Shell model approach for open many-body systems

GANIL - ORNL Theory Collaboration

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1. Introduction
2. Shell model for open quantum systems
3. Certain salient features of the continuum coupling: threshold behavior
4. Two-proton radioactivity
5. Conclusions

Binding energy systematics → 'Magic' numbers of nucleons : 2, 8, 20, 28,…

↓

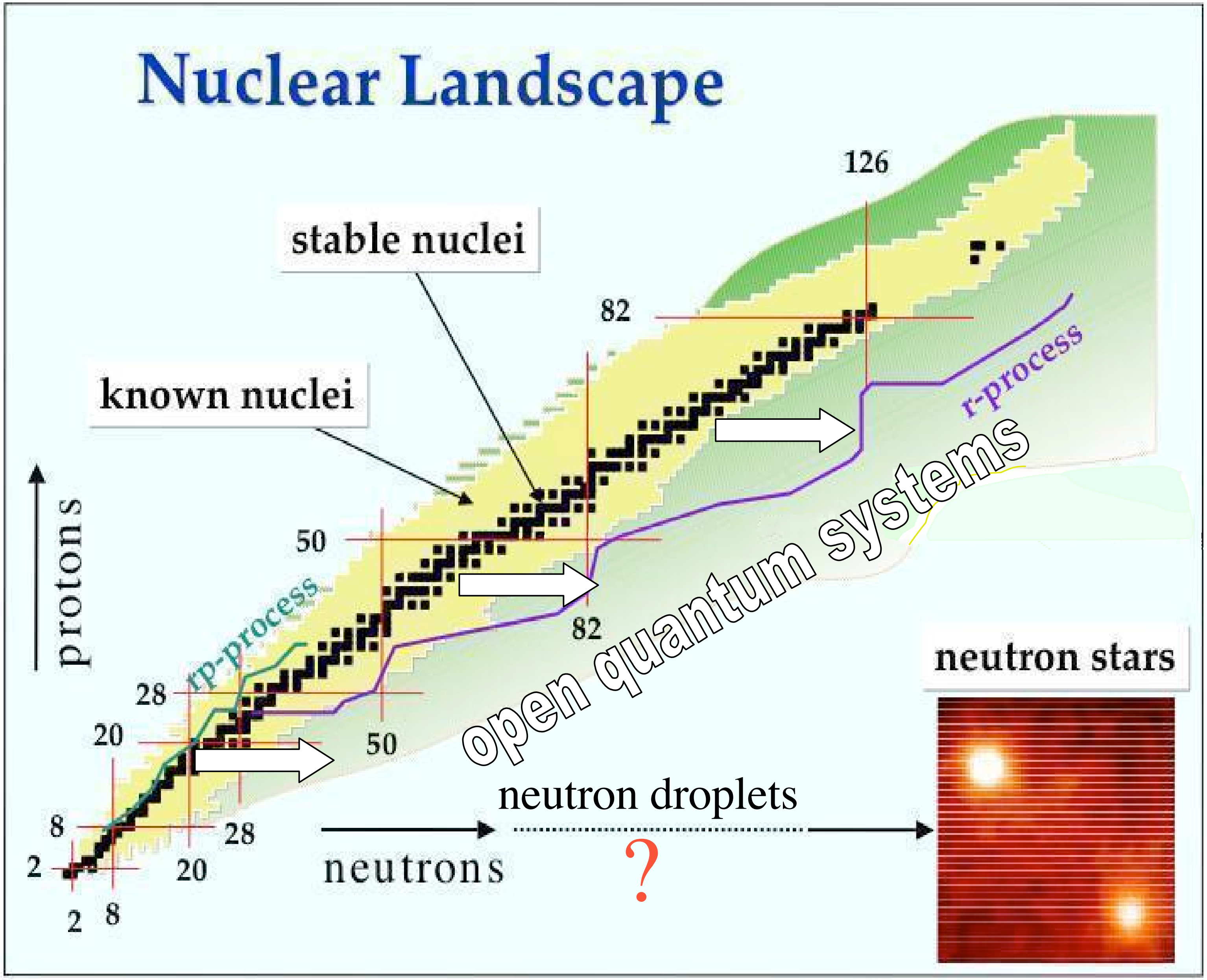
Average one-body potential (1949):
Spherical harmonic oscillator+spin-orbit interaction

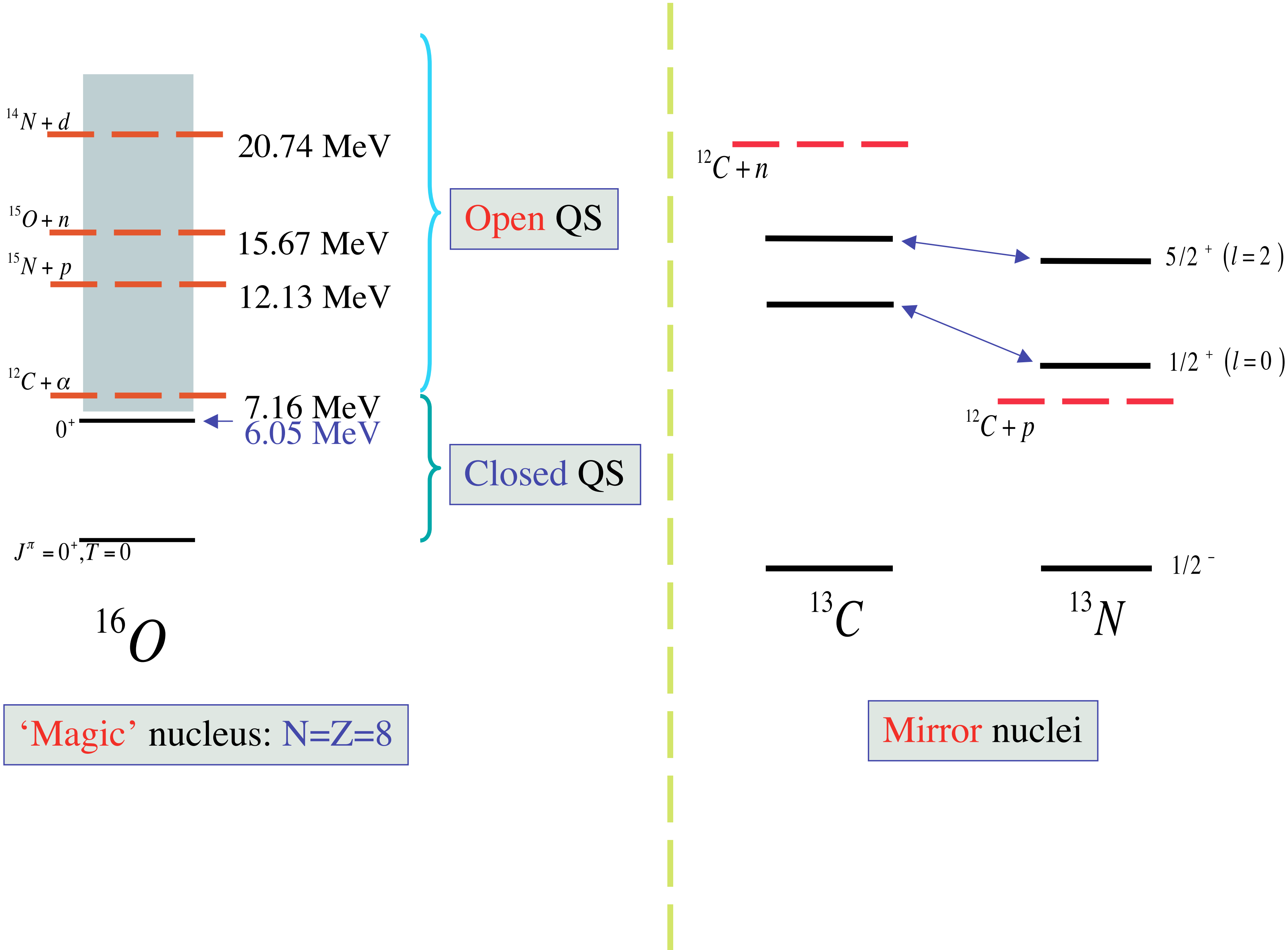
How to describe 'non-magic' nuclei? → Effective interactions

Broken symmetry average potential:
Bohr collective Hamiltonian (1952)
Nilsson potential (1955)
… …

Multiconfigurational Shell Model (1953)

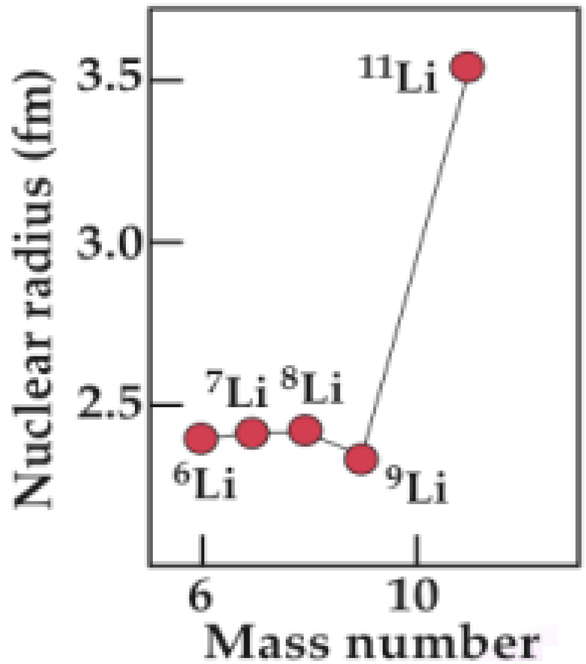
Closed quantum many-body systems


Nuclear Landscape
126
stable nuclei
82
known nuclei
r-process
open quantum systems
50
82
rp-process
neutron stars
28
20
50
8
28
2
20
neutron droplets
neutrons
?
2
8
protons

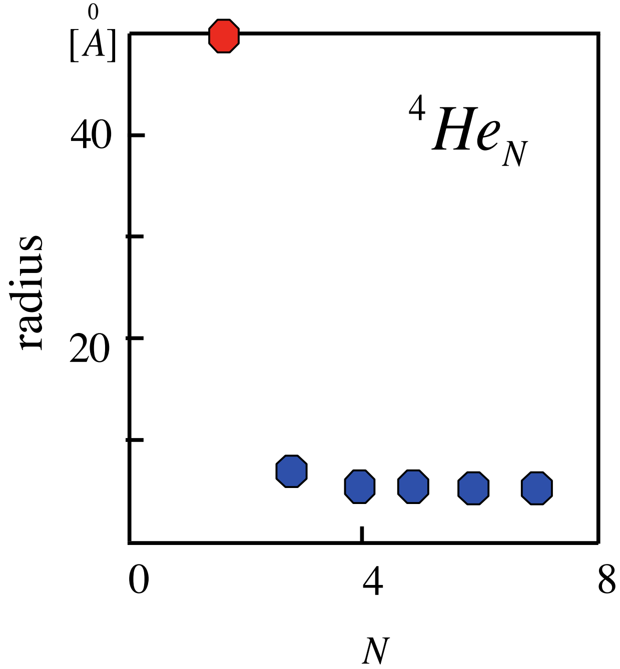


$^{14}N+d$
20.74 MeV
$^{15}O+n$
15.67 MeV
$^{15}N+p$
12.13 MeV
$^{12}C+\alpha$
7.16 MeV
0^+
6.05 MeV
$J^\pi=0^+, T=0$
^{16}O
Open QS
Closed QS
'Magic' nucleus: N=Z=8
$^{12}C+n$
$5/2^+$ $(l=2)$
$1/2^+$ $(l=0)$
$^{12}C+p$
$1/2^-$
^{13}C
^{13}N
Mirror nuclei

Borromean nuclei (1985)



giant ^4He_N -dimer (2000)



Spectra and matter distribution are modified by the proximity of scattering continuum

Open quantum systems

Continuum (real-energy) Shell Model (1977 - 1999 -)	Gamow (complex-energy) Shell Model (2002 -)

Hilbert space formulation : Shell Model Embedded in the Continuum (1999)

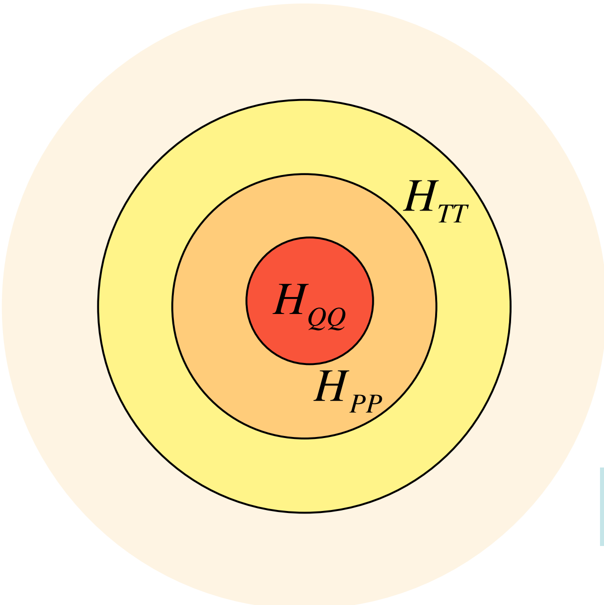
Completness relation for one-body states:

$$\sum_n |u_n\rangle\langle u_n| + \int_0^{+\infty} |u_k\rangle\langle u_k| dk = 1 \quad \not\Rightarrow \quad \sum_k |SD_k\rangle\langle SD_k| \cong 1$$

bound states

non-resonant (real) continuum

but… different emission thresholds open at distinctly different energies



$Q = [A]$

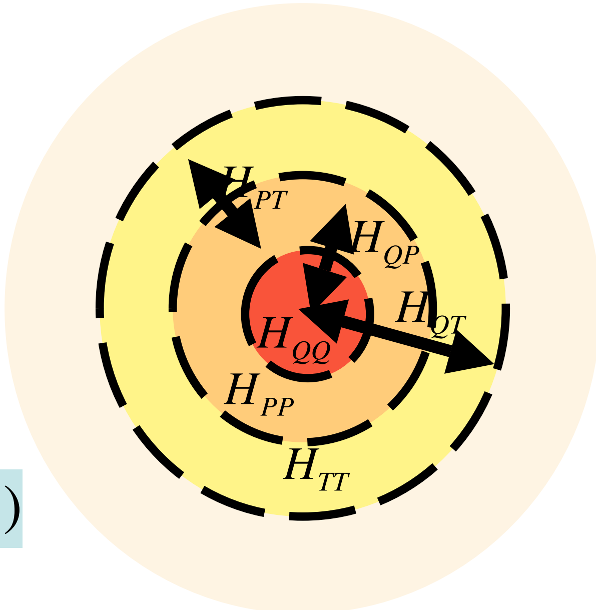
$P = [A-1] \otimes [1]$

$T = [A-2] \otimes [2]$

$\ldots = \ldots$

$Q + P + T + \ldots = 1$

$H_{QQ} \rightarrow H_{QQ}^{eff}(E)$



Incompatible symmetries of H_{QQ} and $H_{QQ}^{eff}(E)$

Rigged Hilbert space formulation : Gamow Shell Model (2002)

$$\hat{H}\Psi = \left(e - i\frac{\Gamma}{2}\right)\Psi \quad : \qquad \Psi(0,k) = 0 \;\; , \;\; \Psi(\vec{r},k) \underset{r\to\infty}{\longrightarrow} O_l(kr)$$

outgoing solution

Eigenvalues : $k_n = \sqrt{\frac{2m}{\hbar^2}\left(e_n - i\frac{\Gamma_n}{2}\right)}$ are the poles of the S-matrix :

- Bound states $(k_n = i\kappa_n)$
- Antibound states $(k_n = -i\kappa_n)$
- Resonances $(k_n = \pm\gamma_n - i\kappa_n)$

Completness relation for one-body states:

(T.Berggren (1968))

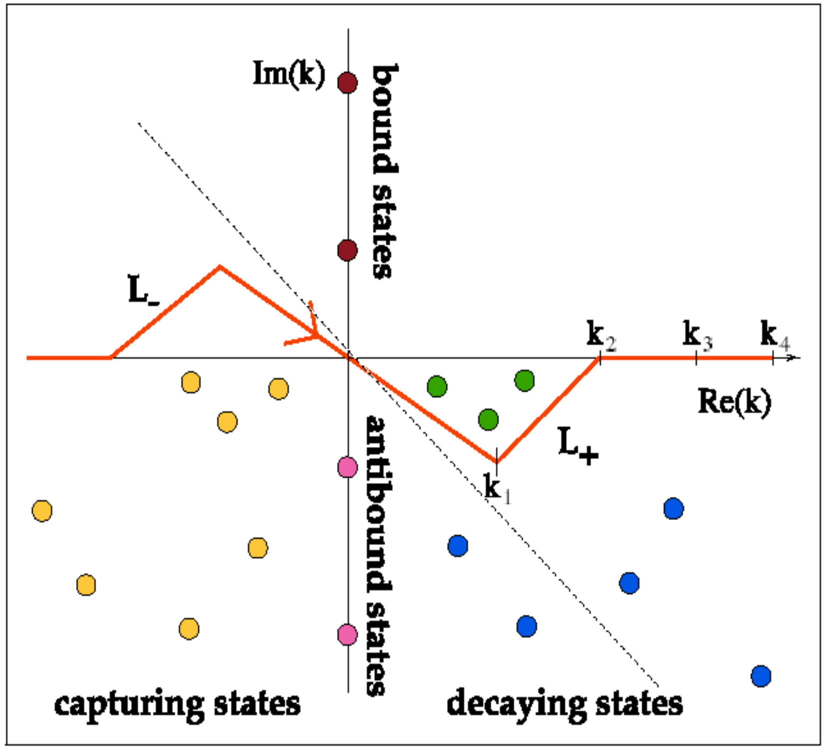
$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \int_{L_+} |u_k\rangle\langle\tilde{u}_k|\, dk = 1$$

bound, anti-bound, and resonance states

non-resonant continuum

$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \sum_{i=1}^{N_d} |u_i\rangle\langle\tilde{u}_i| \cong 1 \; ; \quad \langle u_i|\tilde{u}_j\rangle = \delta_{ij}$$

$$\sum_k |SD_k\rangle\langle SD_k| \cong 1$$



complex-symmetric eigenvalue problem for hermitian Hamiltonian

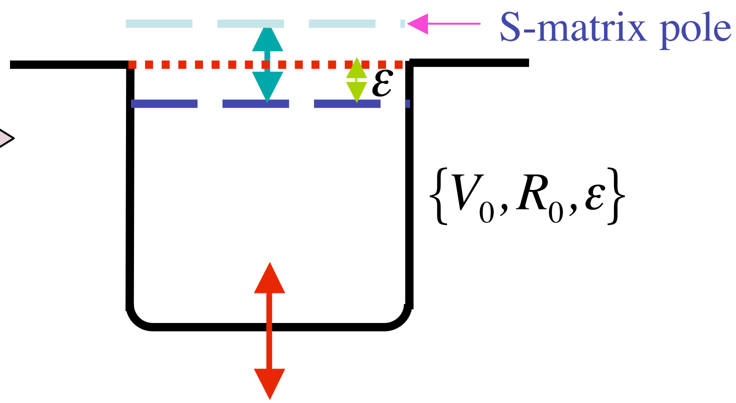
Influence of the poles of the scattering matrix on spectra : simple model

$$E_i^{(corr)}(E,\varepsilon) = \langle \Phi_i | H_{QQ}^{eff}(E,\varepsilon) - H_{QQ} | \Phi_i \rangle$$

Q, P -subspaces

total energy (E); position of S-matrix pole (ε)

- Φ_i - state in Q with angular momentum ℓ
- $H_{cc} = T + V(r)$ - channel Hamiltonian in P ⟹
- $Q - P$ coupling : $w(r) \propto r^{\mu} \left(\mu \geq \ell + 1\right)$ for $r \leq R_0$

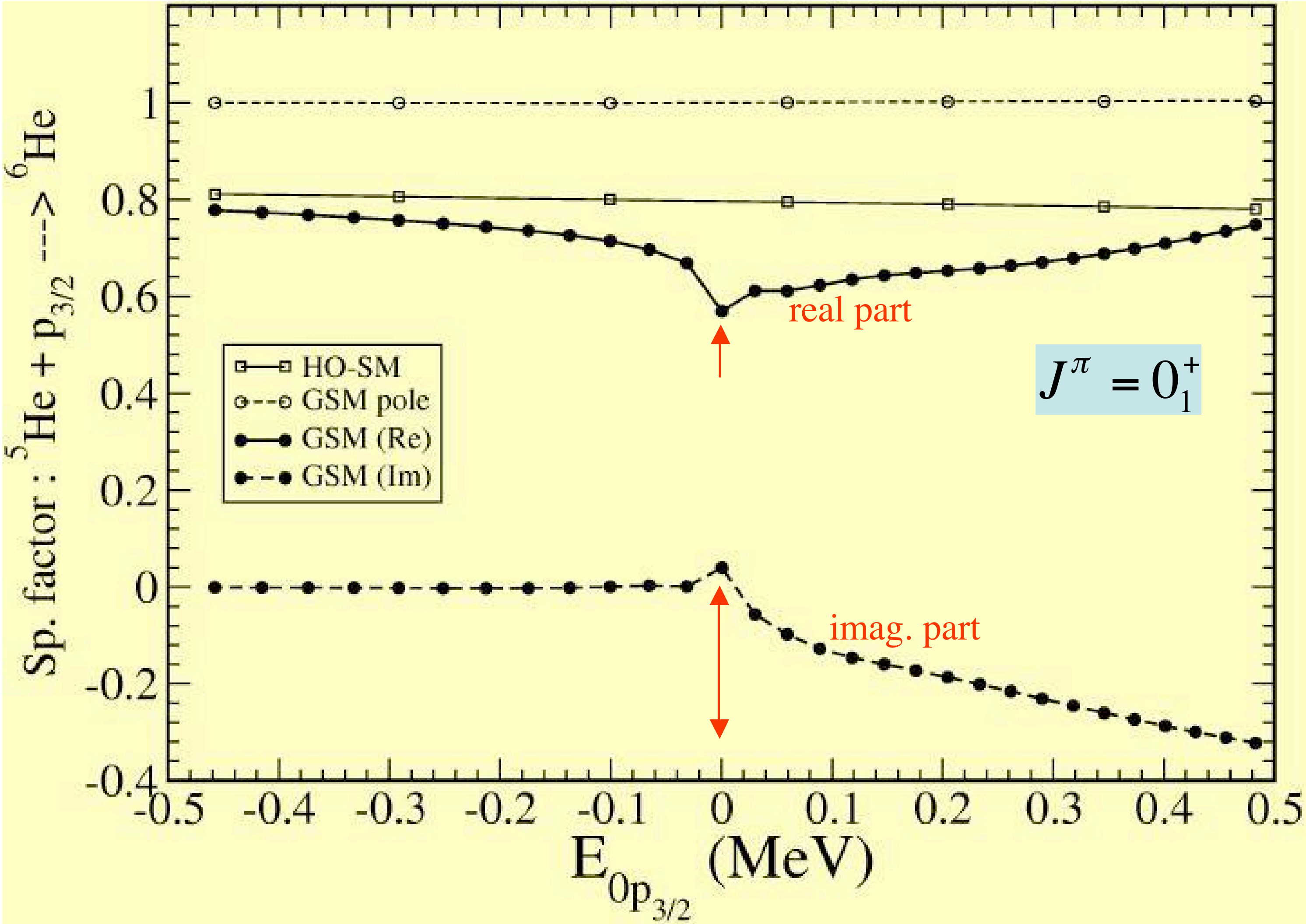


$$E_\ell^{(corr)}\left(E = 0, \varepsilon\right) = -const|\varepsilon|^{-1+\ell/2} + O\left(|\varepsilon|^0\right)$$

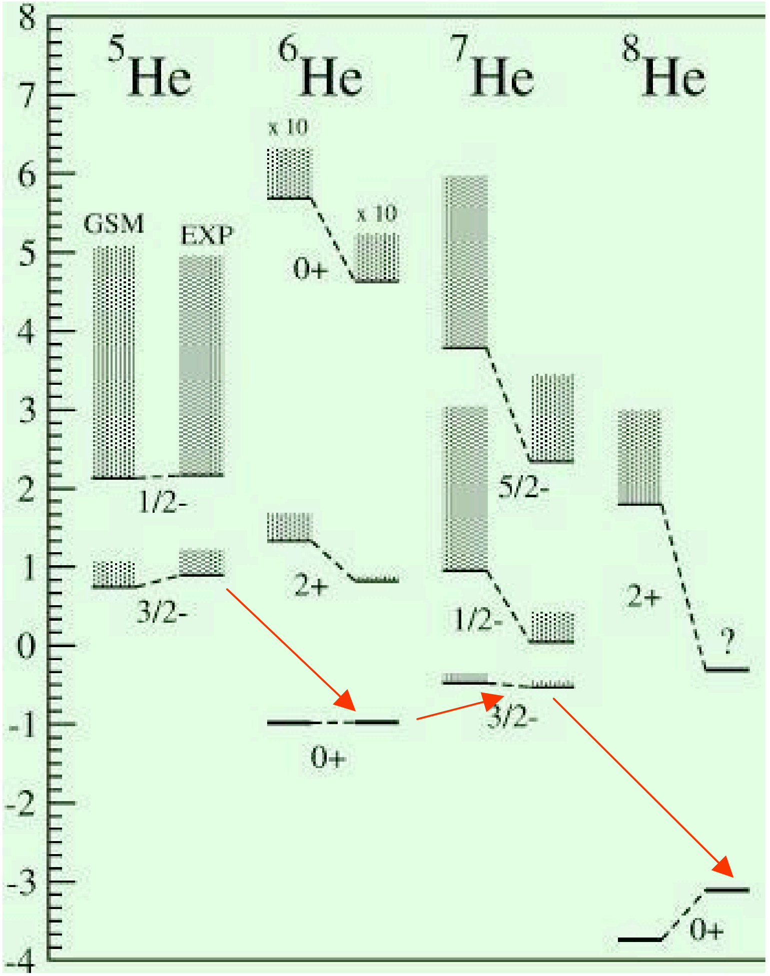
Singularity for $\ell = 0,1$ poles of the S-matrix!

⟹ ‘Non-perturbative’ continuum coupling : instability of the Q subspace

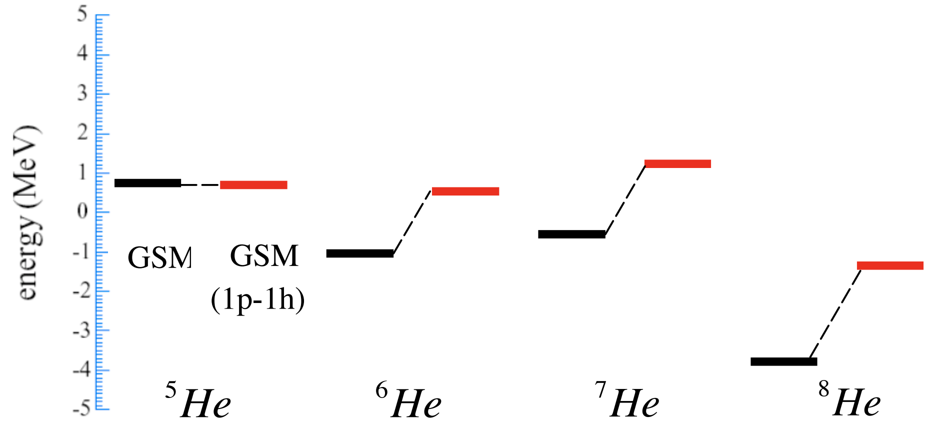
Spectroscopic factors (I)



$$S_{(lj)} = \frac{1}{2J_A + 1}\left[\sum_{n \in (b,r)} \langle \Psi_A \| a^+(n;lj) \| \Psi_{A-1} \rangle^2 + \underbrace{\int_{L^+} \langle \Psi_A \| a^+(k;lj) \| \Psi_{A-1} \rangle^2 dk}_{\text{continuum contribution}} \right]$$



‘Helium anomaly’

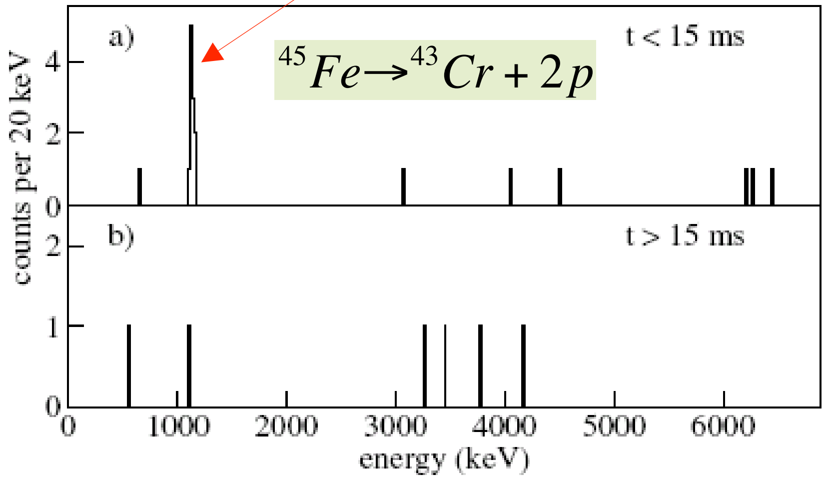


$H = T + U_{WS} + V_{JT}$ → h_{HF} → s.p. basis (continuum)

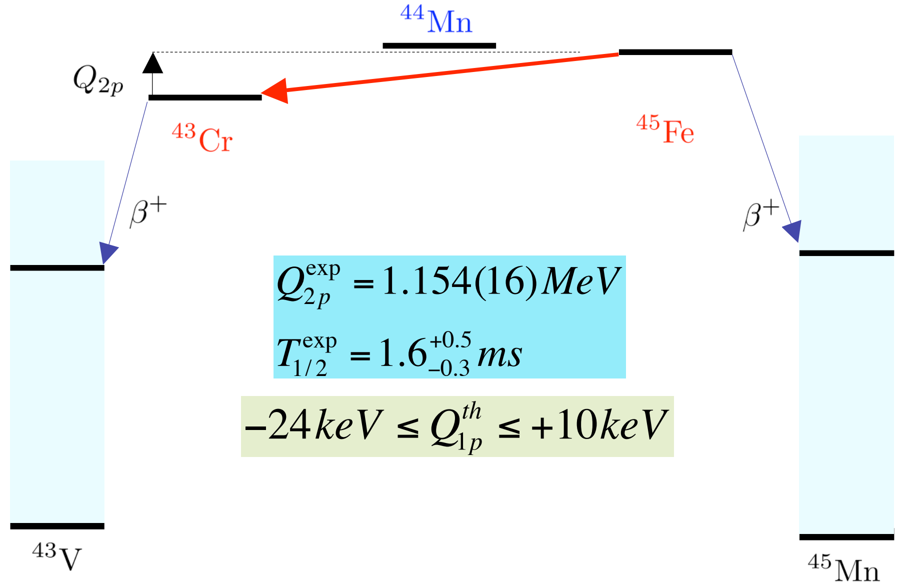
Interaction of nucleons in the continuum states is an essential element of binding mechanism in helium isotopes

2p decay from the ground state of ^{45}Fe, ^{48}Ni and ^{54}Zn

M. Pfutzner et al, Eur. Phys. J. A14 (2002) 279; J. Giovinazzo et al, Phys. Rev. Lett. 89 (2002) 102501



$$^{45}Fe \rightarrow ^{43}Cr + 2p$$

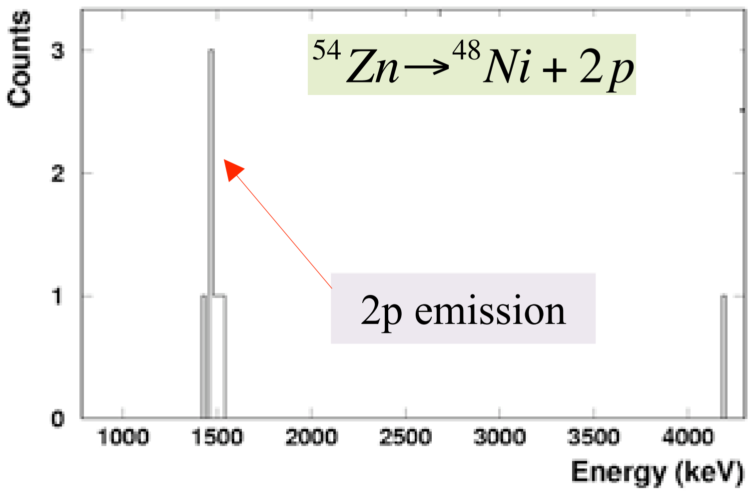


$$Q_{2p}^{exp} = 1.154(16)\,MeV$$

$$T_{1/2}^{exp} = 1.6^{+0.5}_{-0.3}\,ms$$

$$-24\,keV \le Q_{1p}^{th} \le +10\,keV$$

B. Blank et al, Phys. Rev. Lett. 94 (2005) 232501



$$^{54}Zn \rightarrow ^{48}Ni + 2p$$

$$Q_{2p}^{exp} = 1.48(2)\,MeV$$

$$T_{1/2}^{exp} = 3.7^{+2.2}_{-1.0}\,ms$$

Two-proton emission

$$H_{QQ}^{eff}(E) = H_{QQ} + H_{QP}G_P^{(+)}(E)H_{PQ} + [H_{QT} + H_{QP}G_P^{(+)}(E)H_{PT}]\tilde{G}_T^{(+)}(E)[H_{TQ} + H_{TP}G_P^{(+)}(E)H_{PQ}]$$

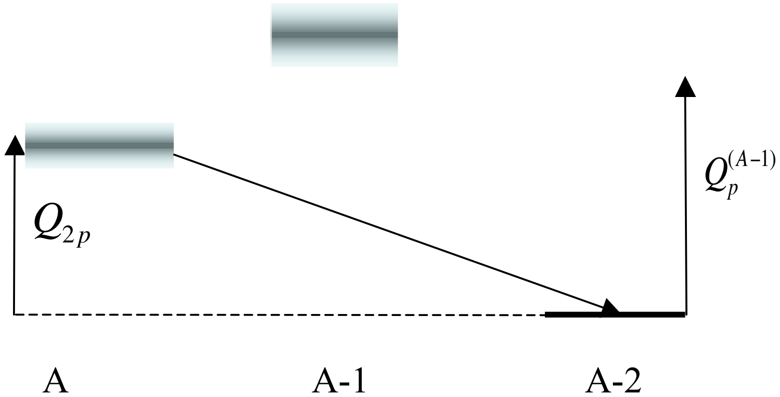
coupling with one particle in the continuum

coupling with two particles in the continuum

I. Direct emission:

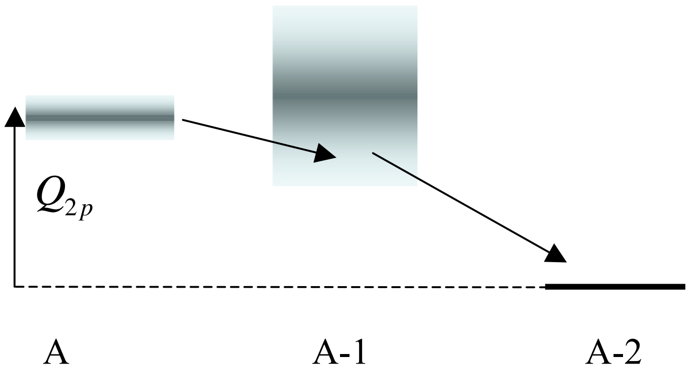
$$\{H_{QP}\ ,\ H_{PT}\} \rightarrow 0$$

$$\rightarrow\quad H_{QQ}^{eff}(E) = H_{QQ} + H_{QT}G_T^{(+)}(E)H_{TQ}$$



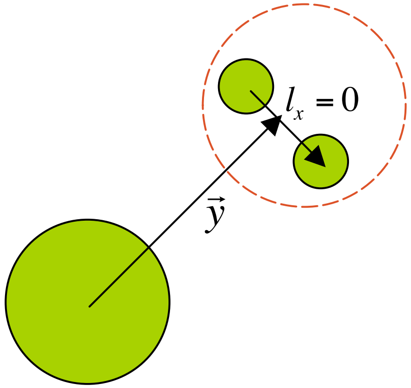
II. Indirect emission :

$$\{H_{QT}\} \rightarrow 0$$



$$\rightarrow\quad H_{QQ}^{eff}(E) = H_{QQ} + H_{QP}G_P^{(+)}(E)H_{PQ} + [H_{QP}G_P^{(+)}(E)H_{PT}]\tilde{G}_T^{(+)}(E)[H_{TP}G_P^{(+)}(E)H_{PQ}]$$

Direct 2p emission



+ final state interaction in terms of $(l_x = 0)$ s-wave phase shift

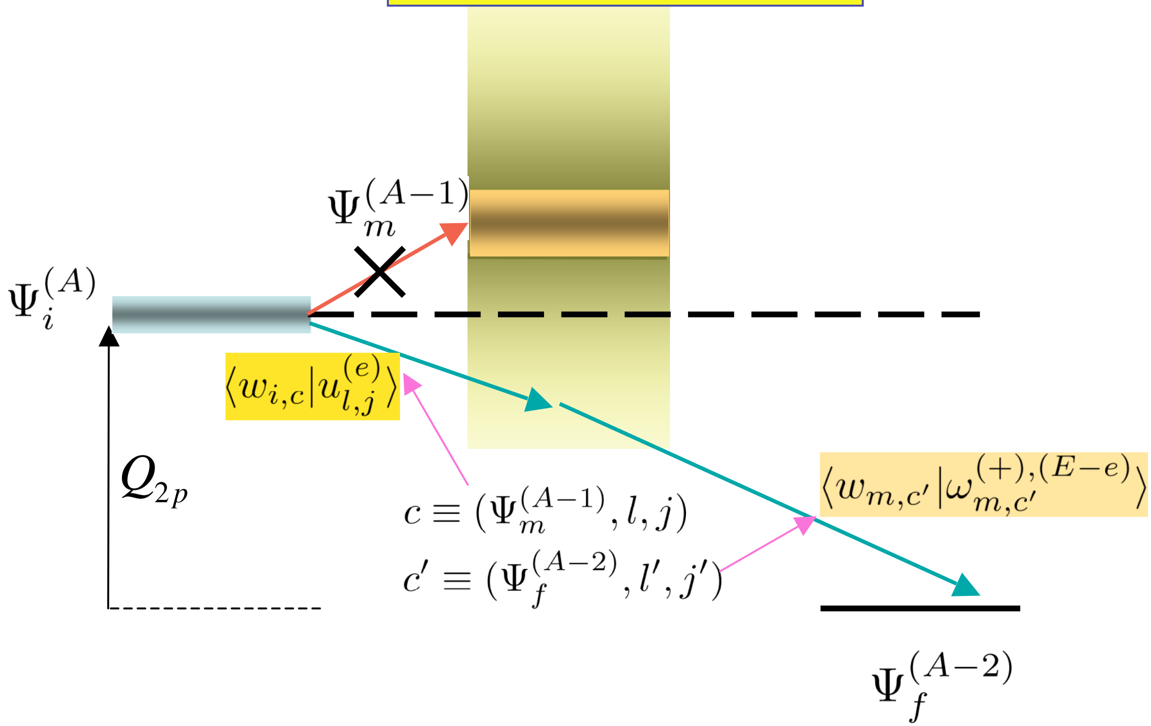
Diproton emission channel:

$$c = \left(\theta^{(\mathrm{int})}_{J_f^{(A-2)}}, l_x = 0, S = 0, L = 0\right)$$

$$\Gamma = \int_0^{Q_{2p}} \Gamma(U)\rho(U)\,dU$$

$$\Gamma(U) = -2Im\left[\left\langle \omega^{T,(+)}_{i,U} \middle| w^T_i \right\rangle\right]$$

Indirect 2p emission



$^{45}_{26}Fe \rightarrow ^{43}_{24}Cr + 2p$ $\qquad J_i^\pi = 3/2^+, J_f^\pi = 3/2^+$

Direct 2p decay

IOKIN eff. interaction

SMEC:

Q_{2p}(MeV)	$T_{1/2}$(ms) no-mixing	$Q_{1p} = -0.1$(MeV)	$Q_{1p} = +0.05$(MeV)
1.138	21.5	19.80	19.77
1.154	13.3	12.30	12.28
1.170	8.4	7.72	7.71

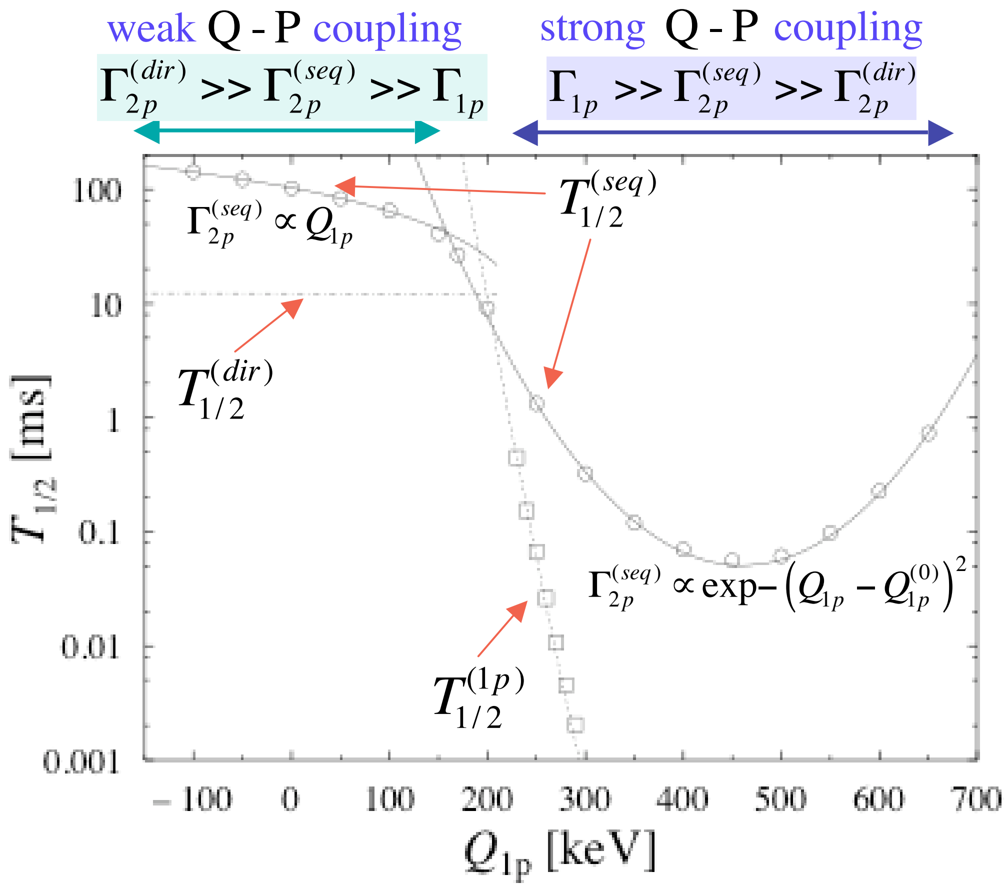
Exp: $T_{1/2} = 1.6^{+0.5}_{-0.3}(ms)$

Indirect 2p decay

SMEC:

Q_{2p}(MeV)	$T_{1/2}$(ms) $Q_{1p} = -0.1$(MeV) no-mixing	$Q_{1p} = -0.1$(MeV)	$Q_{1p} = +0.05$(MeV)
1.138	273.2	199.2	115.6
1.154	175.2	127.6	74.8
1.170	113.8	82.8	49.0

$^{45}_{26}Fe \rightarrow ^{43}_{24}Cr + 2p$



$^{48}_{28}Ni \rightarrow ^{46}_{26}Fe + 2p$

Exp: $T_{1/2} = 8.4^{+12.8}_{-7.0} (ms)$

SMEC: $T_{1/2}^{(dir)} = 6.2^{+4.1}_{-2.5} (ms)$ $T_{1/2}^{(seq)} = 19.3^{+10.8}_{-6.8} (ms)$

$^{54}_{30}Zn \rightarrow ^{52}_{28}Ni + 2p$

Exp: $T_{1/2} = 3.7^{+2.2}_{-1.0} (ms)$

SMEC: $T_{1/2}^{(dir)} = 13.8^{+8.4}_{-5.1} (ms)$ GXFP1 eff. interaction

Conclusions

- Continuum shell model : Gamow (complex-energy) Shell Model or Shell Model Embedded in the Continuum, provide a consistent description of the structure of weakly bound nuclei

- New exotic phenomena in weakly bound nuclei : continuum anti-odd-even staggering effect, modification of ‘magic numbers’, spin-orbit splitting, halos & correlations, symmetry-breaking effects due to the proximity of continuum, influence of the scattering matrix poles on the spectra and wave functions (the spectroscopic factors), new kinds of radioactivity (e.g. 2p-radioactivity), …

  Nuclear structure enters in a new, exciting era!