NUCLEAR HIGH-RANK SYMMETRIES THROUGHOUT THE PERIODIC TABLE

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In the Program for Today:

- High-Rank Point-Group Symmetries in Nuclei: a Summary

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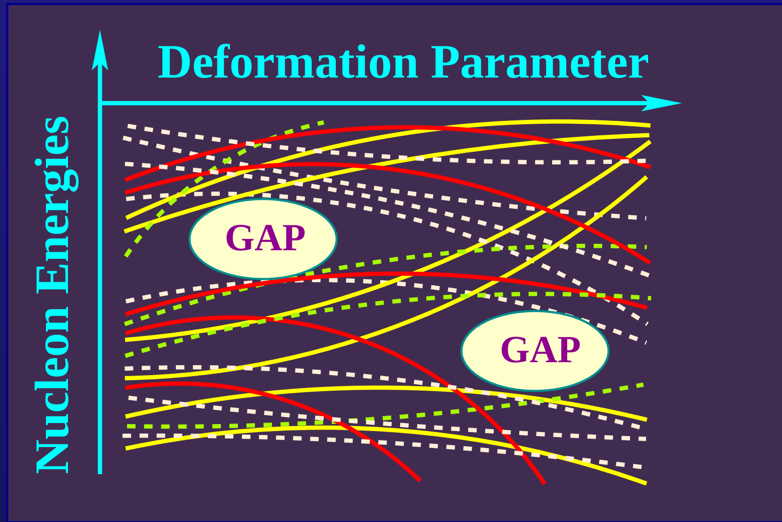
- Abundance of Tetrahedral Nuclei Throughout Periodic Table
- New Universal Parametrisations of the Nuclear Mean-Fields

Nuclear Mean-Field and Exotic Deformations

Deformation-parameter axis represents usually several degrees of freedom. The presence of the sufficiently strong gaps may (but does not need to) signify the onset of the shape coexistence.

Here we will be interested in special shell gaps: those corresponding to the exotic, high-rank symmetries.



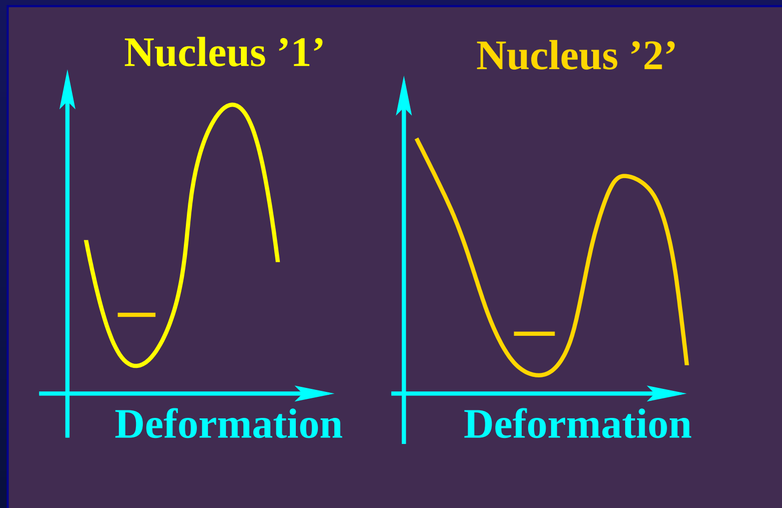


Figure 1: Single particle gaps and total energies

Exotic Symmetries - High-Rank Point Groups

- Consider Hamiltonian $\hat{\mathcal{H}} = \hat{\mathcal{H}}(\vec{r}, \vec{p}, \vec{s}; \hat{\alpha})$ with $\hat{\alpha} \equiv \{\alpha_{\lambda,\mu}\}$

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$$[\hat{\mathcal{H}}, \hat{\mathcal{O}}_k] = 0 \quad \text{with} \quad k = 1, 2, \ldots f.$$

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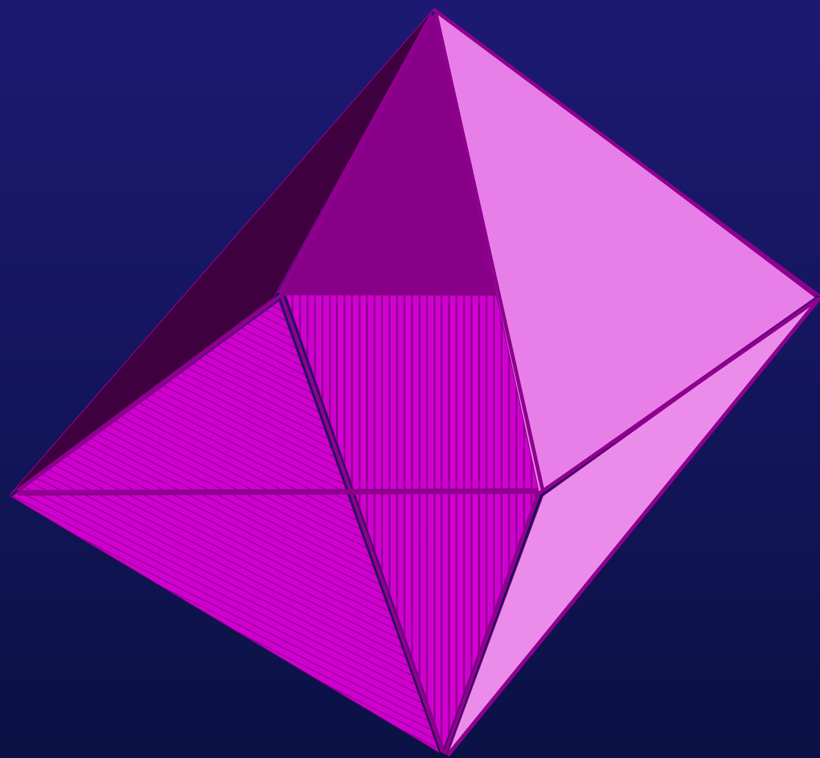
appear in multiplets: d_1-fold degenerate, d_2-fold degenerate, ... etc.

Introducing Octahedral Symmetry

- Octahedral symmetry is most commonly associated with a shape of an octahedron ('diamond').



An octahedron has 8 equal walls. Its shape is invariant with respect to 48 symmetry elements including inversion. However, the nuclear surface cannot be represented in the form of a diamond...

- ... but rather in a form of a regular expansion:

$$\mathcal{R}(\vartheta,\varphi) = R_0 c(\{\alpha\})[1+\sum_{\lambda}^{\lambda_{max}} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda,\mu} Y_{\lambda,\mu}(\vartheta,\varphi)]$$

A Basis for Octahedral Symmetry

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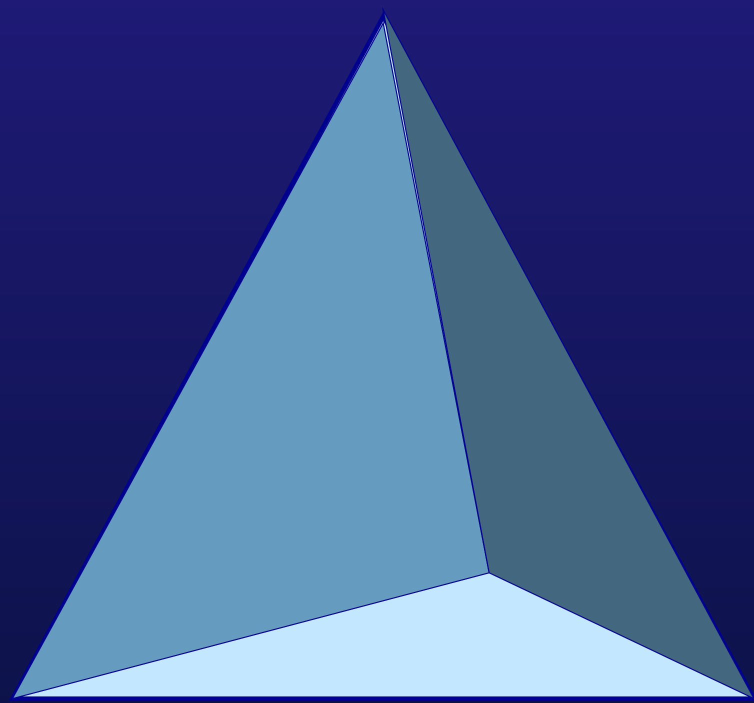
- The third order is characterised by $\lambda = 8$

$$\alpha_{80} \equiv o_8; \quad \alpha_{8,\pm 4} \equiv \sqrt{\frac{28}{198}} \cdot o_8; \quad \alpha_{8,\pm 8} \equiv \sqrt{\frac{65}{198}} \cdot o_8$$

Introducing Tetrahedral Symmetry

- Tetrahedral symmetry is most commonly associated with a shape of a tetrahedron ('pyramid' shape).

A tetrahedron has 4 equal walls. Its shape is invariant with respect to 24 symmetry elements. Tetrahedron is *not* invariant with respect to inversion.



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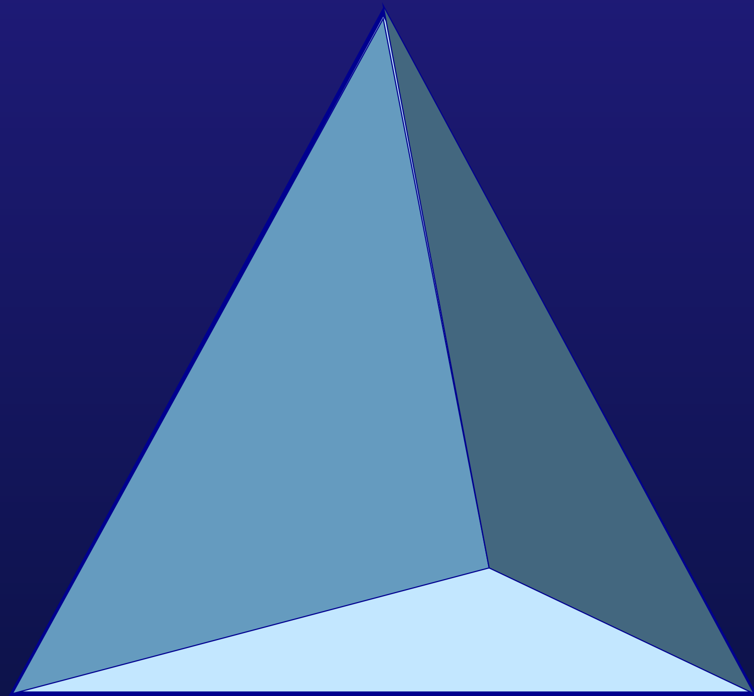

- The first order tetradral deformation is characterised by $\lambda = 3$ and we have

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- The second order is characterised *only* by $\lambda = 7$ *($\lambda = 5$ missing!)*

$$\alpha_{7,\pm 2} \equiv t_7 \quad \text{and} \quad \alpha_{7,\pm 6} \equiv -\sqrt{\frac{11}{13}} \cdot t_7$$

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- The third order is characterised by $\lambda = 9$

$$\alpha_{9,\pm 2} \equiv t_9 \quad \text{and} \quad \alpha_{9,\pm 6} \equiv +\sqrt{\frac{13}{3}} \cdot t_9$$

Symmetry-Oriented Mean-Field Approach [1]

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*) Dudek, Góźdź, Schunck, Acta Phys. Polon. B34, 2491 (2003)

Tetrahedral Symmetry - Surprises

- Usually it is expected that the higher the multipolarity of the deformation the less important the energy contribution

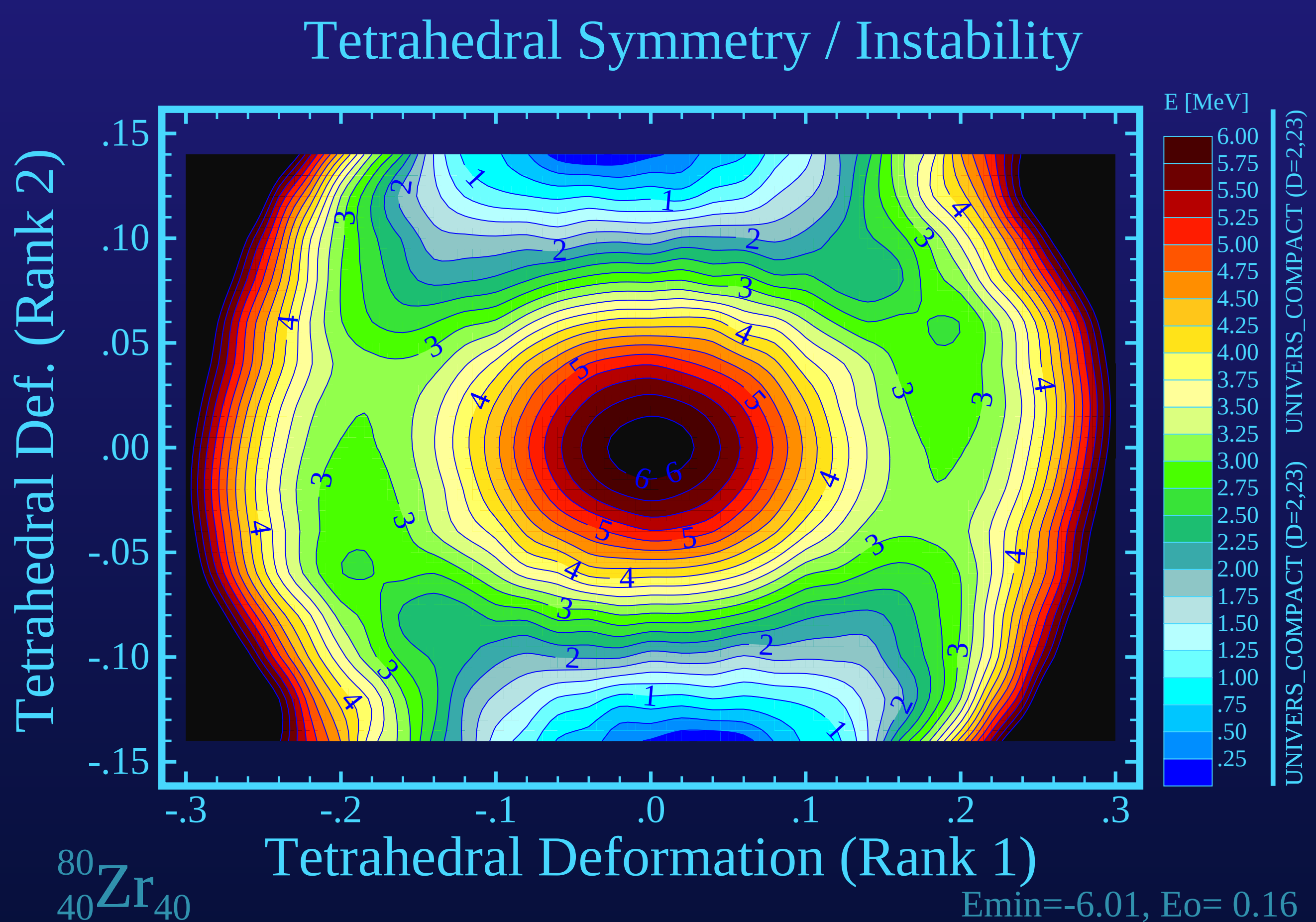
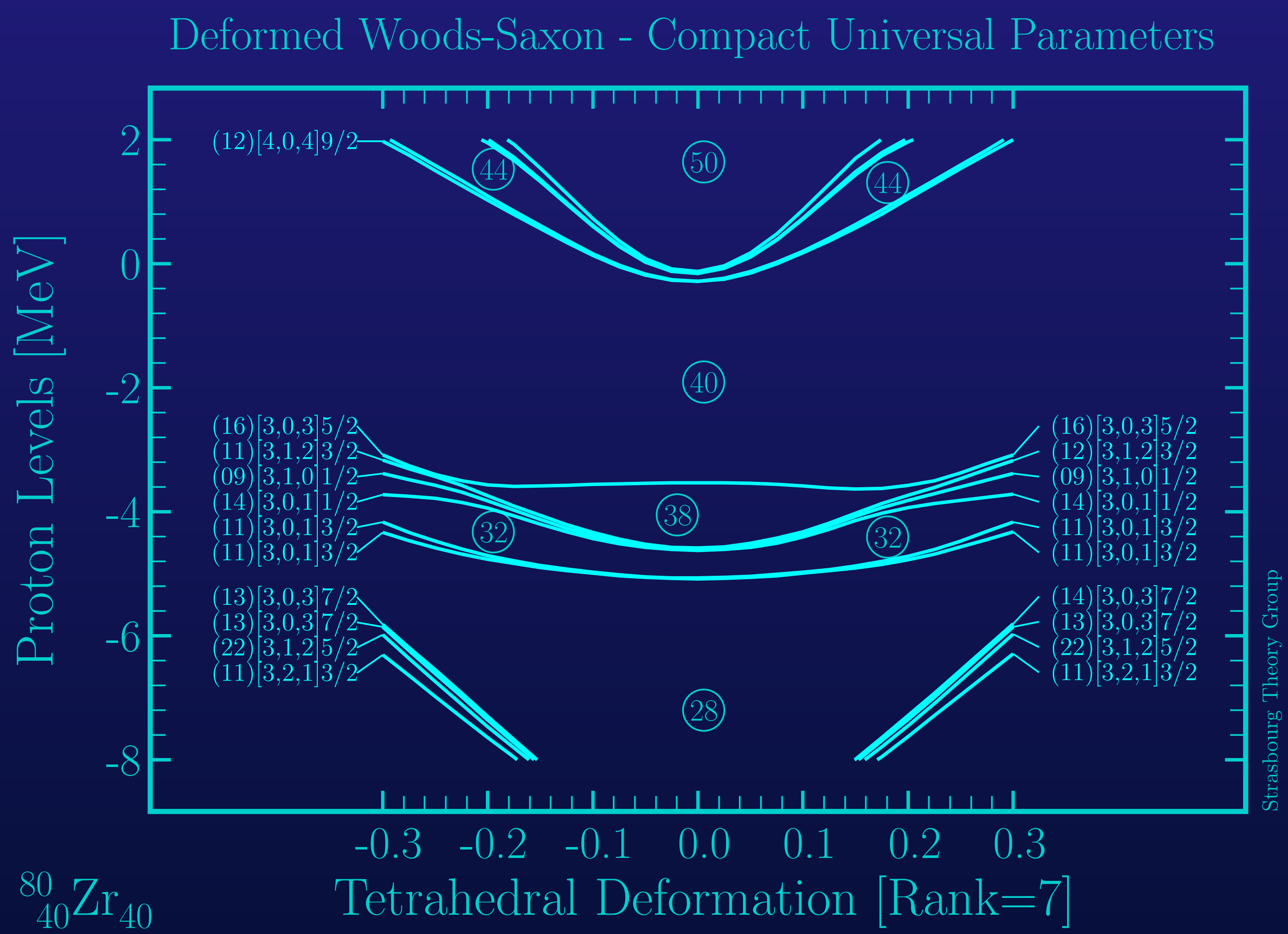


Figure 4: Total energy according to Universal-Compact parametrisation; Strutinsky and Yukawa-Folded techniques. Neighbouring nuclei manifest similar features.

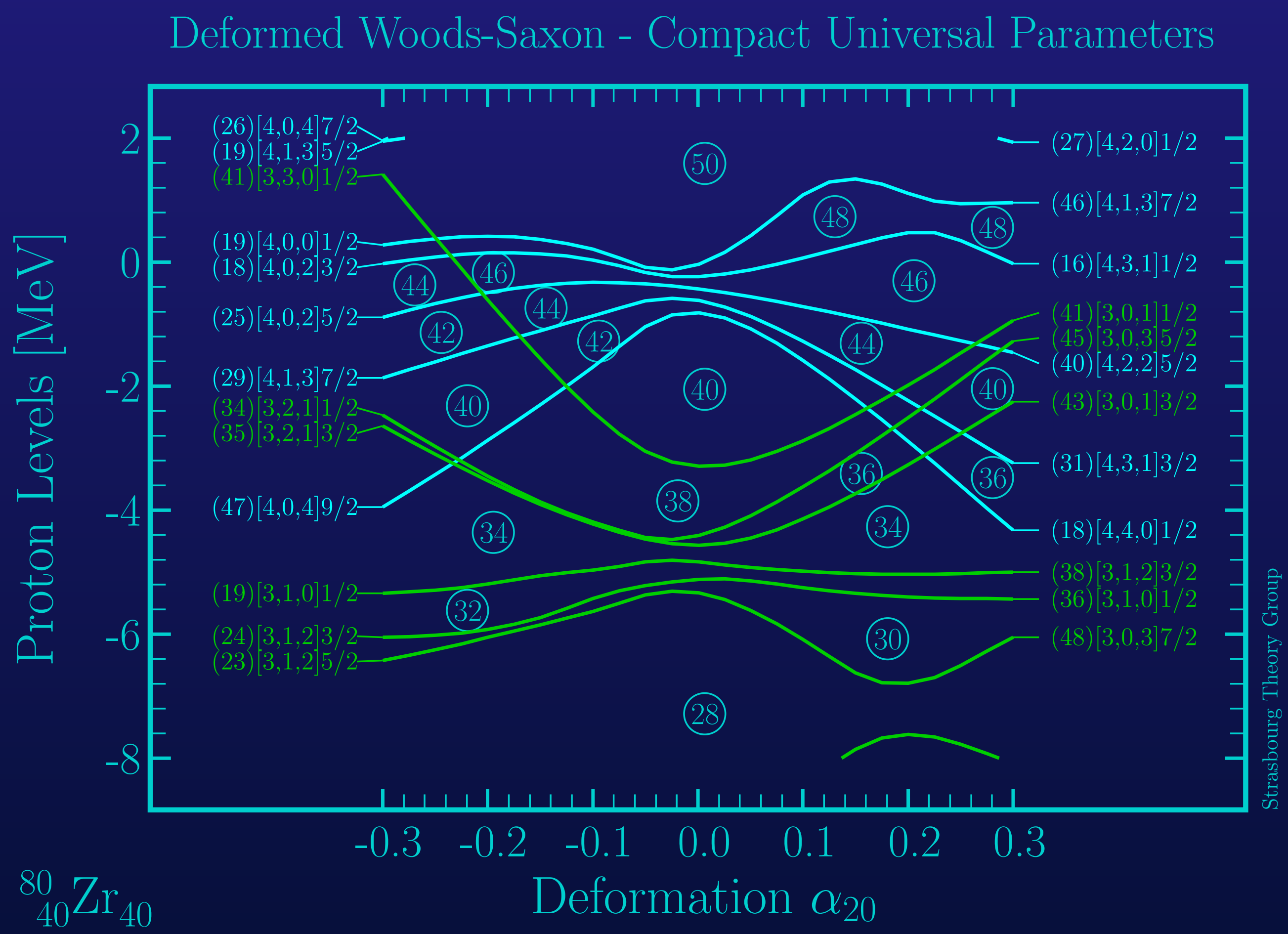
Powerful Impact of the Symmetry-Oriented Bases

- Consider tetrahedral-symmetry shells driven by rank=7 shapes



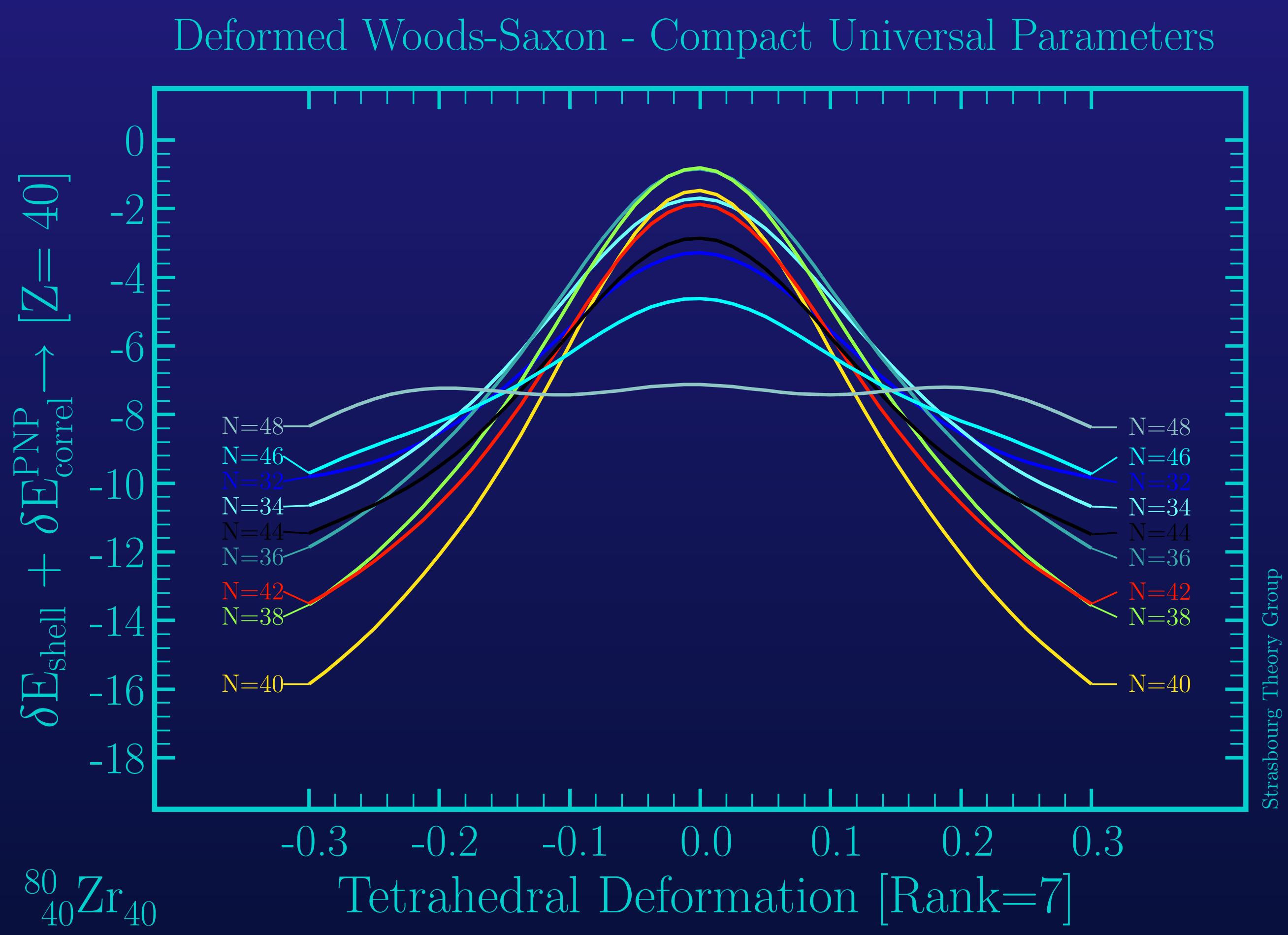
Powerful Impact of the Symmetry-Oriented Bases

- ... and compare them with the 'miserable quadrupole structures':



Extremely Profitable Symmetry-Explorations

- The shell and pairing (here PNP) effects are extremely strong:



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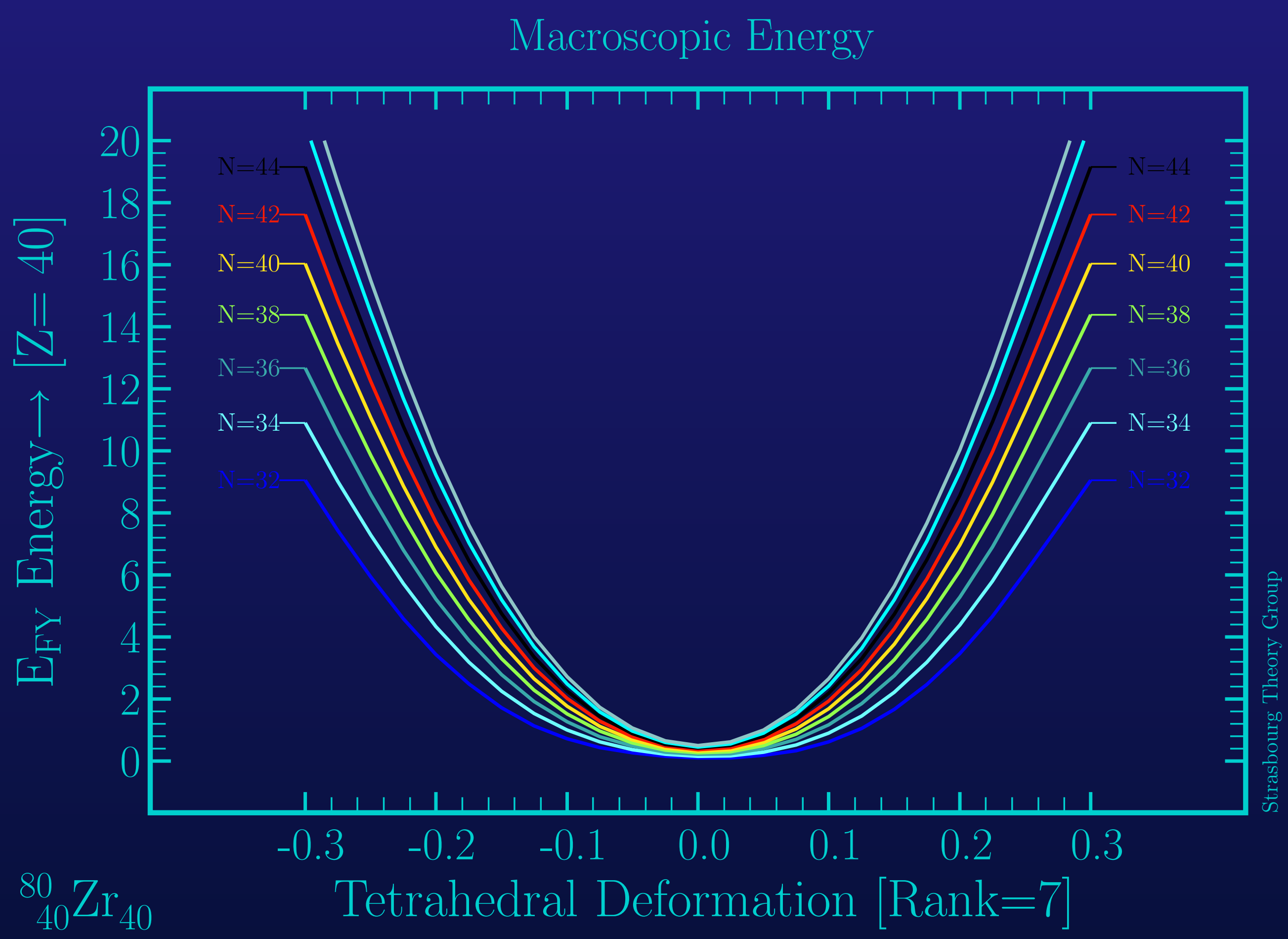
- The quantum effects must compete against the macroscopic ones:



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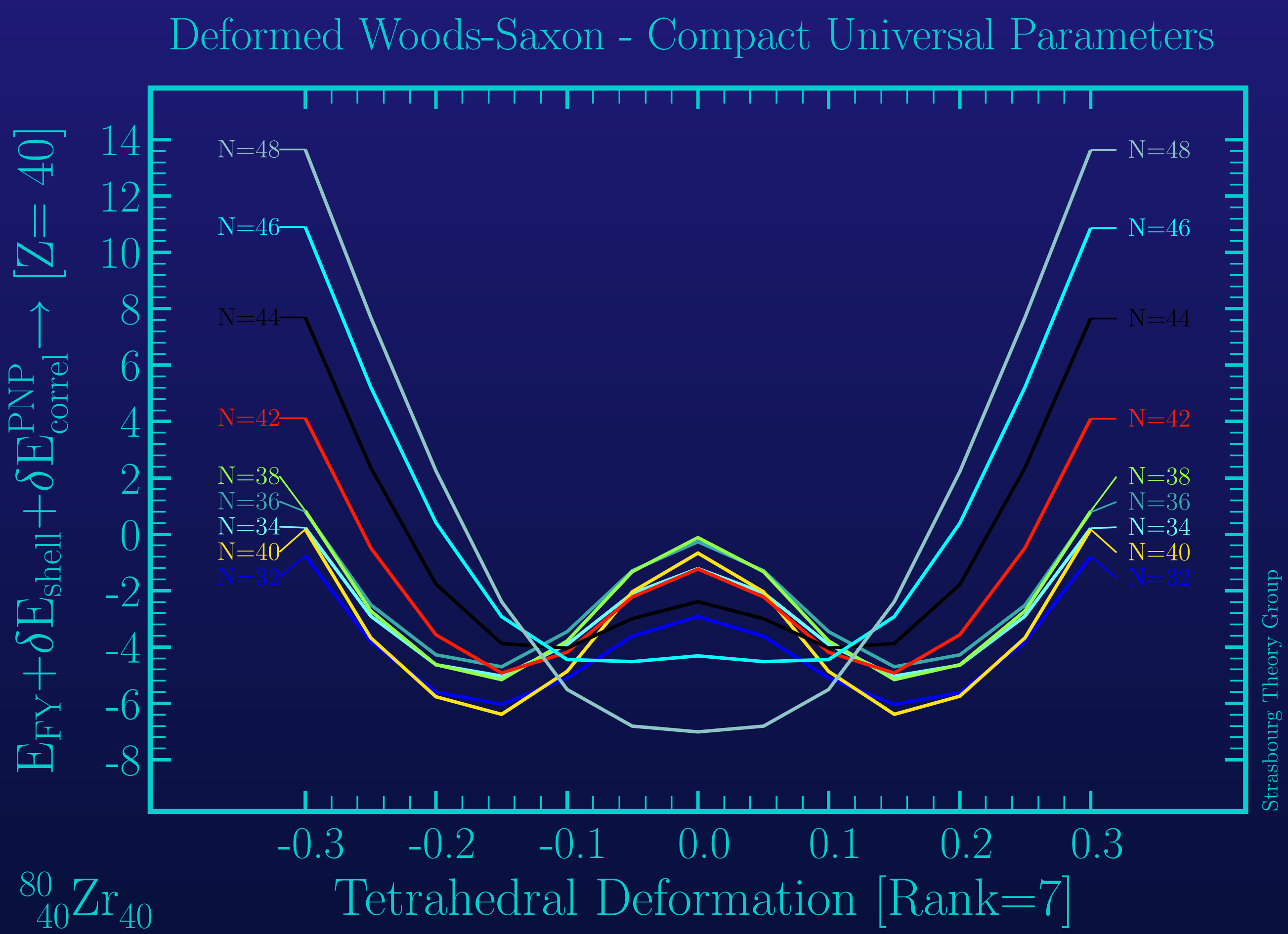
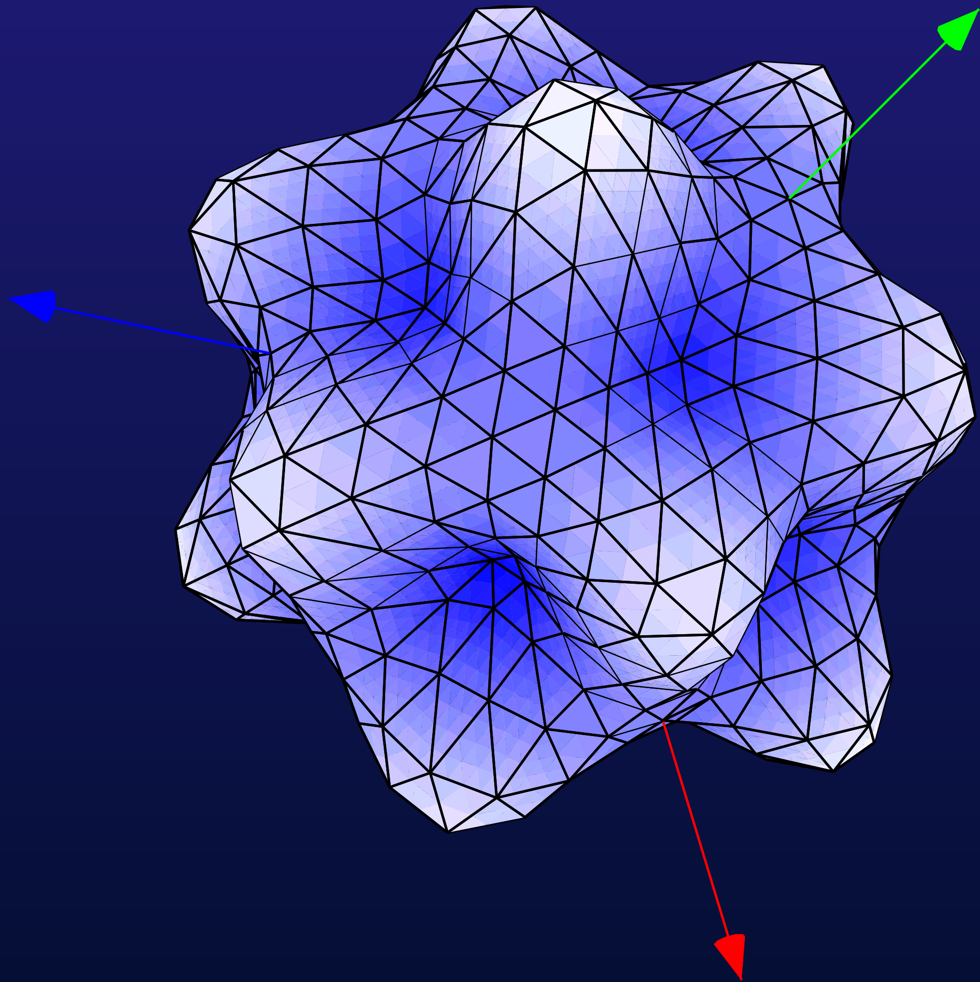
- ... so that there remains a lot of room for a compromise:



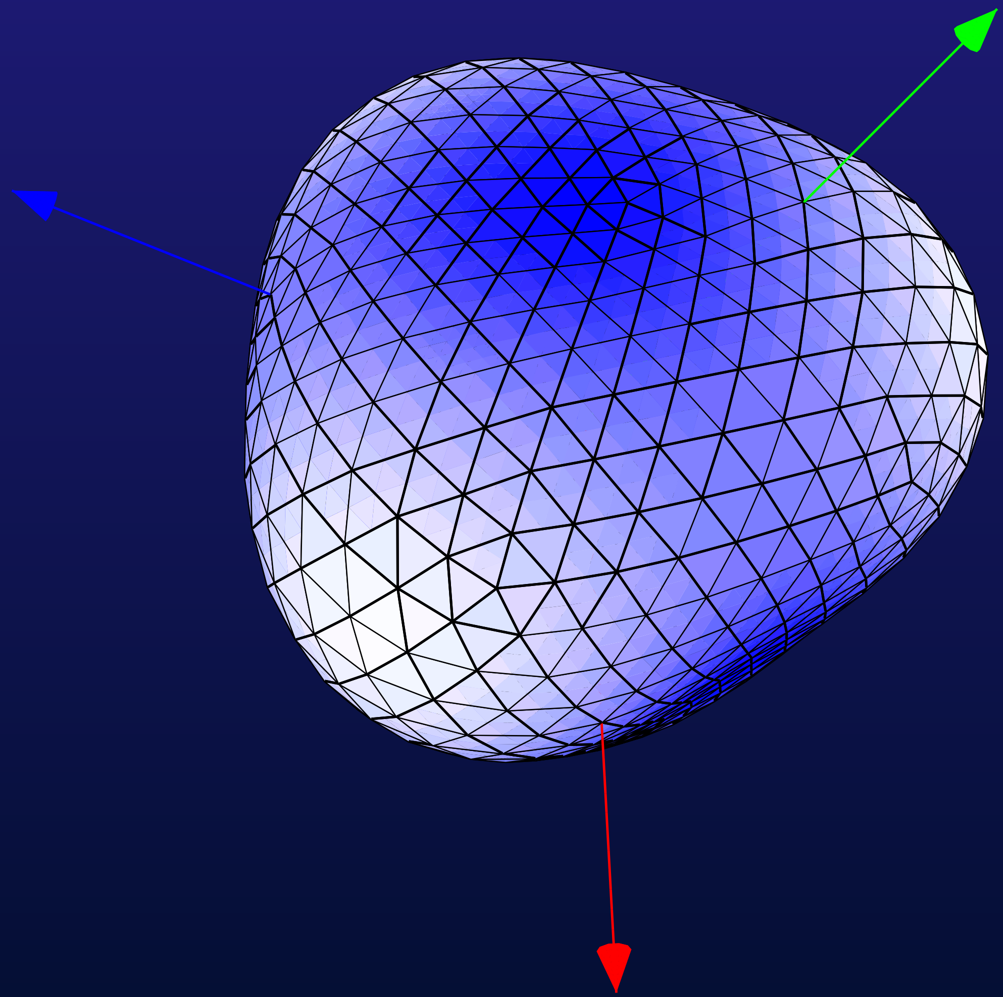
Underlying Shapes Are Exotic Indeed...

- Slightly exaggerated view of a $t_2 \sim 0.16$ nucleus: here $t_2 = 0.24$



Symmetry-Oriented Mean-Field Approach [2]

- Consider a nuclear surface with a tetrahedral deformation:



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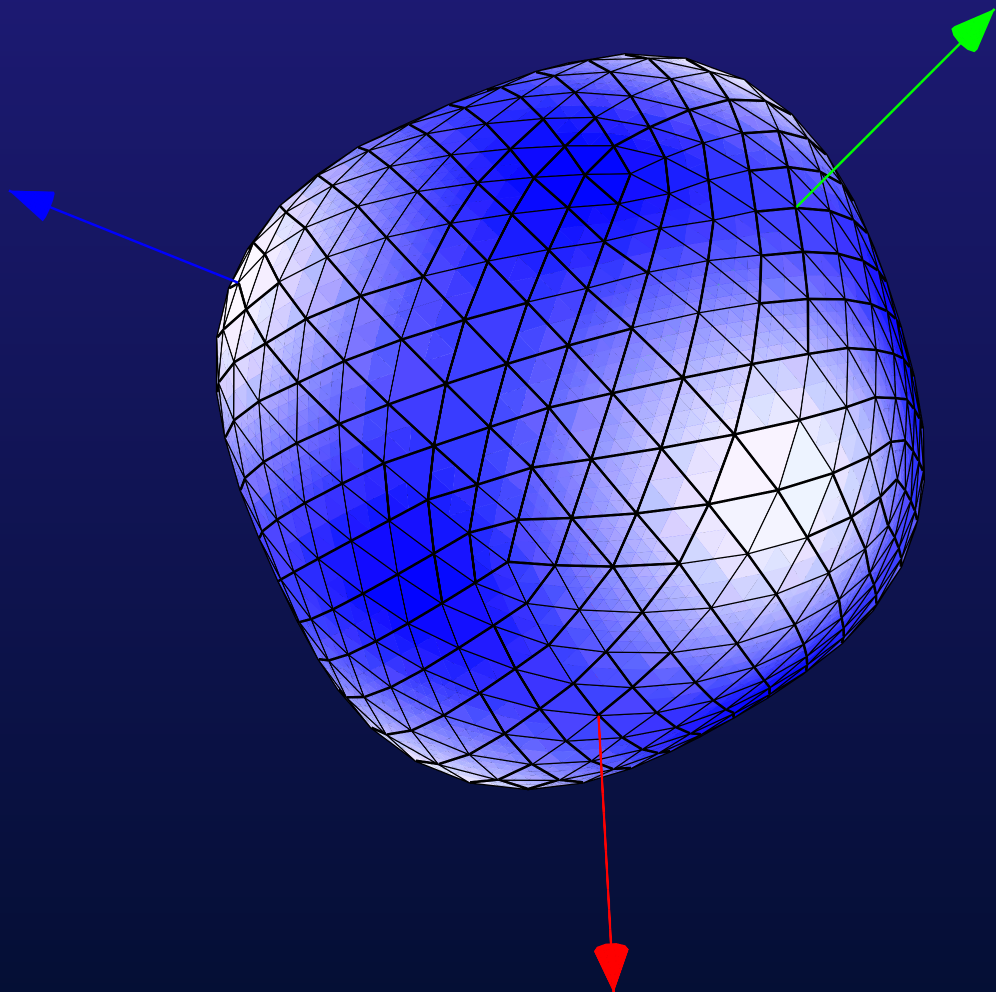
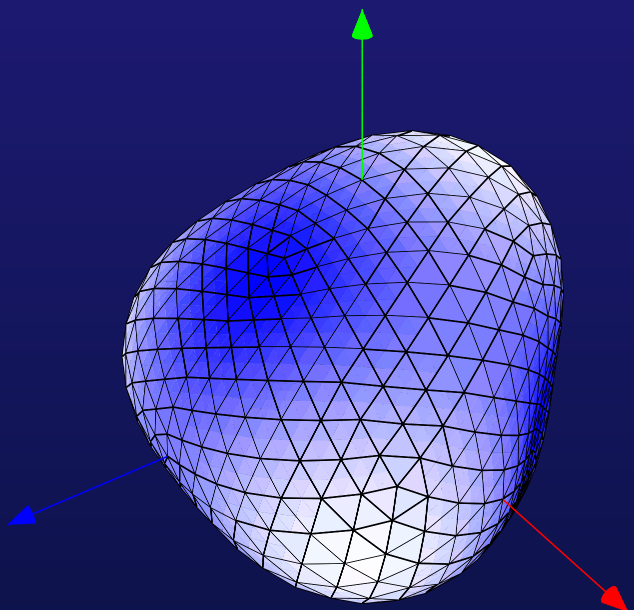
- ... and another nuclear surface with an octahedral deformation:



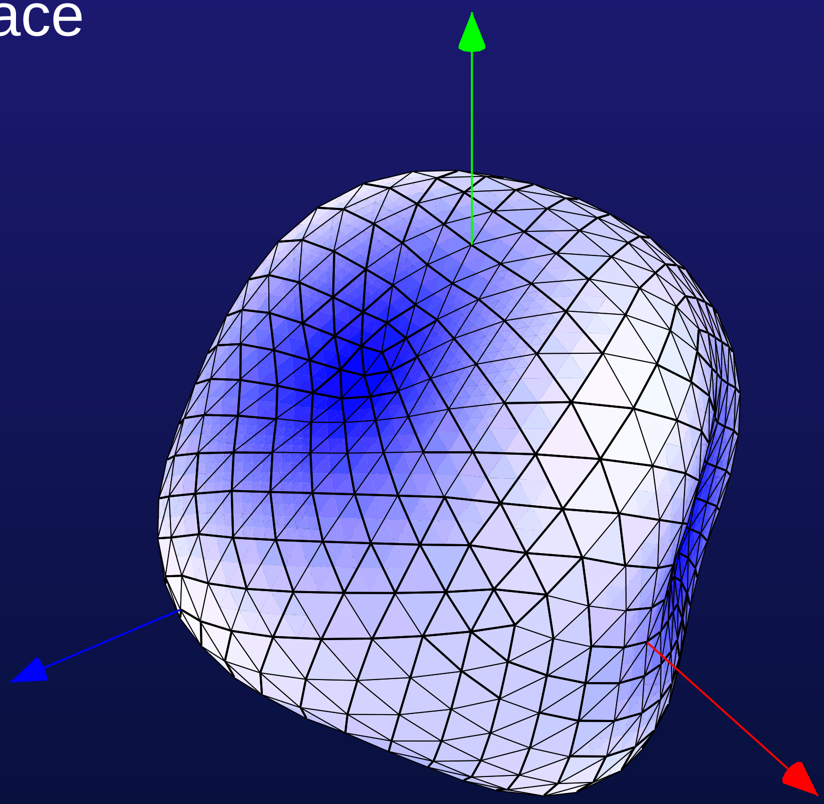
Symmetry-Oriented Mean-Field Approach [2]

- ... or even better, compare them directly ...



Symmetry-Oriented Mean-Field Approach [2]

- A superposition of appropriately oriented tetrahedral-symmetric surface with an octahedral-symmetric surface is a tetrahedral-symmetric surface



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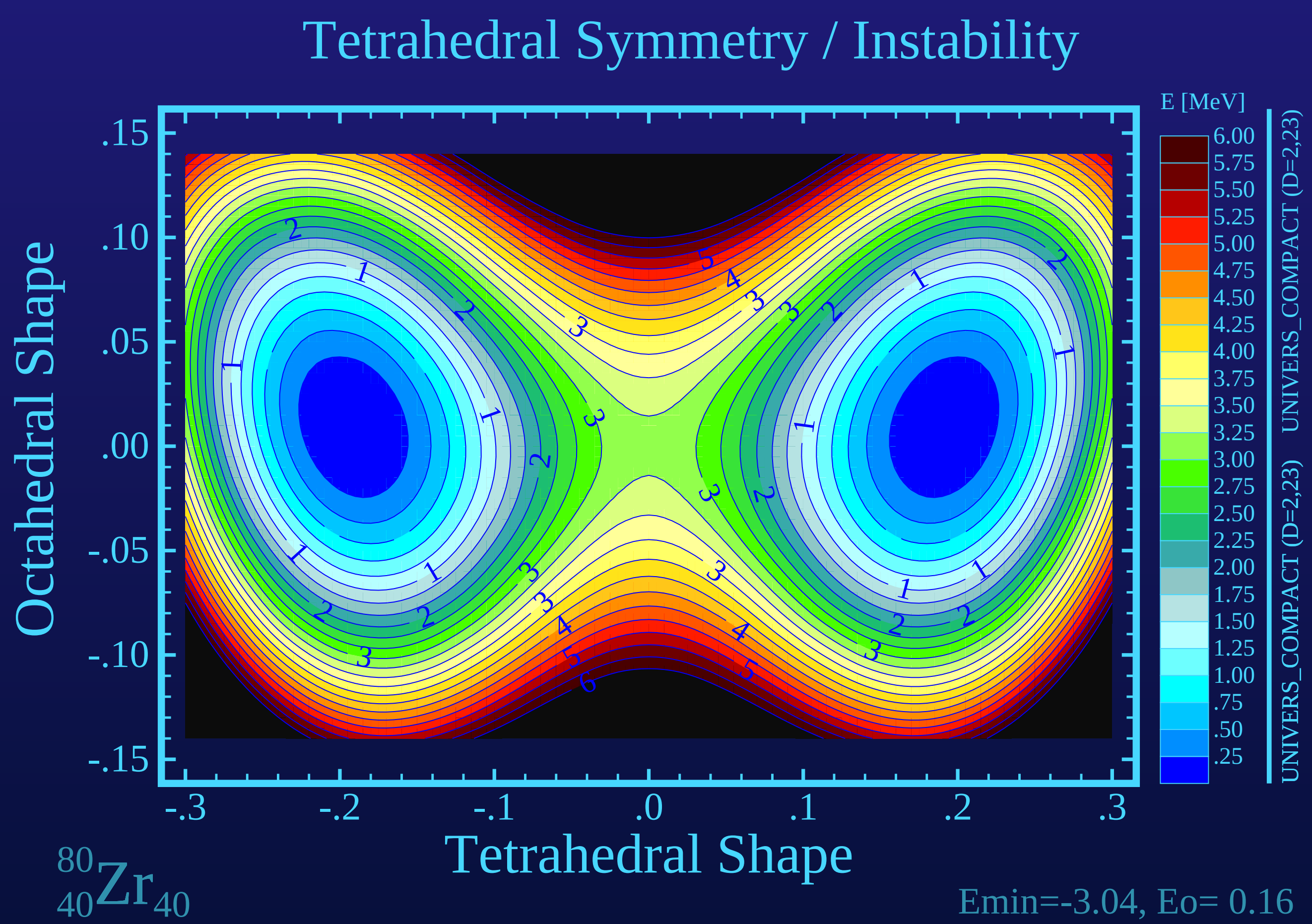

Figure 5: Total energy according to Universal-Compact parametrisation; Strutinsky and Yukawa-Folded techniques.

Combined Tetrahedral and Octahedral Deformations

- Tetrahedral minima can be lowered by the octahedral deformations

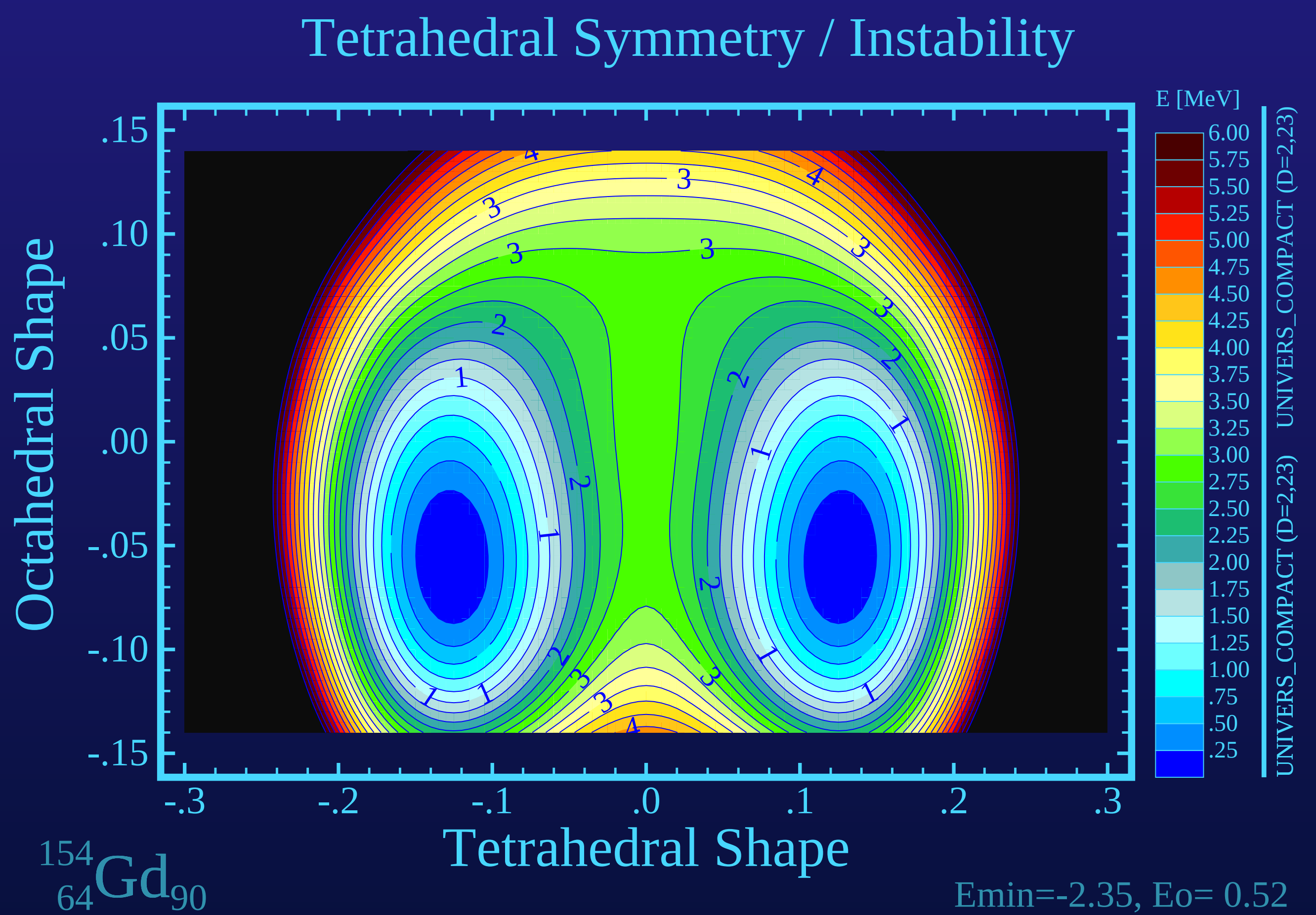

Figure 6: Octahedral deformation lowers the tetrahedral minimum by about 500 keV.

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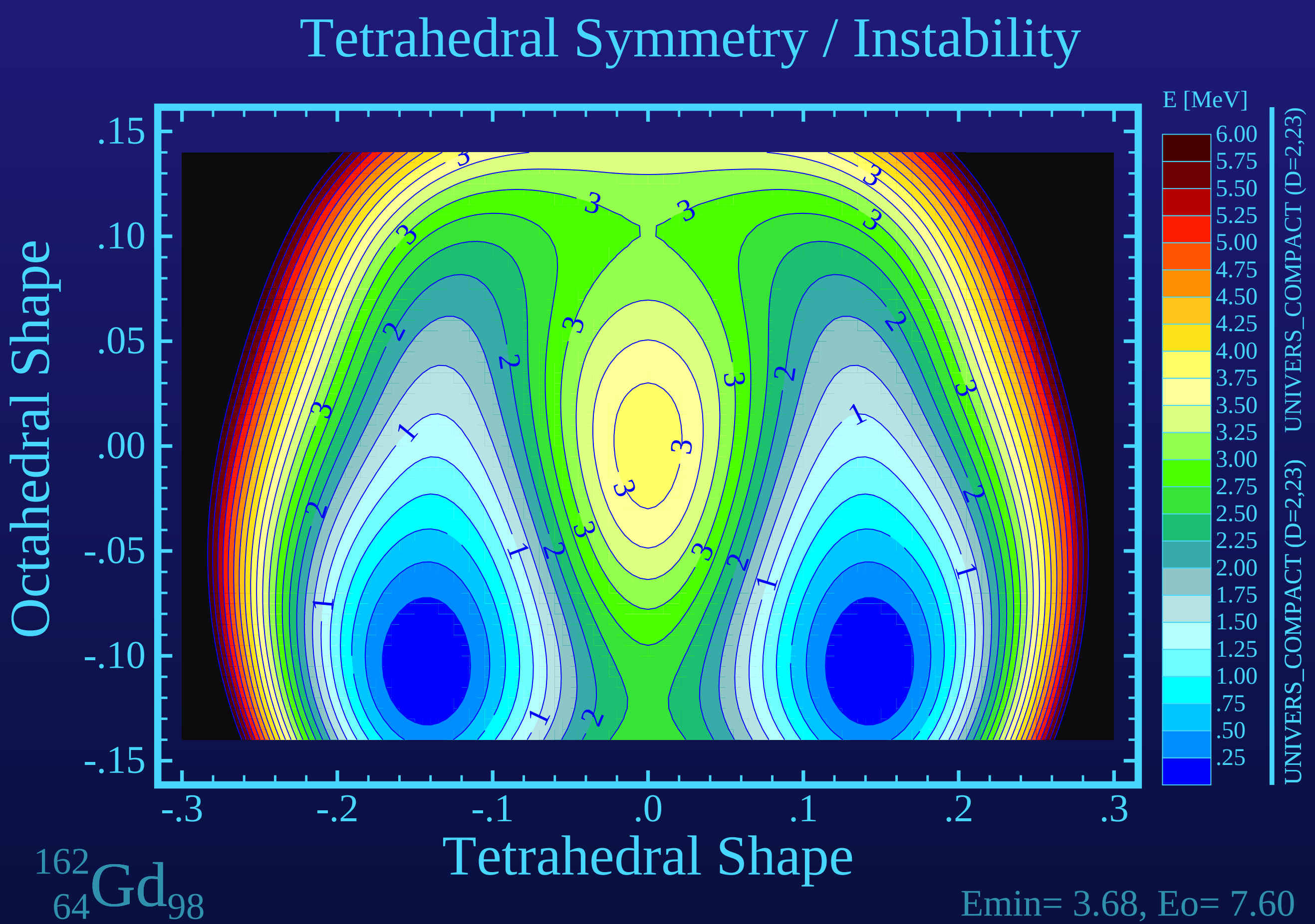

Figure 7: Octahedral deformation lowers the tetrahedral minimum by about 1.2 MeV.

Tetrahedral Symmetry: In Which Nuclei?

- Using α_{32} deformation, tetrahedral magic gaps were predicted at:

$$Z_t = 16, 20, 32, 40, 56, 70, 90, 100, 126$$

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- ... and by a few mass units in heavy and very heavy nuclei so that e.g. $Z = 70 \rightarrow Z = 64$; $Z = N = 56$ remain very weak, etc.

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OR: KRYPTO-SYMMETRY

Octahedral Symmetry - Realistic Spectra

- Example of the *proton* spectra with the Woods-Saxon potential.

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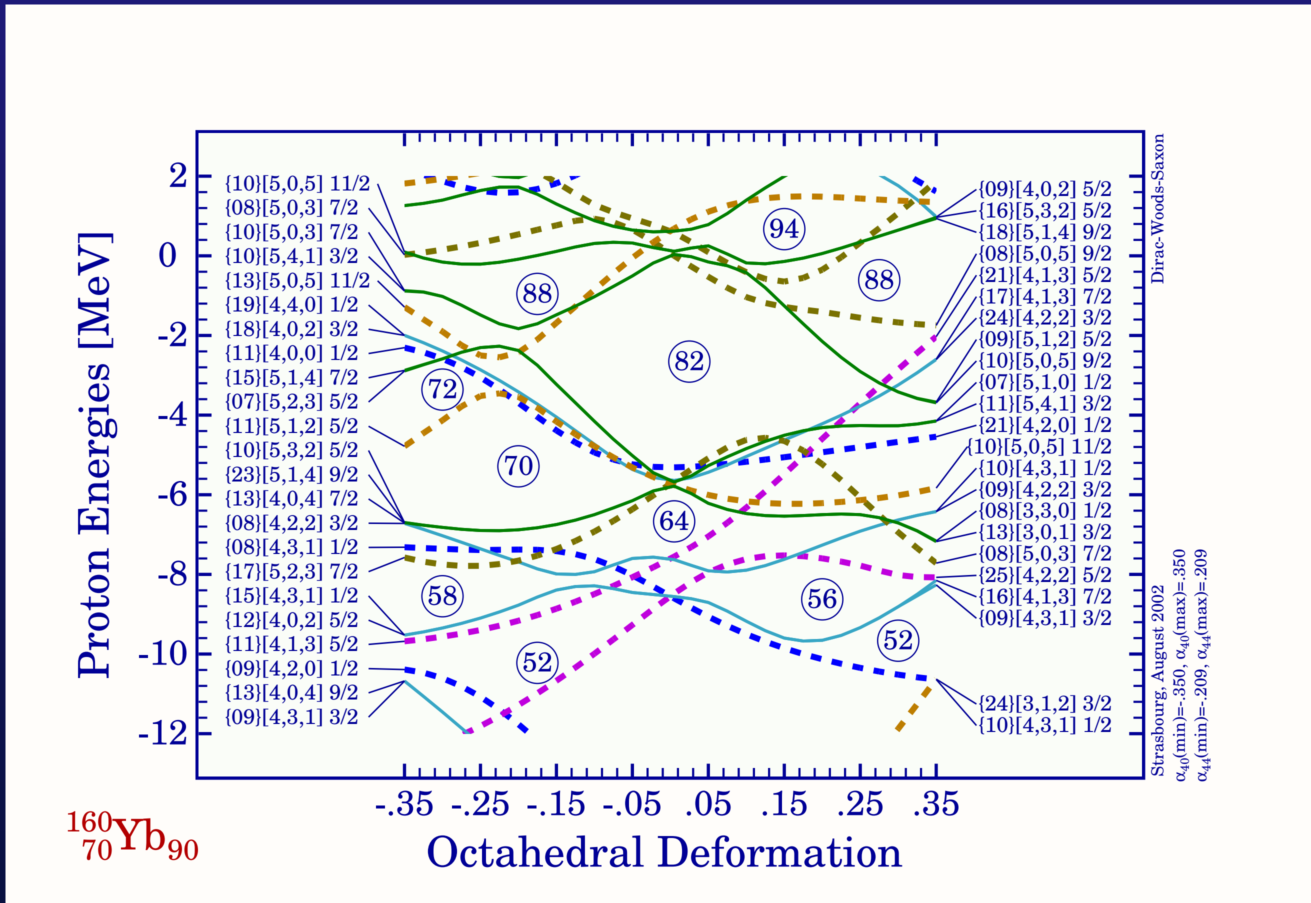

Figure 2: Full lines correspond to 4-dimensional irreps - they are marked with double Nilsson labels. There are *six* families of levels in total. Observe extremely large (over three MeV) octahedral gap at Z=70.

Octahedral Symmetry - Realistic Spectra

- Example of the *neutron* spectra with the Woods-Saxon potential.

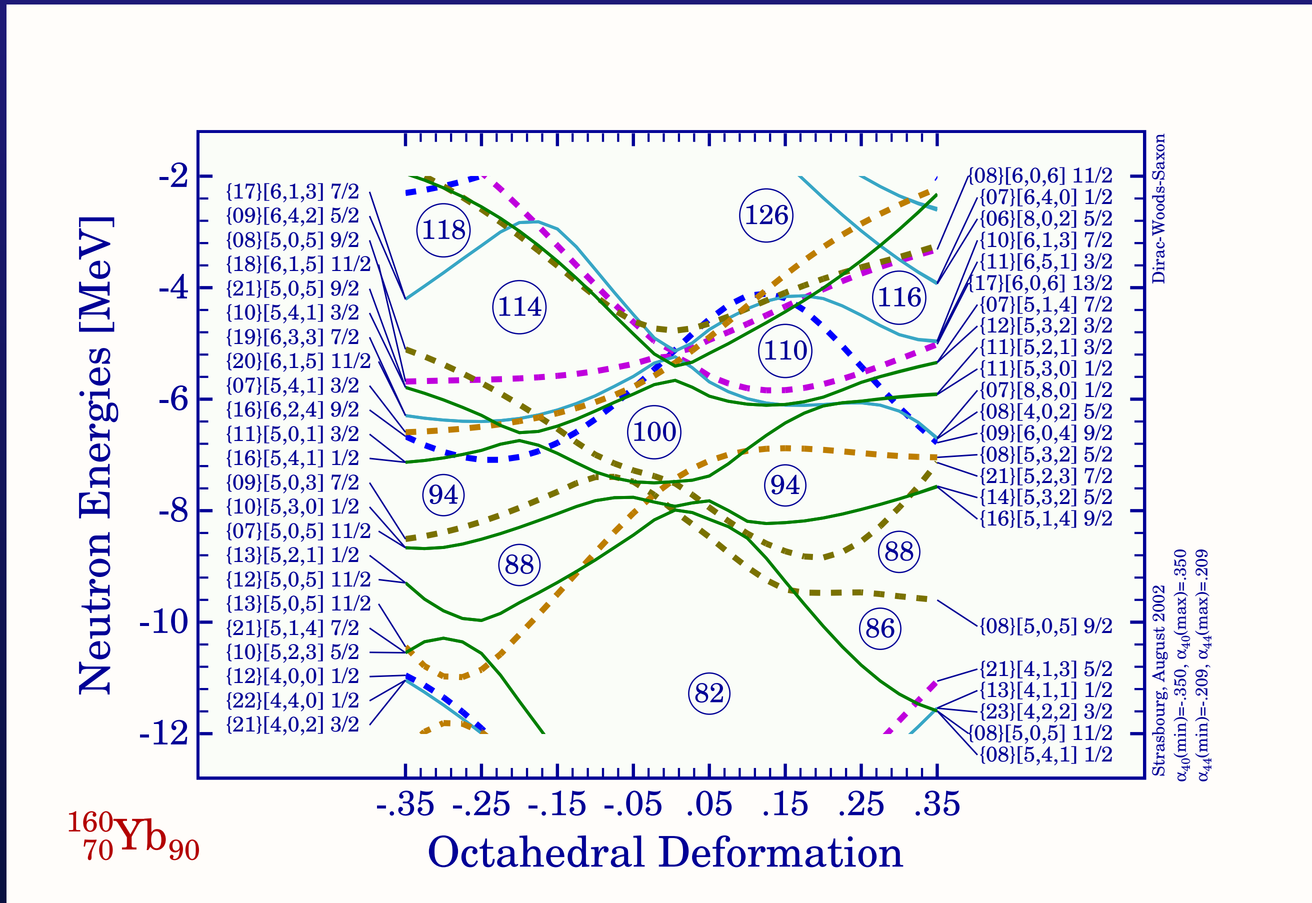

Figure 3: Full lines correspond to 4-dimensional irreps - they are marked with double Nilsson labels. There are *six* families of levels in total. Observe extremely large (over three MeV) octahedral gap at N=114.

High-Symmetries and Challenges

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(Unprecedented Quantum Features)

Properties	High Symmetries		'Usual' symmetries
or features	Tetrahedral	Octahedral	Ellipsoid
No. Sym. Elemts.	48	96	4 + ...
Parity	NO	YES	YES
New Degeneracies	4, 2, 2	$\underbrace{\mathbf{4,2,2}}_{\pi=+}$ $\underbrace{\mathbf{4,2,2}}_{\pi=-}$	$\overbrace{\pi=+}^{\mathbf{2}}$ $\overbrace{\pi=-}^{\mathbf{2}}$
New Q. Numbers	3	$\mathbf{3} + \mathbf{3}$	2: $\pi=\pm 1$

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New Q. Numbers	3	$3\;+\;3$	2: $\pi=\pm1$

We call these new quantum numbers $\tau\rho\,\iota - \tau\iota\mu\upsilon\kappa o\sigma$ (tri-timeric)

'possessing three values'

Very Heavy Nuclei

- An example of coexistence: Tetrahedral and Octahedral Symmetry

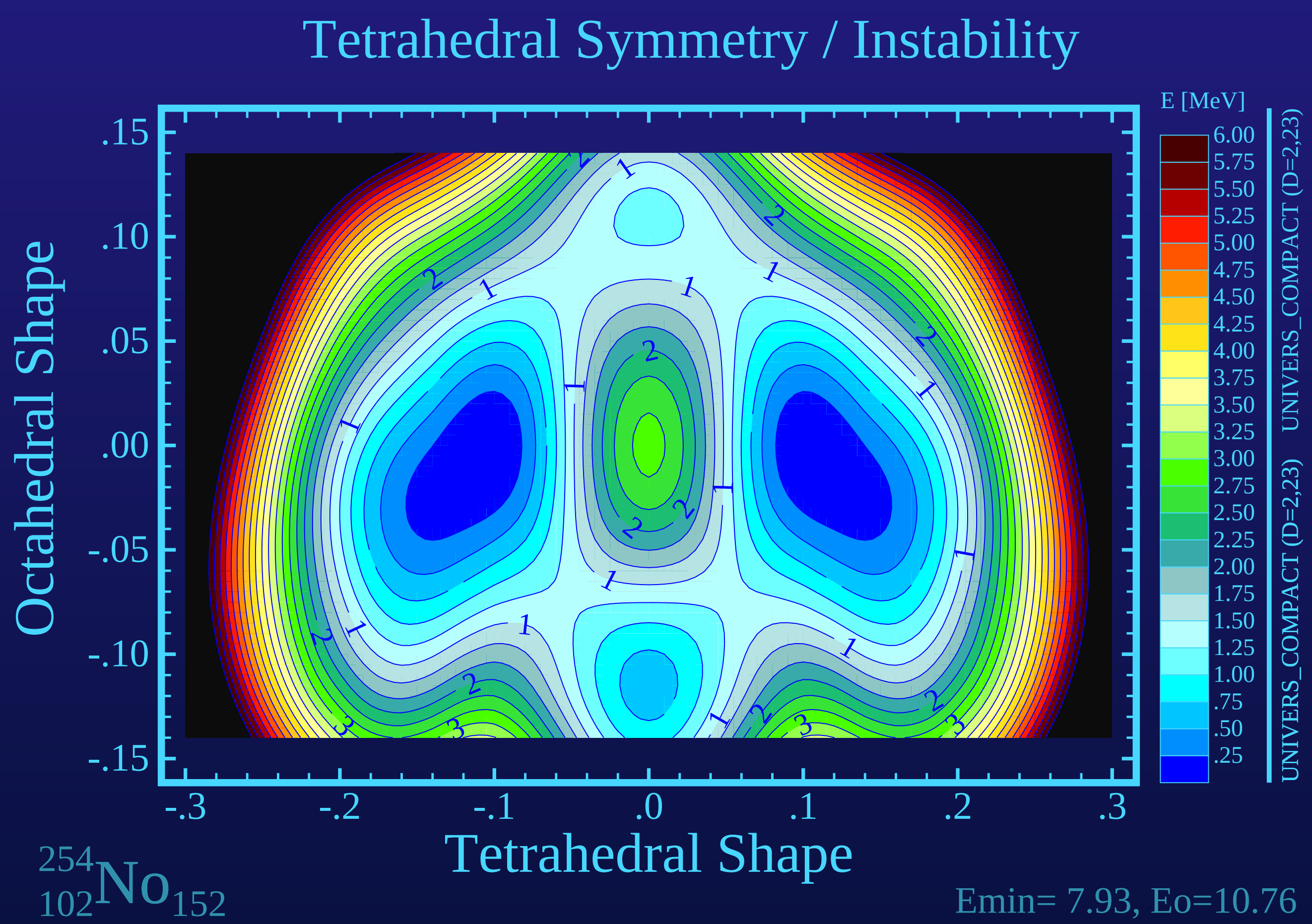

Figure 8: Observe co-existence of formally 3-4 minima of pure-T_d and pure-O_h symmetries.

Towards Super-Heavy Nuclei

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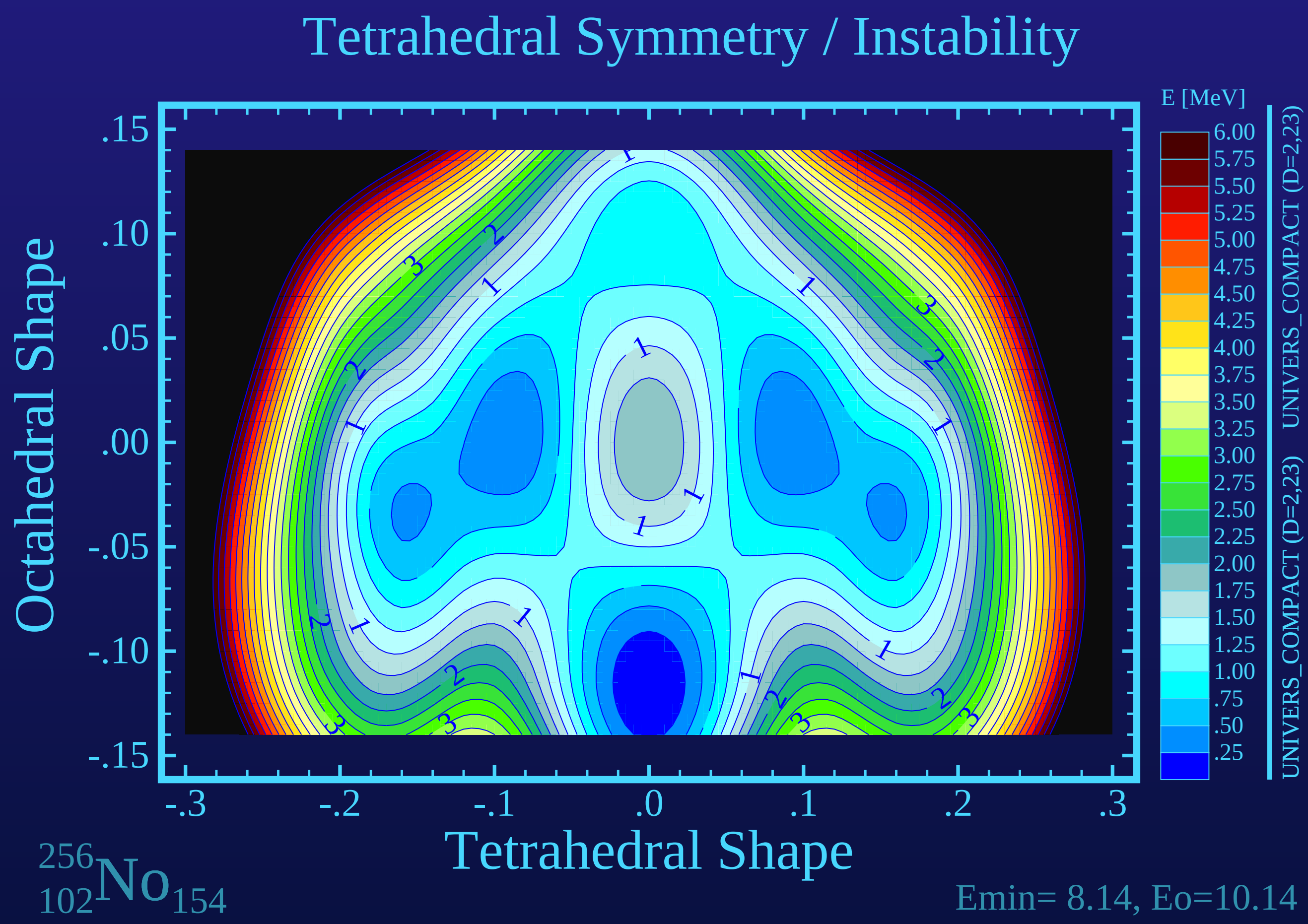

Figure 9: Mixed T_d and O_h susceptibility.

Towards Super-Heavy Nuclei

- An example of coexistence: Tetrahedral and Octahedral Symmetry

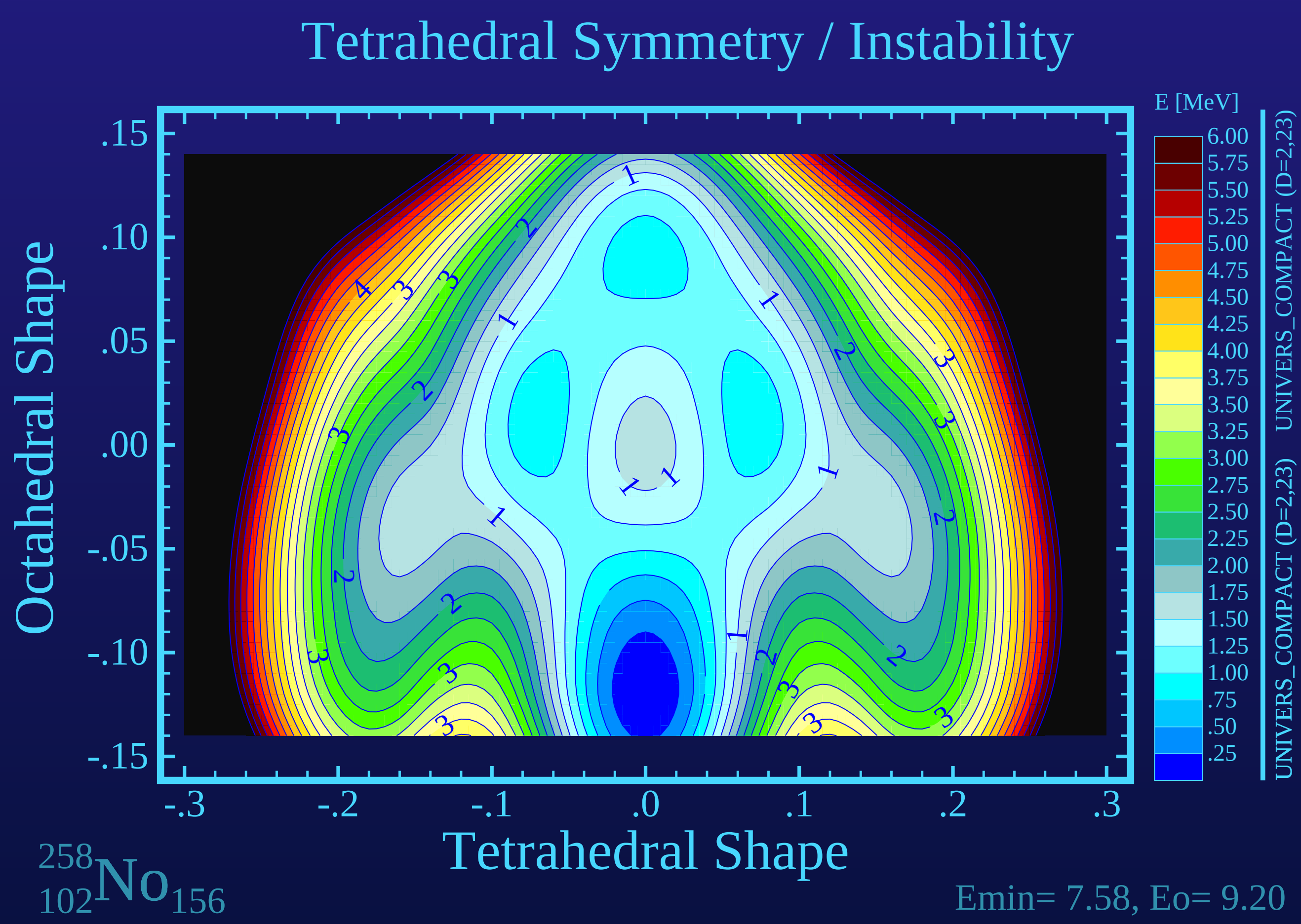

Figure 10: Mixed T_d and O_h susceptibility.

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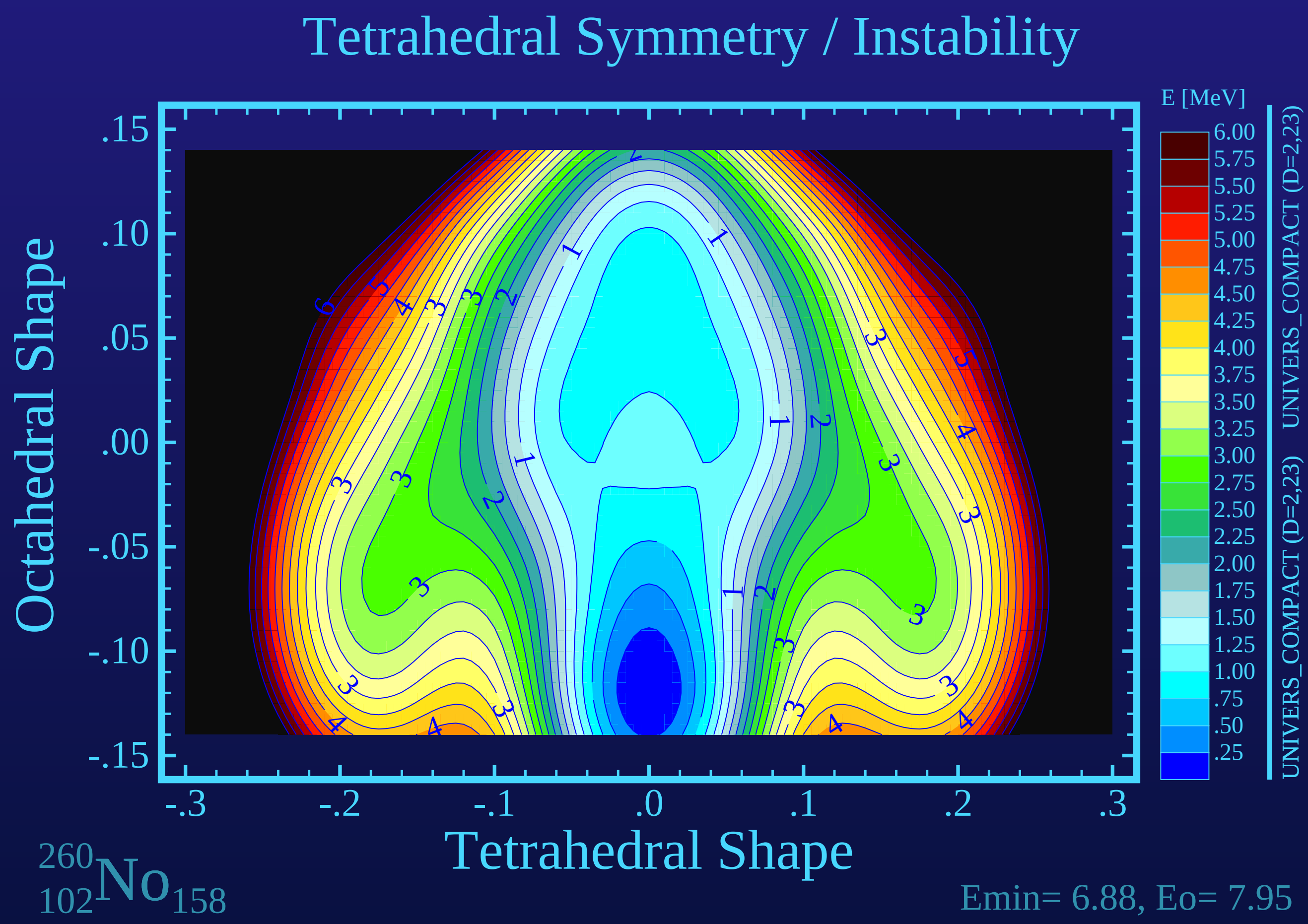

Figure 11: Large amplitude octahedral oscillations?

Towards Super-Heavy Nuclei

- An example of coexistence: 'Tetrahedral vs. Tetrahedral' Symmetry

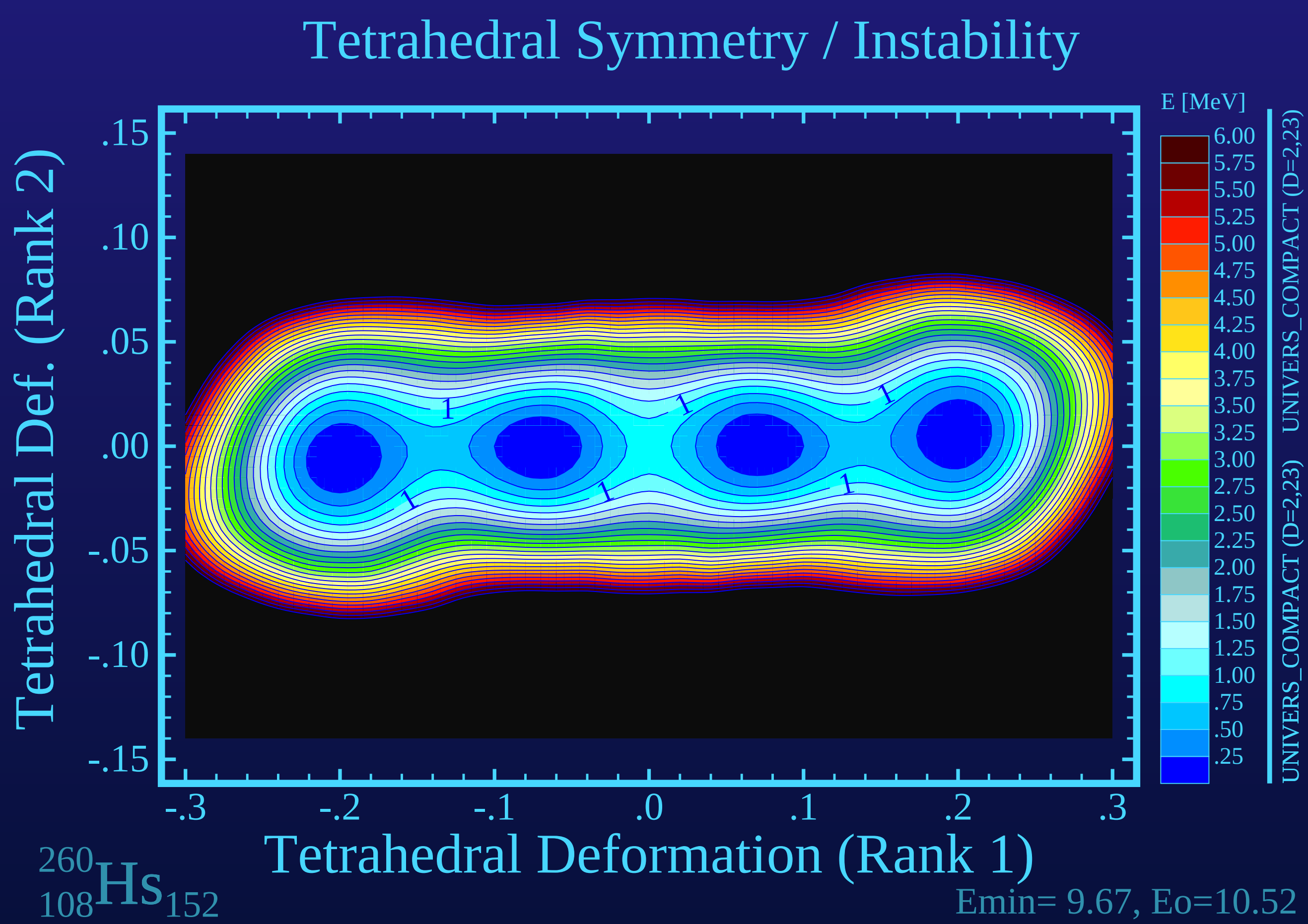

Figure 12: Formally 4 T_d-symmetry minima... however ...

Towards Super-Heavy Nuclei

- An example of shape coexistence in the presence of Tetrahedral and Octahedral Symmetries

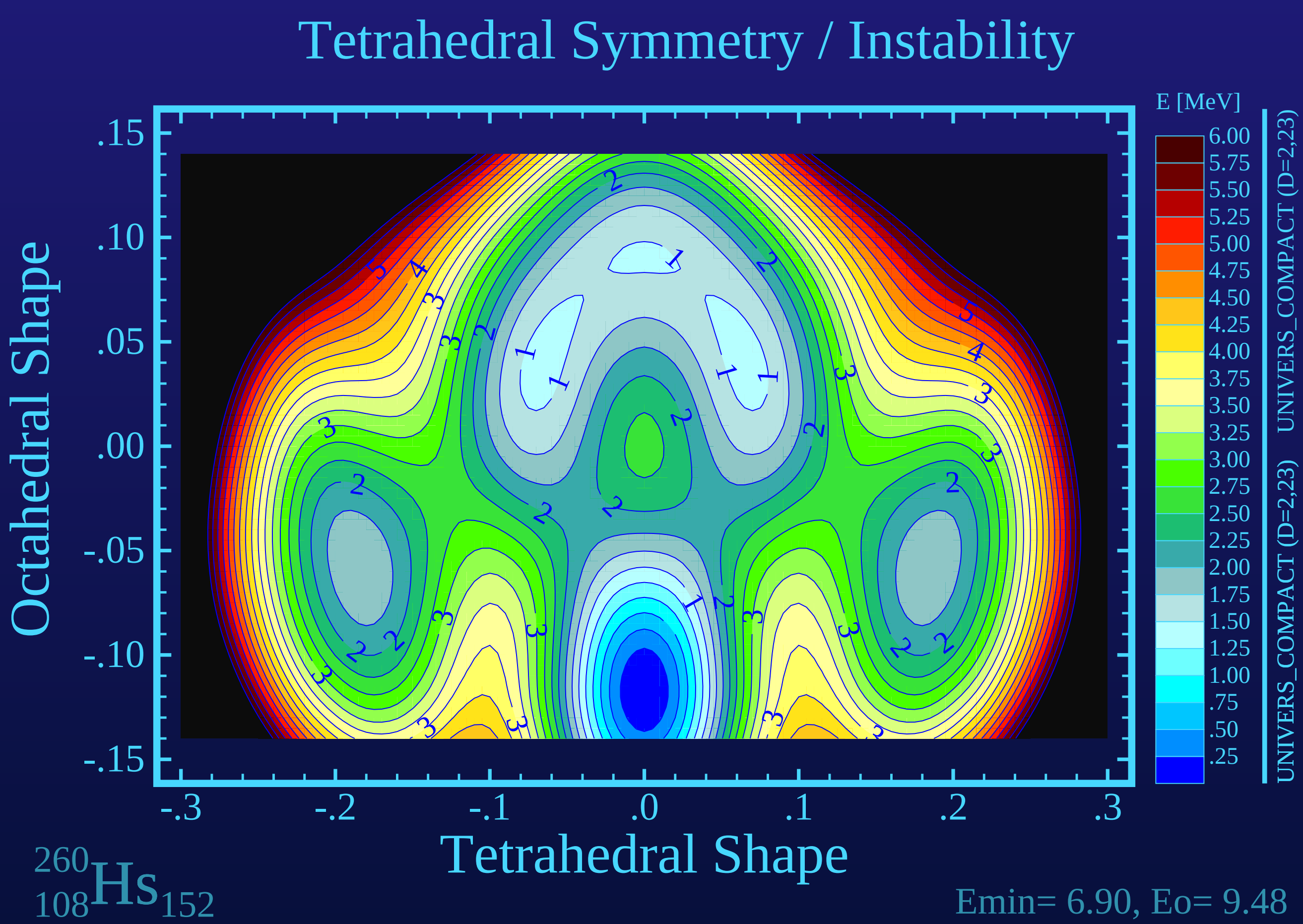

Figure 13: One pure O_h-symmetry minimum, two minima with 'mixed' T_d- and O_h-symmetries and a 'mixed area'.

Towards Super-Heavy Nuclei

- An example of coexistence: Tetrahedral and Octahedral Symmetry

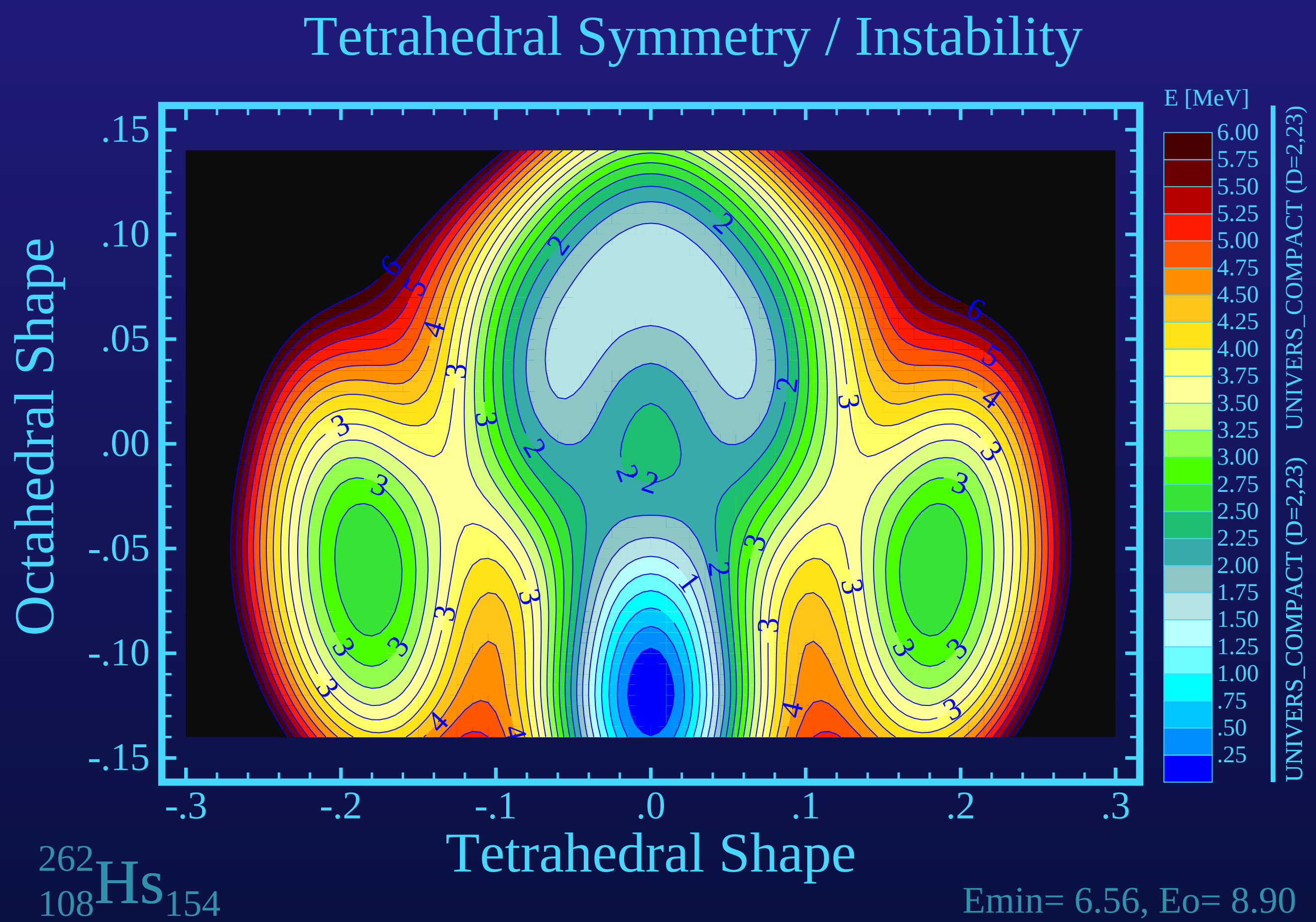

Figure 14: One pure O_h-symmetry minimum, two minima with 'mixed' T_d- and O_h-symmetries and a 'mixed area'.

Towards Super-Heavy Nuclei

- An example of coexistence: Tetrahedral and Octahedral Symmetry

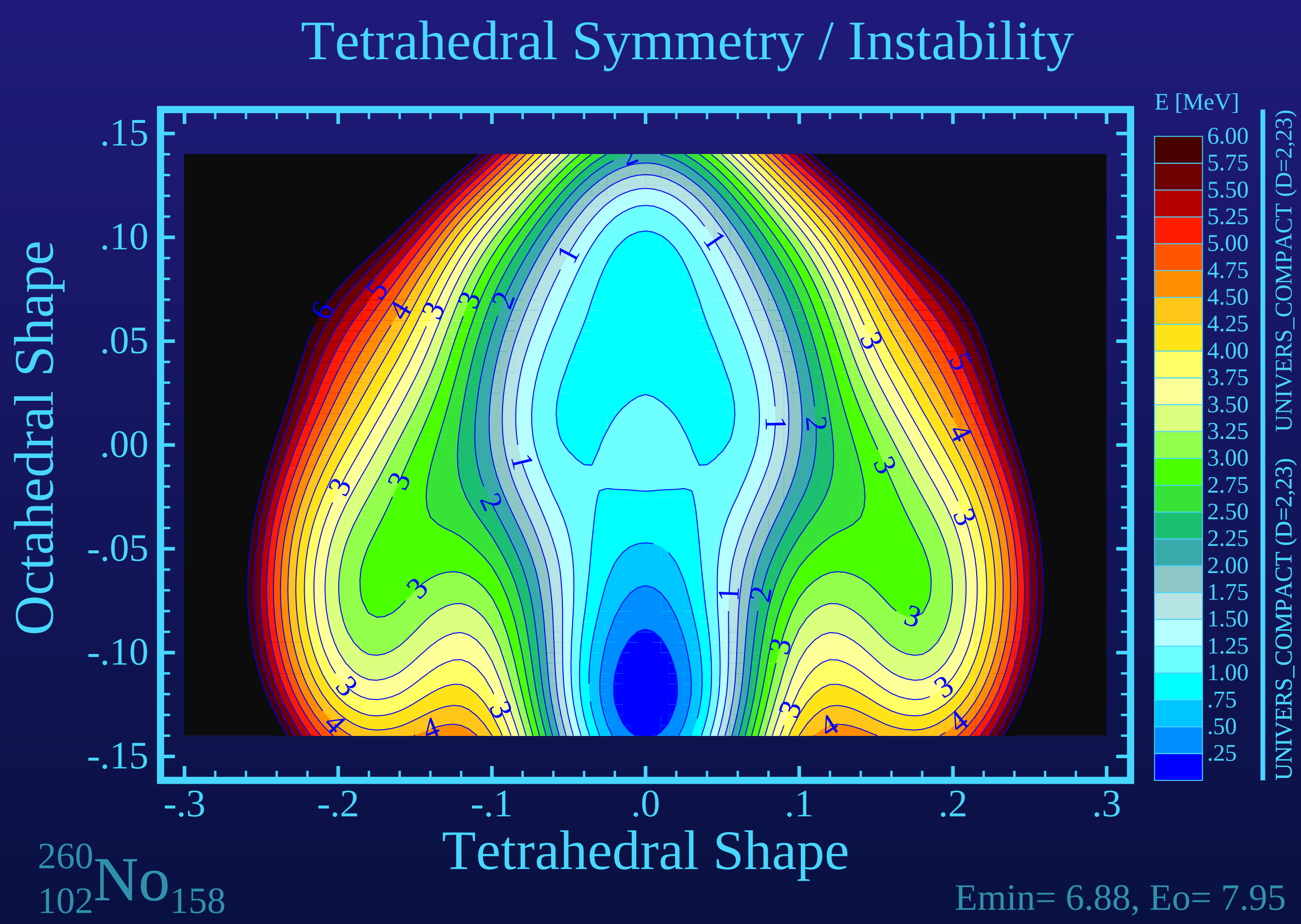

Figure 15: A new type of transitional nuclear configurations.

Towards Super-Heavy Nuclei

- An example of coexistence: Tetrahedral and Octahedral Symmetry

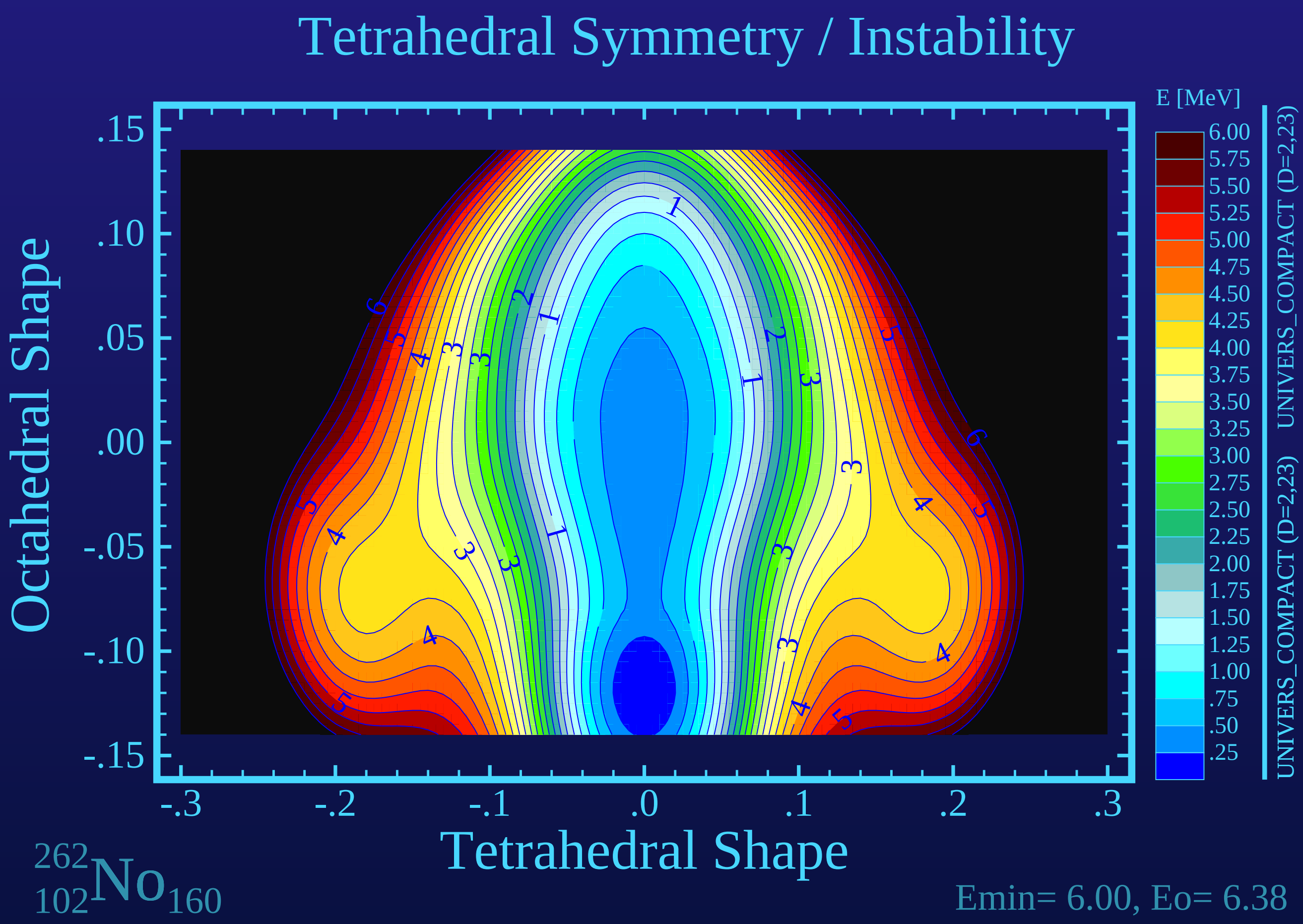

Figure 16: Low energy octahedral vibrations?

First Observations and Suggestions

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- A few degrees of freedom should be considered simultaneously in the mesh-type mean-field calculations

Abundance Scheme for Tetrahedral Symmetry

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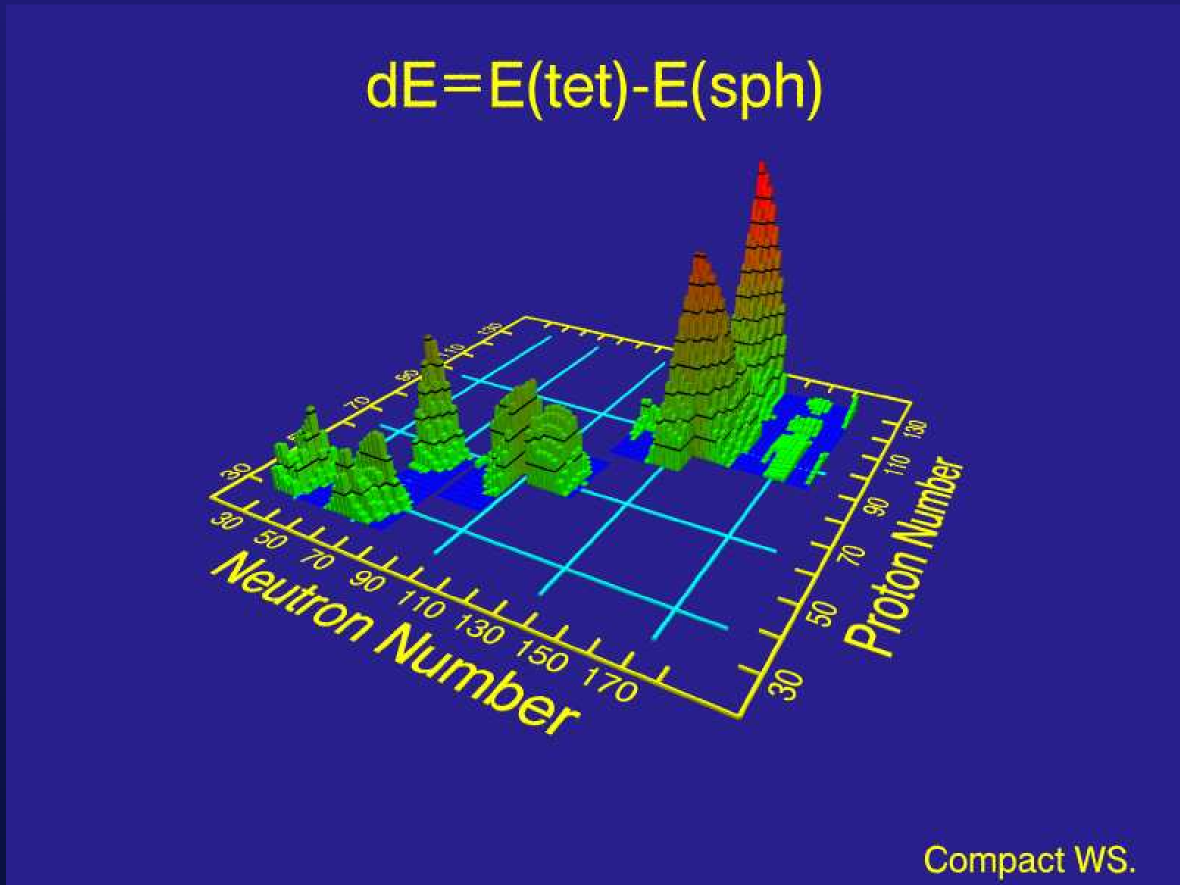

Figure 17: Observe the new optimal positions of the magic numbers: (Z=N=38), (Z=38,N=64), (Z=64,N=98), (Z=98,N=136), (Z=98,N=172).

Abundance Scheme for Tetrahedral Symmetry

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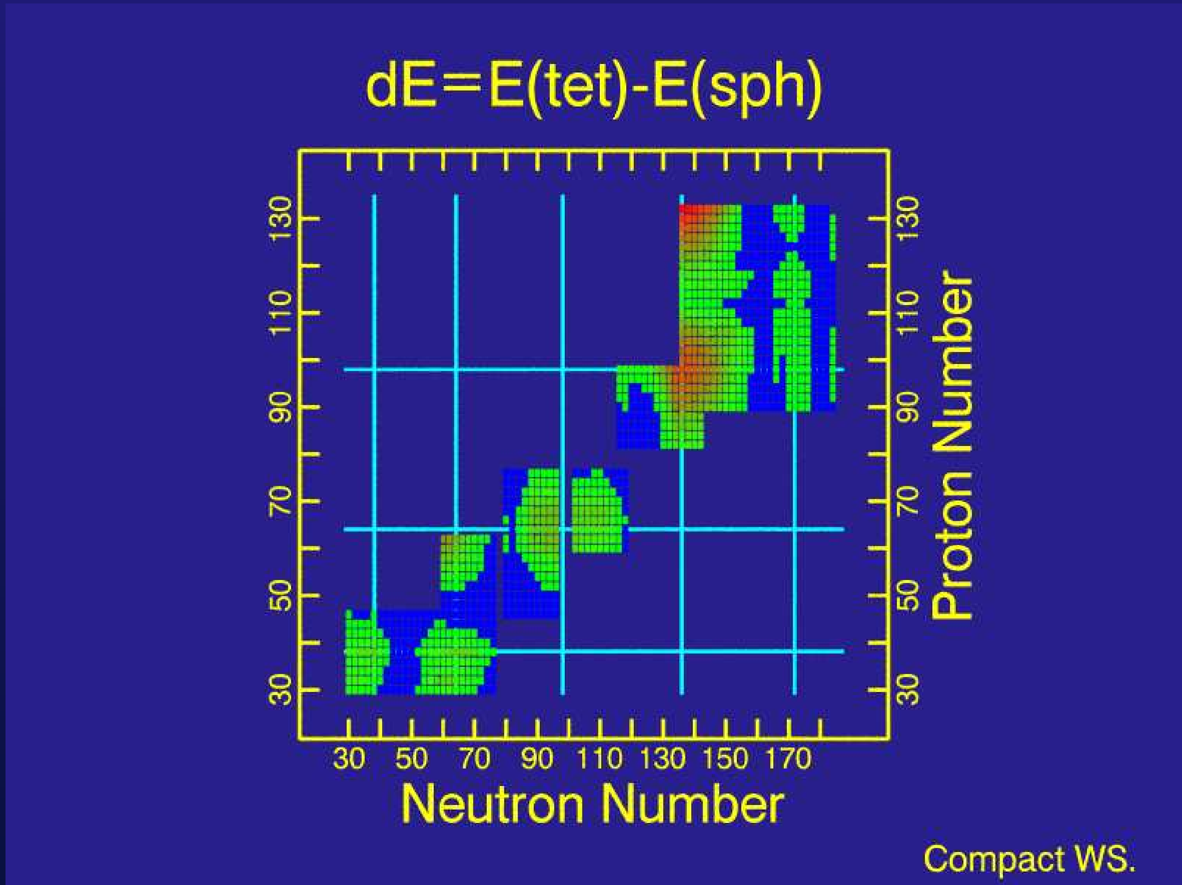

Figure 18: Observe the new optimal positions of the magic numbers: (Z=N=38), (Z=38,N=64), (Z=64,N=98), (Z=98,N=136), (Z=98,N=172).

Remarks about Experimental Signatures [1]

- Single-particle energy levels belong to three irreducible representations, one of them four-dimensional.

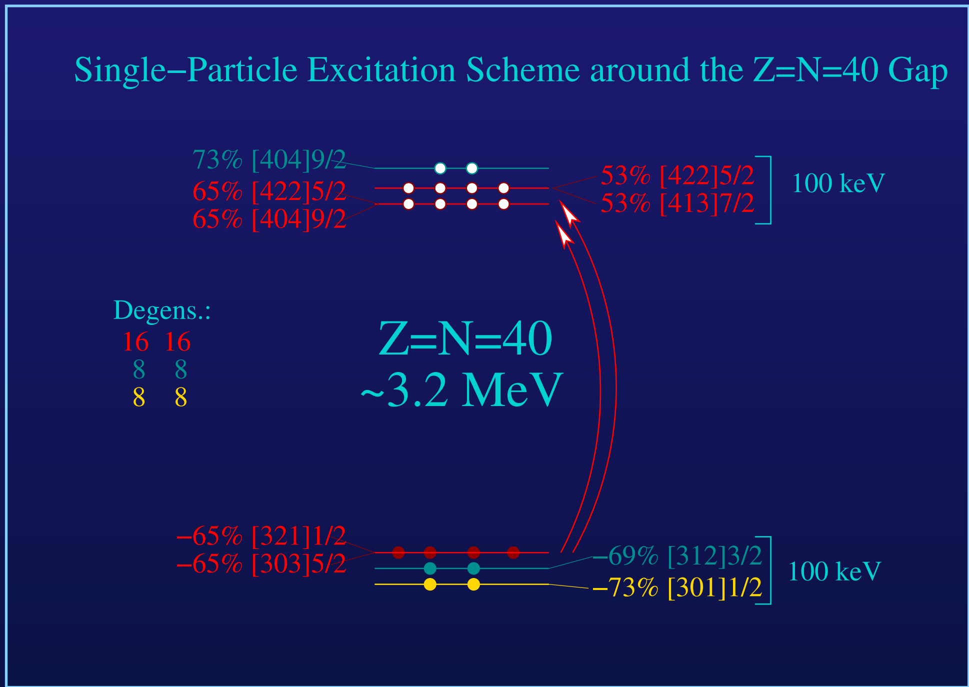

Figure 19: The percentages display the parity contents. In the nuclei with Z or N at, or around, 40 there are numerous degenerate excitations to be expected, with the degeneracies ranging from 8 to 32 (!) in the ideal symmetry cases.

Remarks about Experimental Signatures [2]

- The strongest tetrahedral symmetry effects are expected at low spins, at 1 to 3 MeV above the ground-states

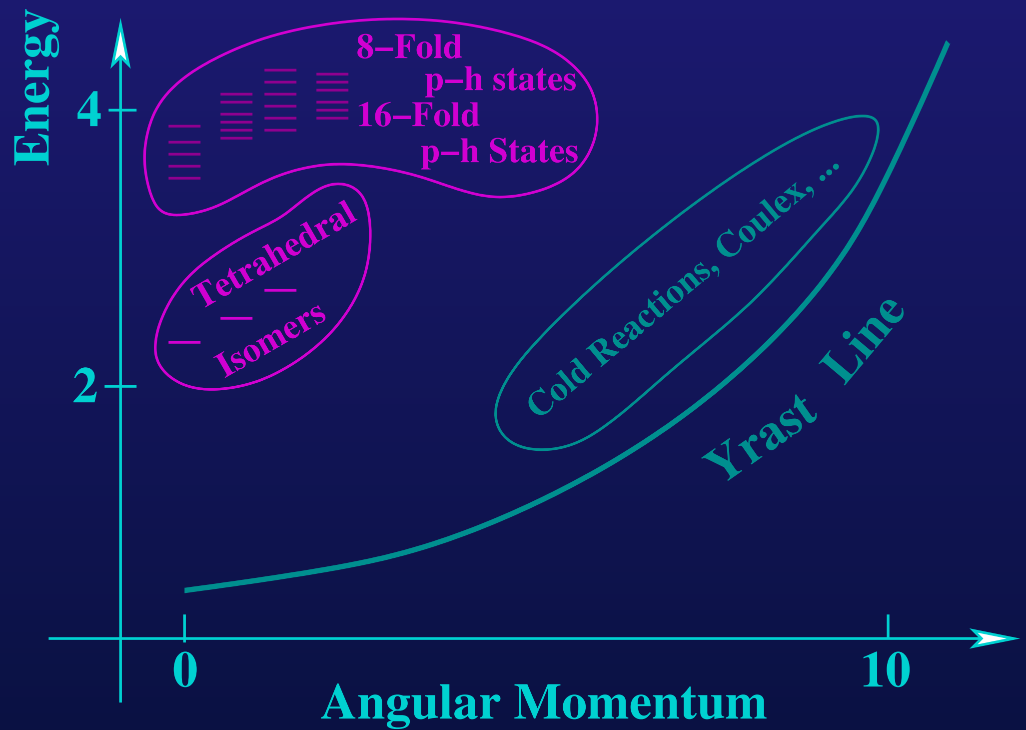

Figure 20: We would like to populate relatively highly-excited states at very low (or low) spins. Reactions with light projectiles could be a choice here.

Remarks about Experimental Signatures [3]

- Predicted isomeric minima are separated from the ground-state minima by the barriers of a few hundreds of keV to a few MeV

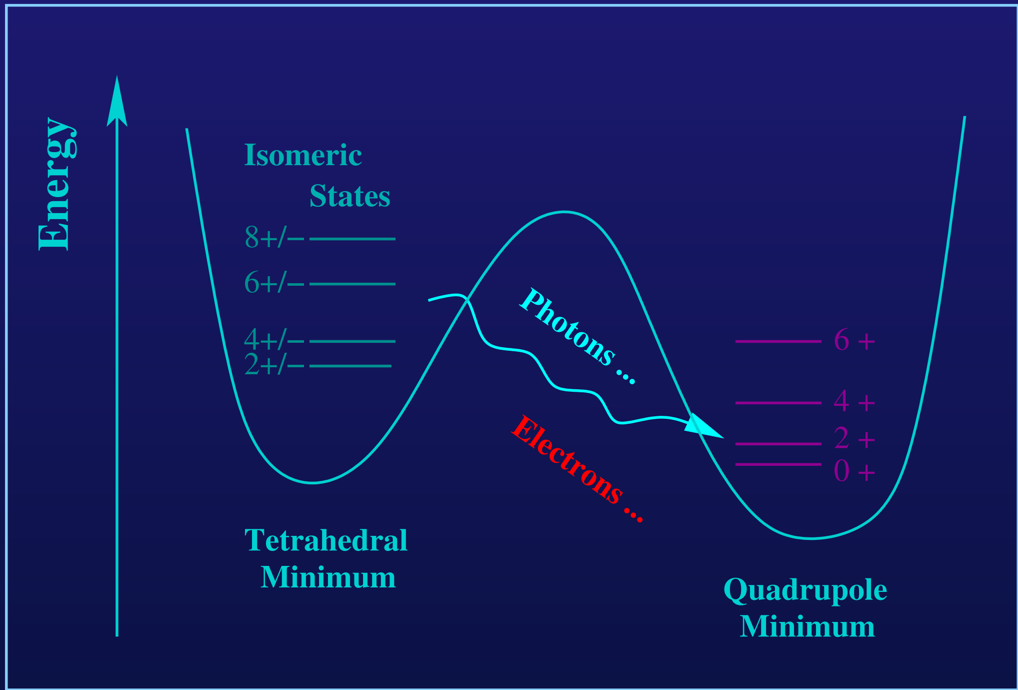

Figure 21: We expect the isomers of the structure that resemble that of the 'yrast traps' in oblate nuclei. Implication: a (model-dependent) test valid in nuclei that do not produce oblate minima!

Remarks about Experimental Signatures [4]

- Consider very heavy and/or super-heavy nuclei

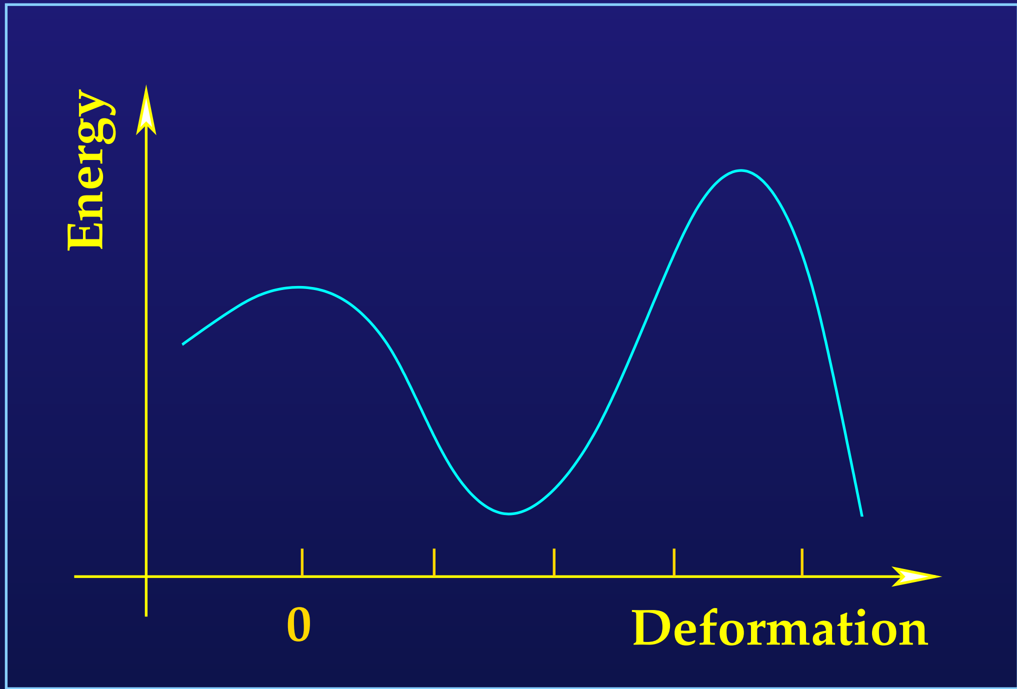

Figure 22: The stability against fission is modelled by a 'fission barrier' usually understood in terms of the quadrupole elongation.

Comments about Stability against Fission

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- In qualitative terms, we have $B \sim 1/\Delta^2$ and $B \sim \langle\, | \frac{\partial H}{\partial \alpha_{\lambda\mu}} |\, \rangle$

Remarks about Experimental Signatures [5]

- The presence of the tetrahedral minima changes drastically the accessible phase space of the problem:

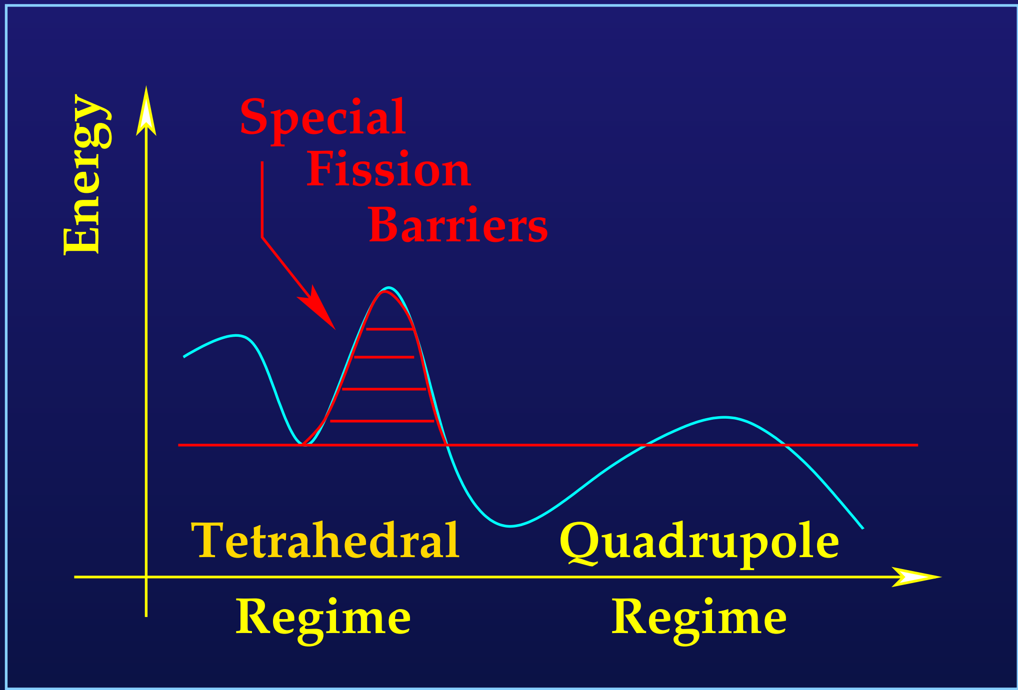

Figure 23: In the case of the tetrahedral minimum there is the whole new area in the deformation space that needs to be traversed towards fission.