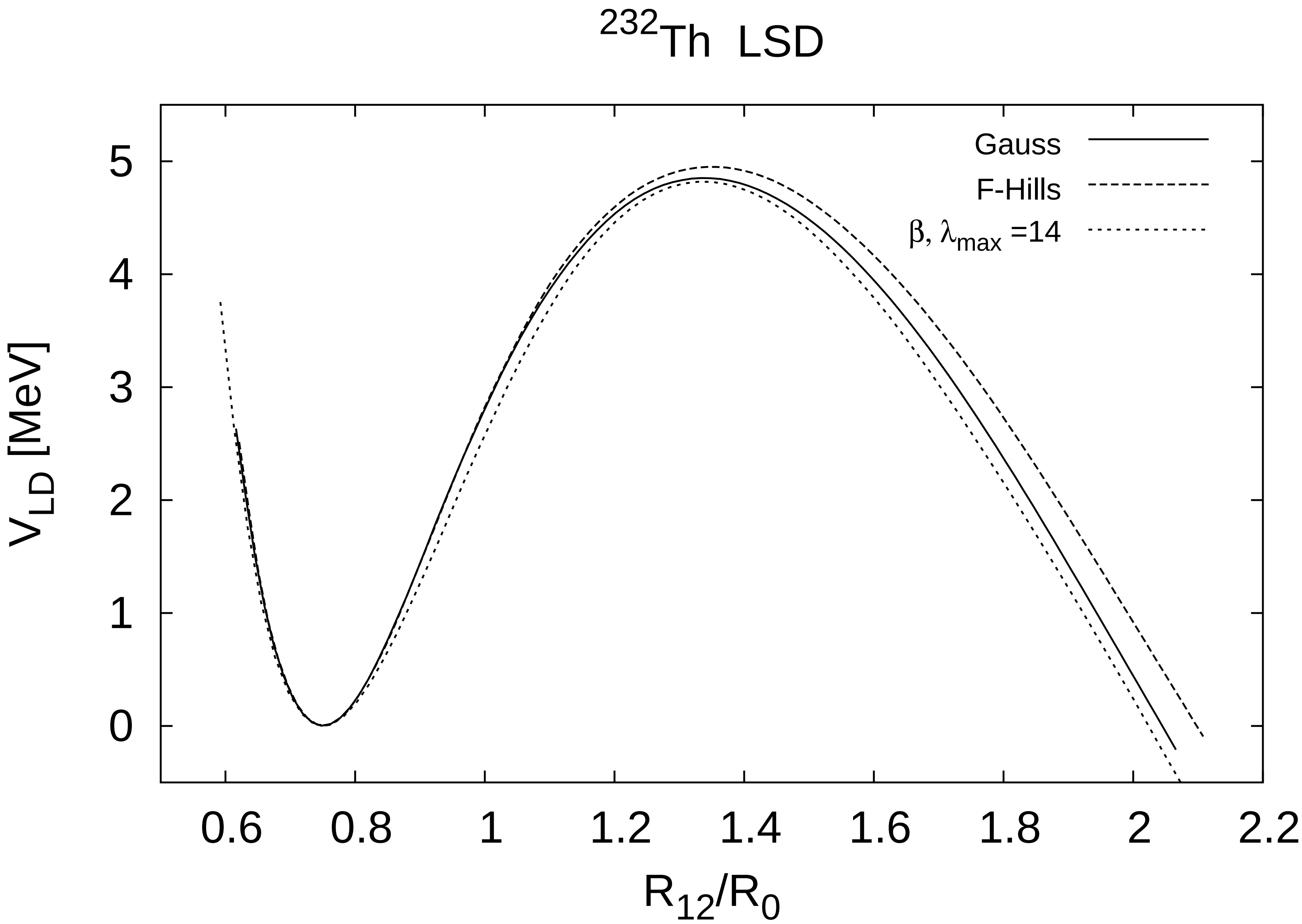


Strange effects around scission

•



Funny-Hills are still attractive![a]



[a] M. Brack, J. Damgaard, A.S. Jensen, H.C. Pauli, V.M. Strutinsky, C.Y. Wong, Rev. Mod. Phys. **44**, 320 (1972).

Generalized TKS expansion

Following the idea of Trentalange et al.[a] we expand the shape of nucleus in a series of Legendre polynomials P_n:

$$\tilde{\rho}_s^2(z) = R_0^2 \sum_{n=0}^{\infty} \alpha_n P_n(\frac{z - z_{sh}}{z_0}) \ ,$$

where $\tilde{\rho}_s(z)$ is the distance from the symmetry axis (z axis) to the surface of the nucleus.

The condition for the tips at $\tilde{\rho}_s^2(z \pm z_{sh}) = 0$ gives:

$$\alpha_0 = - \sum_{n=2,4,}^{\infty} \alpha_n \ , \qquad \alpha_1 = - \sum_{n=3,5,}^{\infty} \alpha_n \ .$$

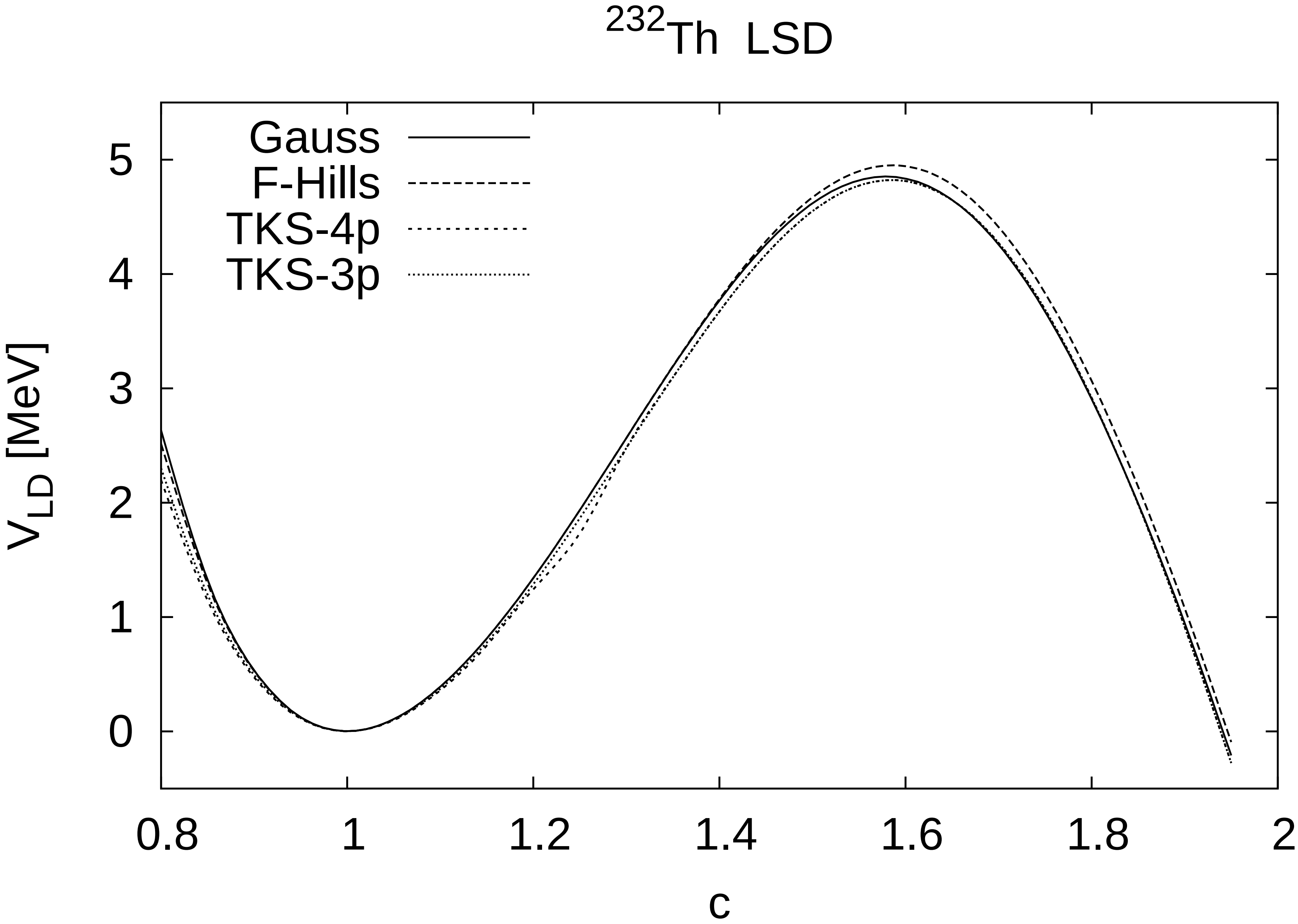
The mass-center and volume conservation conditions lead to:

$$z_{sh} = -\frac{1}{3}\frac{\alpha_1}{\alpha_0} z_0 = -\frac{2}{9}\frac{\alpha_1}{\alpha_0^2} R_0 \ , \qquad z_0 = \frac{2}{3}\frac{R_0}{\alpha_0} \ .$$

---

[a] S. Trentalange, S.E. Koonin, A.J. Sierk, Phys. Rev. **C22**, 1159 (1980).

Generalized TKS expansion



Properties of the TKS expansion

- Spherical form of nucleus corresponds to $\alpha_0 = -\alpha_2 = \frac{2}{3}$,
- Oblate shapes are obtained for $\alpha_2 < -\frac{2}{3}$
  and the prolate ones for $-\frac{2}{3} < \alpha_2 < 0$.
- When $\alpha_2 \to 0$ the nucleus becomes infinitely long.
- Relative length of nucleus is: $c = z_0/R_0 = \frac{2}{3\alpha_0}$.
- The odd λ deformations describe left-right asymmetry.
- Redundant zeros of the $\tilde{\rho}_s^2(z)$ function in the interval
  $(-z0 + z_{sh}, z_0 + zsh)$ make big problems (negative sign of $\tilde{\rho}_s^2$!!).

Funny with the Gaussian neck

Funny-Hills shape definition reads:

$$\tilde{\rho}_s^2(z) = \begin{cases} R_0^2 c^2 \; (1-u^2)\left(A + \alpha u + B u^2\right)\,, & \text{for} \;\; \mathrm{B} \geq 0 \\ R_0^2 c^2 \; (1-u^2)\,(A + \alpha u)\exp\left(B c^3 u^2\right) & \text{for} \;\; \mathrm{B} \leq 0\,, \end{cases}$$

where $u = (z - z_{sh})/z_0$, $z_0 = c\,\mathrm{R}_0$ and $z_{sh} = -c^3 \alpha z_0/5$.

Our new shape definition with the Gaussian neck:

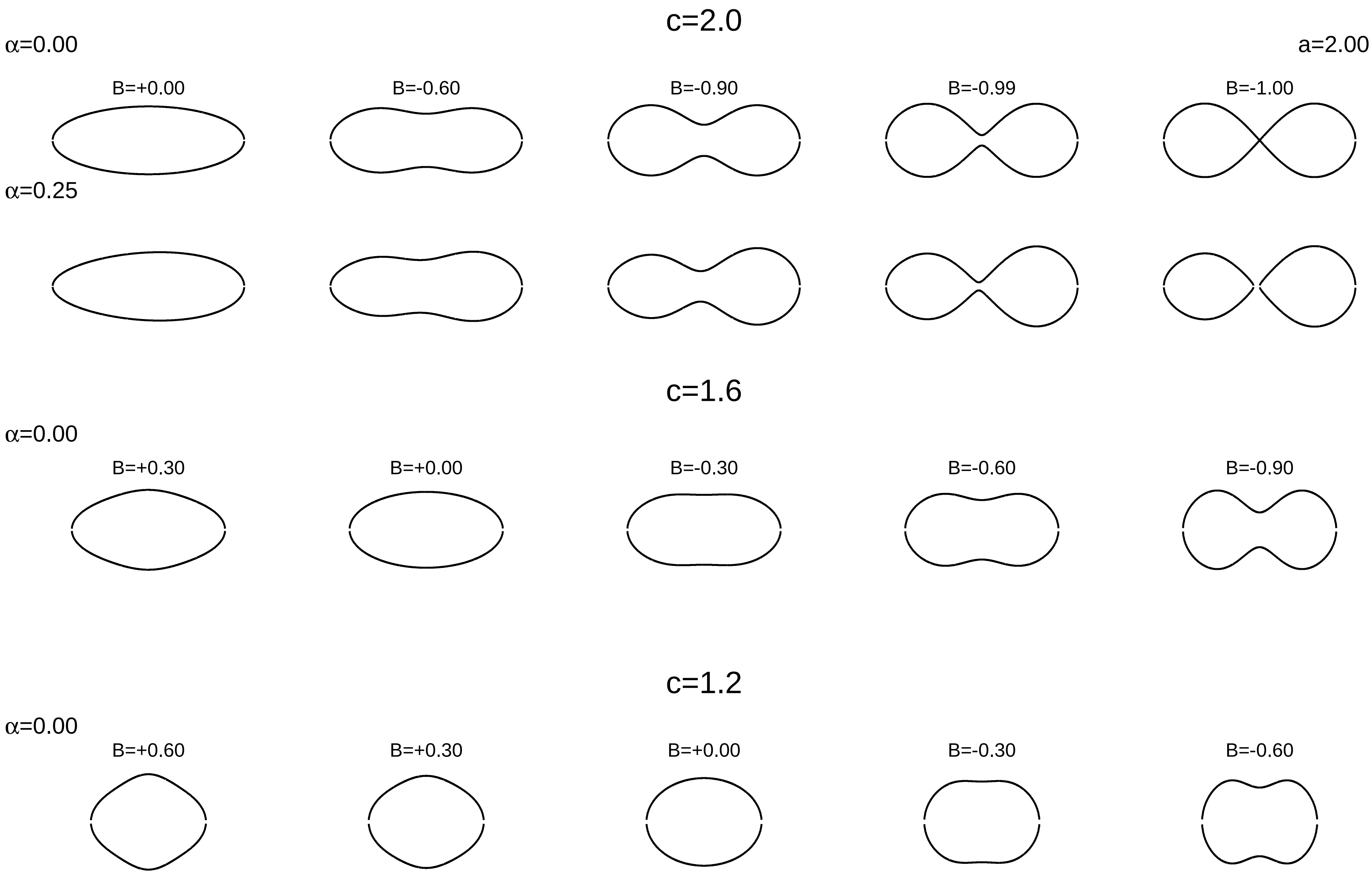
$$\tilde{\rho}_s^2\,(z) = \frac{R_0^2}{c\,f(a,B)}\,(1-u^2)\,(1 + \alpha u - B\,e^{-a^2 u^2})\;,$$

with $f(a,B)$ ensuring the volume conservation

$$f(a,B) = 1 - \frac{3\,B}{4\,a^2}\left[e^{-a^2} + \sqrt{\pi}(a - \frac{1}{2a})\,\mathrm{Erf}(a)\right]$$

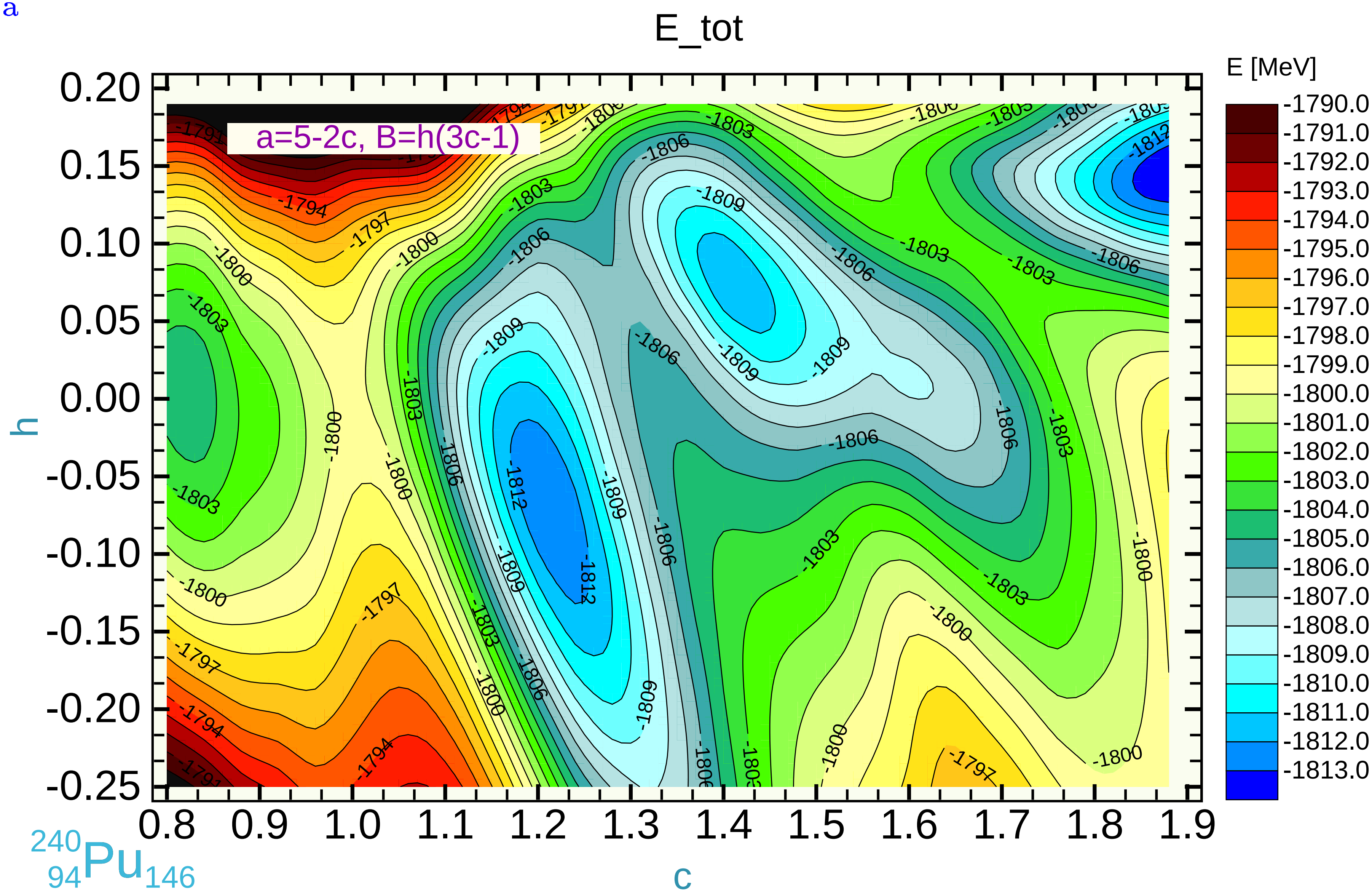
is free from the redundant zeros in the interval $u \in (-1,1)$.

Funny with the Gaussian neck



Example of the PES in (c, h)

[a]



[a] A. Dobrowolski et al. (Yukawa-folded potential plus LSD model)

Nonaxial shapes

A natural extension of the axially symmetric shapes is to assume that the cross section perpendicular to the z-axis has the form of an ellipse

$$\frac{x^2}{\mathcal{A}^2(z)} + \frac{y^2}{\mathcal{B}^2(z)} = 1 \quad \text{with} \quad \pi \tilde{\rho}_s^2(z) = \pi\, \mathcal{A}(z)\, \mathcal{B}(z) \; .$$

Let us introduce a nonaxial deformation parameter[a]:

$$\eta = \frac{\mathcal{B}^2 - \mathcal{A}^2}{\mathcal{A}^2 + \mathcal{B}^2} \; .$$

Finally the square of the distance from the surface point at (z, φ) to the z-axis is given by:

$$\boxed{\rho_s^2(z,\varphi) = \tilde{\rho}_s^2(z)\, \frac{\sqrt{1-\eta^2}}{1+\eta\cos(2\varphi)} \; .}$$

[a] In general the nonaxiality parameter η can depend on z but in the following we assume that it is z-independent.

Relation to β and γ deformations

In the pure ellipsoidal case ($B = 0$) the main half-axis are:

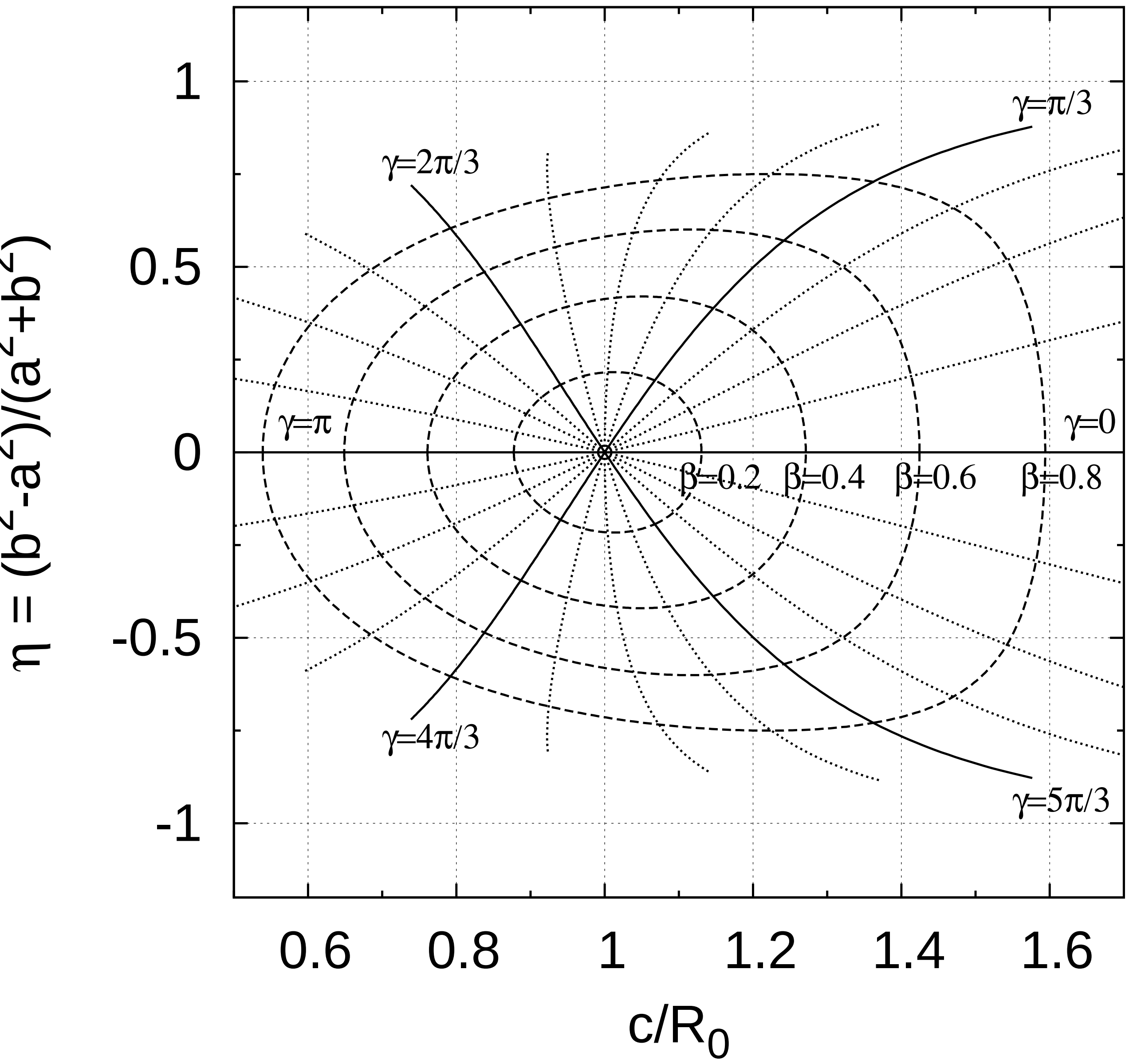
$$\mathcal{A} = R_0/\sqrt{c}\left(\frac{1-\eta}{1+\eta}\right)^{1/4} = R(\beta,\gamma)\left[1 - \sqrt{\frac{5}{4\pi}}\beta\cos(\gamma - \pi/3)\right] ,$$

$$\mathcal{B} = R_0/\sqrt{c}\left(\frac{1+\eta}{1-\eta}\right)^{1/4} = R(\beta,\gamma)\left[1 - \sqrt{\frac{5}{4\pi}}\beta\cos(\gamma + \pi/3)\right] ,$$

$$\mathcal{C} = R_0\, c = R(\beta,\gamma)\left[1 + \sqrt{\frac{5}{4\pi}}\beta\cos(\gamma)\right] ,$$

where β and γ are the quadrupole axial and nonaxial deformation parameters and $R(\beta,\gamma)$ is obtained from the volume conservation condition.

(c, η) to (β, γ) transformation



.

Summary

- **Spherical harmonics expansion of the nuclear surface fails at large deformations.**
- **Trentalange-Koonin-Sierk expansion of the square distance from the symmetry axis to the surface into the Lagrange polynomial series is much more efficient, but is difficult to handle.**
- **Funny-Hills parameterization with two coordinates only, gives the liquid drop barriers which do not exceed those obtained with 5 lowest β_λ deformations.**
- **Modified Funny-Hills parameterization (with the Gaussian neck) describes very well the shapes of fission barriers and it is easy to handle.**
- **New nonaxial deformation ensuring that the z=const cross sections have an elliptic form is proposed.**

Bibliography

1. Lord Rayleigh, *The Theory of Sound*, vol. II, MacMillan, New-York, 1896.
2. M. Brack, J. Damgaard, A.S. Jensen, H.C. Pauli, V.M. Strutinsky, C.Y.Wong, Rev. Mod. Phys. **44**, 320 (1972).
3. S. Trentalange, S.E. Koonin, A.J. Sierk, Phys. Rev. **C22**, 1159 (1980).
4. K. Pomorski, J. Dudek, Phys. Rev. **C67**, 044316 (2003).